Equity Valuation

Equity Valuation

Peter O. Christensen

School of Economics and Management
Aarhus University
Aarhus, Denmark
pochristensen@econ.au.dk

Gerald A. Feltham

Sauder School of Business
University of British Columbia
Vancouver, BC, Canada
gfeltham@shaw.ca

the essence of knowledge

Boston – Delft

Foundations and Trends® in Accounting

Published, sold and distributed by:
now Publishers Inc.
PO Box 1024
Hanover, MA 02339
USA
Tel. +1-781-985-4510
www.nowpublishers.com
sales@nowpublishers.com

Outside North America:
now Publishers Inc.
PO Box 179
2600 AD Delft
The Netherlands
Tel. +31-6-51115274

The preferred citation for this publication is P. O. Christensen and G. A. Feltham, Equity Valuation, Foundation and Trends® in Accounting, vol 4, no 1, pp 1–112, 2009

ISBN: 978-1-60198-272-8

Editorial Scope

Foundations and Trends® in Accounting will publish survey and tutorial articles in the following topics:

- Auditing
- Corporate Governance
- Cost Management
- Disclosure
- Event Studies/Market Efficiency Studies
- Executive Compensation
- Financial Reporting
- Financial Statement Analysis and Equity Valuation
- Management Control
- Performance Measurement
- Taxation

Information for Librarians

Foundations and Trends® in Accounting, 2009, Volume 4, 4 issues. ISSN paper version 1554-0642. ISSN online version 1554-0650. Also available as a combined paper and online subscription.

Foundations and Trends® in
Accounting
Vol. 4, No. 1 (2009) 1–112

DOI: 10.1561/1400000008

Equity Valuation

Peter O. Christensen[1] and Gerald A. Feltham[2]

[1] *School of Economics and Management, Aarhus University, Aarhus, Denmark, pochristensen@econ.au.dk*

[2] *Sauder School of Business, University of British Columbia, Vancouver, British Columbia, Canada, gfeltham@shaw.ca*

Abstract

We review and critically examine the standard approach to equity valuation using a constant risk-adjusted cost of capital, and we develop a new valuation approach discounting risk-adjusted fundamentals, such as expected free cash flows and residual operating income, using nominal zero-coupon interest rates. We show that standard estimates of the cost of capital, based on historical stock returns, are likely to be a significantly biased measure of the firm's cost of capital, but also that the bias is almost impossible to quantify empirically. The new approach recognizes that, in practice, interest rates, expected equity returns, and inflation rates are all stochastic. We explicitly characterize the risk-adjustments to the fundamentals in an equilibrium setting. We show how the term structure of risk-adjustments depends on both the time-series properties of the free cash flows and the accounting policy. Growth, persistence, and mean reversion of residual operating income created by competition in the product markets or by the accounting policy are key determinants of the term structure of risk-adjustments.

Contents

1

Introduction

The valuation of uncertain income streams is at the heart of financial and accounting research as well as in almost all areas of business. Financial statement analysis and equity valuation challenge market prices of traded stocks, and the valuation of a firm's equity is the key in any equity deal be it an acquisition, a merger, or a private equity deal. The increase in the capitalization of private equity funds also calls for good models of valuing equity when there is no market price. Capital budgeting is another prominent example in which the valuation of uncertain investment returns is at the heart of the issue. The key question in all of these settings is how to determine the value today of a stream of uncertain future cash flows or earnings. The main issues involved are how to account for taxes, inflation, growth, and not least for risk.

The typical template for equity valuation starts with the gathering, organization and analysis of current and past information, i.e., knowing the business and its markets, for the purpose of forecasting future cash flows, accounting numbers, and value creation. The specific valuation model guides what needs to be forecasted, and it transforms the forecasts into a value today through discounting for time and for risk. Our focus is on the valuation model.

In practice, a distinction is made between operating and financial activities, assuming value creation derives from the operations while financial activities are zero-NPV activities. This implies that the value of a firm's equity can be determined as the value of the operations minus the (almost readily observable) current value of the financial debt. Hence, the need for forecasting can be limited to forecasting the free cash flows, i.e., the difference between operating cash receipts and investments in operating assets, or to forecasting the accounting numbers for the operations. However, if there is a tax shield on debt, value is also created through the financing.

There are two basic approaches to deal with tax shields. The most common approach in equity valuation textbooks, such as Penman (2007) and Lundholm and Sloan (2004), is to lower the discount rate, i.e., use an after-tax cost of capital to discount the free cash flows or the operating accounting numbers. The alternative approach, also referred to as the "adjusted-net-present-value" (APV) method (see, for example, Grinblatt and Titman, 2002), treats the tax shield as part of the operations and uses a before-tax cost of capital for discounting. While the two approaches yield identical results under idealized conditions, we show that the latter method is much more flexible, and that it is conceptually consistent with the distinction between value creating operating activities and (before-tax) zero-NPV financial activities. Using this method, the free cash flows are indeed equal to the actual net payments to the debt and the equityholders. Moreover, the often daunting task in financial statement analysis of re-allocating taxes between the operations and the financial activities may be avoided — all corporate taxes are part of the operations.

Most equity valuation textbooks are silent about the impact of inflation and about the distinction between nominal and real discount rates, possibly except when it comes to discussing assumptions about reasonable long-term growth rates in cash flows or accounting numbers. Real discount rates are very hard to estimate empirically, since there are only very few traded securities denominated in real consumption units. Hence, typical valuation models use nominal discount rates and, therefore, forecasts of future free cash flows or accounting numbers must also be in nominal terms. If inflation is assumed to be deterministic, then the distinction between real and nominal terms causes no additional problems in valuation, except that one should be clear that nominal (real) forecasts must be discounted with nominal (real) discount rates. However, if future inflation rates are uncertain, one must recognize that some types of businesses are a better hedge against inflation than others. In other words, there is a separate risk premium for inflation risk which needs to be quantified. We show how the inflation risk premium may be determined and estimated.

Growth in cash flows and accounting numbers can be created through inflation, by increasing the size of the business, and even through the choice of accounting policies. However, in determining the value of a firm's equity today, the issue is whether future growth is creating value or destroying it. Nothing is easier than generating growth, but value creating growth requires positive NPV investments — a good equity valuation model must protect you from paying too much for growth in cash flows and in earnings.

Value creating growth is closely related to the sustainability and the creation of competitive advantages in the firm's product markets. It is economically and empirically well-documented that high (or low) abnormal operating performance is likely to be followed by decreasing (increasing) performance, for example, due to entry (or exit) of competitors. In other words, operating performance is likely to be mean reverting to possibly industry-specific growth rates (see, for example, the discussion in Penman, 2007, Chapters 14 and 15).

This phenomenon is well-recognized in equity valuation textbooks when it comes to forecasting expected growth in, for example, accounting earnings (through so-called "fade" rates). It is less recognized that mean reversion in the earnings process also affects the uncertainty about future earnings, i.e., a higher degree of mean reversion lowers the uncertainty of future earnings and, therefore, the risk premium used in calculating the value of the firm's equity today should be lower.

Unfortunately, standard equity valuation models, which are based on a constant risk-adjusted cost of capital estimated from stock returns, do not provide a link between the risk premium in the discount rate and the time-series properties of the firm's underlying cash flows or accounting numbers.

We develop a new equity valuation model, which provides a direct link between the time-series properties of a firm's fundamentals, such as cash flows and accounting numbers, and the associated risk-adjustments. Instead of risk-adjusting the discount rate in the denominator, our equity valuation model risk-adjusts the firm's expected fundamentals in the numerator. These risk-adjustments to the expected fundamentals are determined by how the fundamentals covary with the valuation index (also known as the normalized pricing kernel).

In order to illustrate our approach, consider the valuation of a sequence of cash flows $\{c_\tau\}_{\tau>t}$ for a given sequence of the valuation index $\{q_{\tau t}\}_{\tau>t}$, and assume, for simplicity, that the riskless spot interest rate ι is constant. The value of the sequence of cash flows at date t is given by (see, for example, Rubinstein, 1976; Feltham and Ohlson, 1999)

$$V_t = \sum_{\tau=t+1}^{\infty} \frac{\mathrm{E}_t[c_\tau] + \mathrm{Cov}_t[c_\tau, q_{\tau t}]}{(1+\iota)^{\tau-t}},$$

where $\mathrm{E}_t[\cdot]$ and $\mathrm{Cov}_t[\cdot,\cdot]$ denote conditional expectations and covariances, respectively, given the information available at the valuation date t. Clearly, the risk-adjustments, $\mathrm{Cov}_t[c_\tau, q_{\tau t}]$, to the expected cash flows depend on the time-series properties of both the cash flows and the valuation index.

In order to illustrate a setting in which the growth in the cash flows is mean reverting to an industry-specific growth rate, suppose the cash flows follow a first-order auto-regressive process with mean reversion to a deterministic trend with growth rate $\mu < \iota$, i.e., the trend at date τ is given by $c_t^o(1+\mu)^{\tau-t}$, where c_t^o is the level of the trend at date t. Denote the deviation from the trend at date τ by $c_\tau^\Delta = c_\tau - c_t^o(1+\mu)^{\tau-t}$ and, thus, c_τ^Δ follows the AR(1) process

$$c_\tau^\Delta = \omega_c c_{\tau-1}^\Delta + \varepsilon_\tau, \qquad \omega_c \in [0,1),$$

where $\{\varepsilon_\tau\}_{\tau>t}$ are zero-mean and serially uncorrelated normally distributed innovations with variance $\mathrm{Var}_t[\varepsilon_\tau] = \sigma_c^2$. The auto-regression parameter ω_c measures the persistence of the deviations from the trend or, in other words, the degree of mean reversion to the trend is higher, the lower is ω_c. Solving recursively for c_τ^Δ and substitution of c_τ^Δ yield the cash flow at date τ,

$$c_\tau = c_t^o(1+\mu)^{\tau-t} + [c_t - c_t^o]\,\omega_c^{\tau-t} + \sum_{u=0}^{\tau-1-t} \omega_c^u \varepsilon_{\tau-u},$$

with mean

$$\mathrm{E}_t[c_\tau] = c_t^o(1+\mu)^{\tau-t} + [c_t - c_t^o]\,\omega_c^{\tau-t},$$

and variance

$$\mathrm{Var}_t[c_\tau] = \sigma_c^2 \frac{1-\omega_c^{2(\tau-t)}}{1-\omega_c^2}.$$

Hence, the current deviation from the trend, $c_t - c_t^o$, is expected to decay (or "fade"), and to decay faster for higher degrees of mean reversion (i.e., for lower values of ω_c). Of course, long-term cash flows are more uncertain than near-term cash flows, but the uncertainty of future cash flows is lower for higher degrees of mean reversion, i.e., $\mathrm{Var}_t[c_\tau]$ is lower for lower values of ω_c.

The risk-adjustment to the expected cash flows does not depend on the uncertainty of the cash flows *per se*, but rather on how these cash flows covary with the valuation index. Assume the valuation index is given by (in order to be consistent with a constant riskless spot interest rate)

$$q_{\tau t} = \prod_{u=t+1}^{\tau} d_u,$$

where $\{\delta_u = \ln d_u\}_{u>t}$ are serially uncorrelated normally distributed innovations with $\delta_u \sim N\big(-\tfrac{1}{2}\sigma_\delta^2, \sigma_\delta^2\big)$ and constant contemporaneous covariance with the cash flow innovations, i.e., $\mathrm{Cov}_t[\varepsilon_u, \delta_u] = -\sigma_{c\delta}$. Note that

$$q_{\tau t} = \exp[\delta_{\tau t}], \quad \delta_{\tau t} \equiv \sum_{u=t+1}^{\tau} \delta_u,$$

with $\delta_{\tau t} \sim N\big(-\tfrac{1}{2}(\tau-t)\sigma_\delta^2, (\tau-t)\sigma_\delta^2\big)$ such that $\mathrm{E}_t[q_{\tau t}] = 1$. Using that the cash flows c_τ and $\delta_{\tau t}$ are jointly normally distributed, we can use Stein's Lemma to determine the risk-adjustment to the expected cash flow as $\mathrm{Cov}_t[c_\tau, q_{\tau t}] = \mathrm{Cov}_t[c_\tau, \delta_{\tau t}]$. Using that $\mathrm{Cov}_t[\varepsilon_u, \delta_u] = -\sigma_{c\delta}$, the risk-adjustments are

$$\mathrm{Cov}_t[c_\tau, q_{\tau t}] = -\sigma_{c\delta}\frac{1-\omega_c^{\tau-t}}{1-\omega_c}.$$

Hence, the risk-adjustment for long-term expected cash flows is larger than for near-term expected cash flows, but the risk-adjustments are lower for higher degrees of mean reversion, i.e., for lower values of ω_c.

This simple example illustrates that mean reversion in the firm's fundamentals is a key ingredient for not only making forecasts of the firm's fundamentals but also for making risk-adjustments. The standard approach of using risk-adjusted discount rates estimated from stock returns in the denominator cannot capture this link. The firm's cash flows and earnings are likely to exhibit mean reversion, and are also likely to vary with the business cycle, whereas in standard asset pricing models, like the CAPM, there are no mean reversion and business cycle variations in expected returns — these temporal variations are "priced out" in current equity prices. This is the key difference between using expected stock returns and fundamentals in valuation.

The separation of discounting for time through riskless interest rates in the denominator and discounting for risk through risk-adjustments to the expected fundamentals in the numerator allows us to directly use almost readily observable term structure of interest rates for discounting for time in the denominator (through the associated nominal zero-coupon interest rates). The determination of the valuation index is, of course, where the challenge lies.

The existence of the valuation index is ensured by an assumption of no-arbitrage in the financial markets. This is the point to which the analyses in Feltham and Ohlson (1999) and Christensen and Feltham (2003, Chapter 9) take us. However, no-arbitrage alone does not tell us much about the valuation index — at this juncture, it is merely a mathematical construct without economic content. For the valuation index to be of any practical relevance in equity valuation, additional economic structure has to be imposed on the model such that the valuation index can be determined in terms of more fundamental economic variables.

There are two basic approaches to determining the valuation index. One common approach in asset pricing is to make an ad hoc assumption that it is given as a linear function of certain pricing factors, such as the Fama–French factors, or the spot riskless interest rate and the Sharpe ratio. The alternative approach is to determine it endogenously in an equilibrium model. We follow the latter route and show how it can be determined in consumption-based capital asset pricing models clearly distinguishing between the real and the nominal valuation index, in order to also determine the risk-adjustment for inflation risk.

A wealth of unresolved empirical issues remains, but it is our hope that careful future empirical and econometric research can resolve some of these issues. This is not supposed to mean that there are no unresolved empirical issues in applying traditional valuation models using risk-adjusted discount rates. The following quote is from Penman (2007, p. 691):

> "*Compound the error in beta and the error in the risk premium and you have a considerable problem. The CAPM, even if true, is quite imprecise when applied. No one knows what the market premium is. And adopting multifactor pricing models adds more risk premiums and betas to estimate. These models contain a strong element of smoke and mirrors.*"

The bad news is that a valuation model is indeed required in order to transform the forecasts of a firm's future cash flows or earnings into a valuation of the firm's equity today. The good news is that we develop a new valuation model which is based on the firm's fundamentals and, thus, may entail fewer elements of "smoke and mirrors," and may prove useful when applied in empirical research and in practice.

The following sections provide a brief overview of the issues covered in the remaining chapters.

1.1 Standard Equity Valuation Models

In *Risk-adjusted Discount Rates* we review and critically examine standard valuation models as they are presented in textbooks and used in practice. These models rely on estimates of expected future cash flows or earnings and a constant risk-adjusted cost of capital estimated from historical stock returns using some form of capital asset pricing model and a weighted average cost of capital formula. We show that this approach is based on analyses for single-period settings, which do not easily carry over to multi-period settings and, more importantly, are likely to induce systematic biases in equity valuation.

The concept of the weighted average cost of capital has its roots in the fact that the expected one-period return of a portfolio of assets (such as debt and equity) is equal to the weighted average of the expected returns on the individual assets. This is a fundamental property used in any single-period portfolio selection problem. In a multi-period setting, however, the firm's cost of capital is the discount rate, which makes the discounted value of the expected future free cash flows equal to the current value of the firm, i.e., the cost of capital is the equivalent of an internal rate of return on the sequence of firm value and expected free cash flows, much like the yield-to-maturity on a long-term coupon-bearing bond.

However, in general, the internal rate of return on a portfolio of assets differs from the weighted average of the internal rates of returns on the individual assets. This is the reason why yield-to-maturities are no longer used in the valuation of fixed income securities. Instead, contemporaneous fixed income analyses are based on the concept of the term structure of zero-coupon interest rates, which discounts future payments with date-specific discount rates reflecting the expected future interest rates and their uncertainty.

The weighted average of internal rates of returns on a portfolio of assets is indeed equal to the internal rate of return on the portfolio if the expected cash flows on the individual assets grow at the same and constant rate in perpetuity. If this is descriptive, then simple equity valuation models, like various extensions of the Gordon growth model, may be applied.

In these models, equity values can be linked to contemporaneous cash flows and accounting numbers based on so-called "stationary linear information dynamics" (see, for example, Christensen and Feltham, 2003, Chapter 10, for a review). This approach has provided useful insights into the impact of accounting policies on the relationship between equity values and contemporaneous accounting numbers. However, in practice, equity valuation uses so-called "full information" forecasting in which explicit forecasts of accounting numbers and free cash flows are made up to a forecast horizon, while the simple valuation models are only used for determining the continuation values at the forecast horizon (see, for example, Penman, 2007, Chapters 14 and 15).

If the cost of capital is estimated based on short-term stock returns, then leverage, interest rates, and risk premia must all be constants. However, interest rates change continuously, and leverage and, thus, equity risk premia, change whenever the stock price changes. We show that an estimate of average expected returns is likely to be an upward biased measure of the cost of capital (due to Jensen's inequality) if expected returns are stochastic. This bias may not be trivial, but the bias is likely to be hard to quantify (see, for example, the discussion in Hughes et al., 2009).

We review recent asset pricing literature, which has attempted to account for stochastic expected returns in the valuation of uncertain cash flow streams. Based on an assumed stochastic process for expected returns or the pricing kernel, this literature aims at determining a term structure of risk-adjusted discount rates (see, for example, Brennan, 1997; Ang and Liu, 2004; Brennan and Xia, 2006). The date-specific risk-adjusted discount rates are convoluted mixtures of the date-specific zero-coupon interest rates, the date-specific market prices of risk, and the systematic risk of the cash flow sequence, but these models do not make use of an observable term structure of interest rates.

We show that in certain cases, a risk-adjusted discount rate can simply not be used to find the current value of an uncertain future cash flow. This approach requires that the expected cash flow and its current value have the same sign and, obviously, this need not be the case. In order to illustrate this fact, a simple example is the valuation of a forward contract. The forward price is determined such that the current value of the contract is equal to zero, and the payoff is the difference between the future spot price and the forward price. Unless the expected spot price is equal to the forward price (which would imply risk-neutral pricing), the only risk-adjusted discount rate, which would yield a current value of zero, is either plus or minus infinity. We show how similar instability problems arise in the valuation of equities. Instead, risk-adjusting the expected payoff on the forward contract yields that the forward price is equal to the risk-adjusted expected spot price. More generally, this is the unifying principle used in the pricing of derivatives, and we show how this principle also can be used to price primary securities like equities.

Equities are the ultimate long-term claims. Why is it that the information in a readily observable term structure of interest rates is not used in standard equity valuation models? In his seminal paper, "The Valuation of Uncertain Income Streams and the Pricing of Options," Mark Rubinstein characterized the relationship between the valuation of cash flow streams and the prices of zero-coupon bonds in a multiperiod setting under uncertainty. The key innovation in his analysis is to separate the discounting for time, through zero-coupon interest rates in the denominator, and the discounting for risk, through risk-adjustments to the expected cash flows in the numerator. As noted above, this is the route we follow for the purpose of valuing equities.

1.2 Cash Flow and Accounting-Based Valuation Models

We frame our valuation question in terms of valuing a firm's common equity. However, it should be noted that our approach is applicable for the general question of how to find the value today of a stream of future cash flows or earnings in other settings, such as in capital budgeting. We teach our students in their first finance class that the value today of an investment is the net present value of the incremental future cash flows it generates. We also teach them that they should be very careful distinguishing cash receipts from revenues and expenditures from costs — the difference being the non-cash accruals. The key, of course, is the impact of the time value of money.

Nevertheless, in recent years there has been an increased interest in accounting-based valuation models which instead of discounting future cash flows determine the net present value by discounting future residual income or abnormal earnings growth anchored by current book value or capitalized one-period ahead earnings, respectively. These accounting-based models provide exactly the same net present values as the discounted cash flows models. The reason for this equivalence is the articulation of financial statements within and across periods. For example, a conservative assessment of the book value of an asset today will eventually show up as higher residual income numbers in future periods — the "error" in current book value is perfectly balanced against the increase in the net present value of future residual income numbers (see, for example, the discussion of "counter-balancing errors" in Christensen and Demski, 2002, Chapter 4).

The cash flow is a very easy concept to understand for first year students, whereas understanding the implications of financial statement articulation requires more training. An advantage of accounting-based models over the discounted cash flow models is that *value creation* is reflected earlier in the accounting-based models. For example, in high growth firms, the free cash flows are typically negative in the short run due to high capital investments in new property, plant, and equipment. Accrual accounting recognizes these investments as assets on the balance sheet and, thus, the firm may still have positive earnings and even positive residual income (which measures earnings in excess of the capital cost of the book value of invested capital). In reality, firms have for several centuries used accrual accounting instead of cash flow accounting (which would fully expense capital investment expenditures), to provide information about value creation and performance of its employees — in a timely manner and for each period separately.

It is important to recognize that value creation (or performance) as reflected in financial statements does not perfectly reflect value creation in market value terms due to the particular accounting rules applied. For example, research and development expenditures are typically fully expensed, and firms do not recognize the net present value of a capital investment as an asset on the balance sheet at the investment date. Instead, these benefits will be recognized as increases in future earnings as they are realized. The existence of these accounting rules can only be understood by considering the stewardship role of accounting and the incentives of managers and firms to misrepresent information. The determination of the particular accounting rules must strike a balance between timeliness, accuracy, and reliability. The beauty of accounting-based valuation models is that they recognize value creation in the short run although imperfectly, and that any "errors" made will be perfectly offset in the net present value of subsequent residual income numbers.

As a practical matter, financial analysts and others evaluating equities do typically not forecast free cash flows directly even if they are using the discounted free cash flow model. A likely reason is that past free cash flow numbers contain considerable transitory noise due to, for example, the particular timing of past capital investment expenditures. Instead, a common approach is to forecast value drivers like sales, profit margins, and asset turnover. From these forecasts, future operating earnings and book values of operating assets can be calculated. These are the key inputs to the accounting-based valuation models. Hence, these models can be applied directly by calculating future residual operating income or abnormal operating income growth from forecasts of operating income, book values of operating assets, and the capital charge on opening book values or last period's free cash flows.

If the discounted free cash flow models are used, the forecasts of free cash flows are obtained from the so-called "operating asset relation," i.e., ending book value of operating assets equals opening book value of operating assets plus operating income minus the free cash flow (which is the amount that can be used to pay the debt and the shareholders). Obviously, these forecasts of free cash flows are no more reliable than the forecasts of the accounting numbers on which they are based.

Hence, the choice between discounted free cash flow models versus accounting-based models is merely a choice of how to present the analysis and the results, i.e., given the same set of forecasts the models are mathematically equivalent. The discounted free cash flow model is nothing else than a special case of the accounting-based models in which the non-cash accruals are reversed in order to yield cash flow accounting.

1.3 A New Equity Valuation Model

Our focus is not to discuss the most efficient ways of forecasting future accounting numbers or free cash flows or to advocate accounting-based valuation models over discounted free cash flow models. Instead, our focus is on discounting cash flows or accounting numbers in a theoretically sound manner in a multi-period setting under uncertainty.

We extend the Rubinstein (1976) analysis for the purpose of developing an easy-to-use equity valuation model consistent with contemporaneous multi-period asset pricing theory. Instead of discounting expected cash flows or earnings with risk-adjusted discount rates, the new approach discounts risk-adjusted expected cash flows or earnings using zero-coupon interest rates from a readily observable term structure of interest rates (which reflects expected future spot interest rates and their risk).

The valuation model recognizes that short- and long-term interest rates are not equal, and that interest rates, expected equity returns, and inflation rates are all stochastic. We explicitly characterize the risk-adjustments to expected cash flows or earnings in an equilibrium setting. We show how the term structure of risk-adjustments depends on both the time-series properties of free cash flows and the accounting policy. The accounting policy affects the time-series properties of earnings, for example, through the income recognition policy. We show that growth, persistence, and mean reversion of residual operating income created by competition in the product markets or by the accounting policy are key determinants of the term structure of risk-adjustments in accounting-based models. We use a set of stylized examples to illustrate the types and the magnitudes of errors which may result from using standard valuation models based on risk-adjusted discount rates. These errors are far from being trivial, and we show how the errors depend on the time-series properties of free cash flows and earnings.

Our starting point is the implications of no-arbitrage in perfect multi-period financial markets. A fundamental property of no-arbitrage pricing is that the valuation operator is a linear functional — also known as the *value additivity principle*. That is, the value of a portfolio of two uncertain cash flow streams must be equal to the sum of the values of the individual cash flow streams. An immediate consequence of the value additivity principle is that the value of a sequence of future uncertain cash flows must be equal to the sum of the values of each of the future uncertain cash flows.

Similarly, the value of a given future uncertain cash flow must be equal to the value of the expected cash flow plus the value of the difference between the uncertain cash flow and the expected cash flow. If calculated at the future date, the latter is referred to as (minus) the *cash flow risk premium*. The difference between the expected cash flow and the cash flow risk premium is the *risk-adjusted expected cash flow*. Since the risk-adjusted expected cash flow is a certain number, its current value must be equal to its discounted value using the current zero-coupon interest rate for the particular future date as the discount rate. Hence, no-arbitrage implies that the current value of a sequence of future uncertain cash flows is equal to the sum of the sequence of future risk-adjusted expected cash flows discounted by the associated zero-coupon interest rates.

In standard finance textbooks, such as Grinblatt and Titman (2002), this method is referred to as the *certainty equivalent method* — with the certainty equivalent being equal to the risk-adjusted expected cash flow. However, one should be careful about the language when it comes to actually determining the certainty equivalents. In the economics of uncertainty, the certainty equivalent of uncertain wealth is defined as the level of wealth which has the same utility as the expected utility of the uncertain wealth, i.e., the wealth level which makes the decision maker indifferent between receiving this level of wealth with certainty and retaining the uncertain wealth. This concept is *not* equal to the risk-adjusted expected cash flow in equilibrium valuation models (as it might appear from reading at least some finance textbooks, see also Damodaran, 2005).

One way to see this is to note that certainty equivalents do not satisfy the value additivity principle and, thus, violate the implications of no-arbitrage. The certainty equivalent of a portfolio of two uncertain cash flows is only equal to the sum of their individual certainty equivalents if these cash flows are stochastically independent and the decision maker's preferences have no wealth effects (i.e., exponential utility). Hence, it is impossible to determine the certainty equivalents of individual securities in a meaningful way even if the investors' utility functions are known — the certainty equivalent of an asset depends on the other risks the decision maker has in his portfolio (and in his other income).

Moreover, the certainty equivalent of optimal invested wealth is the value the decision maker attaches to total wealth (like in a take-it-or-leave-it offer), whereas the equilibrium prices of risky securities are determined such that no investor has any incentive to change his optimal portfolio of securities (on the margin). In a general equilibrium setting with HARA utilities, we show that this implies that the risk-adjusted expected cash flows are lower than the certainty equivalents.

If the language of certainty equivalents is retained for risk-adjusted expected cash flows, the meaning should be clear: the certainty equivalent of an uncertain cash flow is the certain cash flow which has the same equilibrium value as the uncertain cash flow. Of course, this definition of certainty equivalents is moot until it is specified how equilibrium values of uncertain cash flows can be calculated. Unfortunately, no-arbitrage alone provides no guidance to actually determining the risk-adjustments to expected cash flows. No-arbitrage pricing is very useful to find the relative prices between redundant derivative securities and the primary securities. An equilibrium model is required to determine the prices of the primary securities spanning the market risks.

We assume that equilibrium prices are determined in an effectively dynamically complete market (such that equilibrium allocations are Pareto optimal), and that the investors have homogeneous beliefs and time-additive preferences. In that setting, it is well-known (see, for example, Christensen and Feltham, 2003, Chapter 6) that the risk-adjustment to expected cash flows can be determined as the covariance between the cash flow and a valuation index, which is measurable with respect to aggregate consumption at that date.

Furthermore, if it is assumed that aggregate (or log-aggregate) consumption and the cash flow are jointly normally distributed, then Stein's Lemma can be used to separate the valuation index out of the covariance as a factor equal to the expected value of the derivative of the valuation index. Hence, the risk-adjustment is determined as the covariance between the cash flow and (log-)aggregate consumption times this valuation-index factor. The latter can be determined from the market prices of aggregate consumption claims (like in the standard single-period CAPM), or by using a general equilibrium approach, i.e., by assuming a particular set of investor preferences.

In the latter approach, normally distributed aggregate consumption goes well with exponential utilities, whereas log-aggregate consumption goes well with power utilities. The former combination is probably the best known in accounting research, whereas the latter is the preferred combination in finance. Hence, we will show our results for general HARA utilities. We also show how to explicitly account for stochastic inflation through nominal zero-coupon interest rates and a nominal valuation index.

1.4 Outline

The remaining chapters are organized as follows. In *Risk-adjusted Discount Rates* we review and critically examine standard valuation models based on risk-adjusted discount rates with an emphasis on the many implicit assumptions made in this approach. *Multi-period Asset Pricing Theory and Accounting Relations* reviews key results from multi-period asset pricing theory in discrete-time, and shows how equity valuation models can equivalently be based on free cash flows or accrual accounting numbers. Based on these results, we derive an accounting-based multi-period equity valuation model in *An Accounting-based Multi-period Equity Valuation Model* with equilibrium risk-adjustments determined by prices of aggregate consumption claims. *Equity Valuation with HARA Utility* includes a general equilibrium analysis of a setting in which the investors have HARA utility, and aggregate (or log-aggregate) consumption and residual operating income are jointly normally distributed. Appendix B extends the setting to preferences with external habit formation (which recently has gained popularity in asset pricing theory), and Appendix C discusses the relationship between risk-adjusted expected cash flows and certainty equivalents.

In *Equity Valuation with HARA Utility*, Section 5.1 examines a simple setting in which residual operating income and aggregate (or log-aggregate) consumption are given by a first-order vector-autoregressive model with mean reversion to a deterministic exponential trend.

The analysis stresses the importance of both the contemporaneous correlation between residual operating income and aggregate consumption, and the time-series properties of these processes. In this section, we also briefly discuss some of the empirical issues involved and point to new empirical research venues. The setting in Section 5.1 is the basis for the comparison of the general equilibrium analysis and the standard approach of using a time-independent risk-adjustment to the required rates of returns in Section 5.2. Section 5.3 examines how the accounting policy choice affects the time-series properties of residual operating income and, thus, the term structure of risk-adjustments. In Section 5.4 we consider a setting in which the standard approach of using a constant risk-adjusted cost of capital is actually consistent with the general equilibrium model. We skip the usual chapter with concluding remarks. The reader is probably already sufficiently confused at this point, but hopefully at a higher level.

References

Ang, A. and J. Liu (2004), 'How to discount cash flows with time-varying expected returns'. *The Journal of Finance* **59**(6), 2745–2783.

Bansal, R. and A. Yaron (2004), 'Risks for the long run: A potential resolution of asset pricing puzzles'. *The Journal of Finance* **59**(4), 1481–1509.

Beaver, W., P. Kettler, and M. Scholes (1970), 'The association between market determined and accounting determined risk measures'. *The Accounting Review* **45**(4), 654–682.

Brennan, M. J. (1997), 'The term structure of discount rates'. *Financial Management* **26**(1), 81–90.

Brennan, M. J. (2003), 'Corporate investment policy'. In: G. M. Constantinides, M. Harris, and R. Stulz (eds.): *Handbook of the Economics of Finance*. Elsevier, pp. 167–214.

Brennan, M. J., A. W. Wang, and Y. Xia (2004), 'Estimation and test of a simple model of intertemporal capital asset pricing'. *The Journal of Finance* **59**(4), 1743–1776.

Brennan, M. J. and Y. Xia (2006), 'Risk and valuation under an intertemporal capital asset pricing model'. *The Journal of Business* **79**(1), 1–36.

Caks, J. (1977), 'The coupon effect on yield to maturity'. *The Journal of Finance* **32**(1), 103–115.

Campbell, J. Y. and J. H. Cochrane (1999), 'By force of habit: A consumption-based explanation of aggregate stock market behavior'. *Journal of Political Economy* **107**(2), 205–251.

Campbell, J. Y. and R. J. Shiller (1988a), 'The dividend-price ratio and expectations of future dividends and discount factors'. *Review of Financial Studies* **1**(3), 195–228.

Campbell, J. Y. and R. J. Shiller (1988b), 'Stock prices, earnings, and expected dividends'. *The Journal of Finance* **43**(3), 661–676.

Christensen, J. A. and J. S. Demski (2002), *Accounting Theory: An Information Content Perspective*. McGraw-Hill/Irwin.

Christensen, P. O., L. E. de la Rosa, and G. A. Feltham (2009), 'Information and the cost of capital: An ex ante perspective'. Forthcoming in *The Accounting Review*.

Christensen, P. O. and G. A. Feltham (2003), *Economics of Accounting: Volume I — Information in Markets*. Springer Series in Accounting Scholarship. Springer Science+Business Media, Inc.

Christensen, P. O., S. E. Graversen, and K. R. Miltersen (2000), 'Dynamic spanning in the consumption based capital asset pricing model'. *European Finance Review* **4**(2), 129–156.

Christensen, P. O. and J. A. Nielsen (1986), 'The bond-type effect on yield to maturity'. *The Scandinavian Journal of Economics* **89**(2), 193–208.

Claus, J. and J. Thomas (2001), 'Equity premia as low as three percent? Evidence from analysts' earnings forecasts for domestic and international stock markets'. *The Journal of Finance* **56**(5), 1629–1666.

Cochrane, J. H. (2005), 'Financial markets and the real economy'. *Foundations and Trends in Finance* **1**(1), 1–101.

Cochrane, J. H. (2008), 'The dog that did not bark: A defense of return predictability'. *The Review of Financial Studies* **21**(4), 1533–1575.

Cooper, I. A. and K. G. Nyborg (2007), 'Valuing the debt tax shield'. *Journal of Applied Corporate Finance* **19**(2), 50–59.

Cooper, I. A. and K. G. Nyborg (2008), 'Tax-adjusted discount rates with investor taxes and risky debt'. *Financial Management* **37**(2), 365–373.

Cox, J., J. Ingersoll, and S. Ross (1981), 'A re-examination of traditional hypotheses about the term structure of interest rates'. *The Journal of Finance* **36**(4), 769–799.

Cox, J., J. Ingersoll, and S. Ross (1985), 'A theory of the term structure of interest rates'. *Econometrica* **53**(2), 385–467.

Damodaram, A. (2005), 'Valuation approaches and metrics: A survey of the theory and evidence'. *Foundations and Trends in Finance* **1**(8), 693–784.

Duffie, D. (2001), *Dynamic Asset Pricing Theory*. Princeton: Princeton University Press, third edition.

Easton, P. (2009), 'Estimating the cost of capital implied by market prices and accounting data'. *Foundations and Trends in Accounting* **2**(4), 241–364.

Feltham, G. A. and J. A. Ohlson (1995), 'Valuation and clean surplus accounting for operating and financial activities'. *Contemporary Accounting Research* **11**(2), 689–731.

Feltham, G. A. and J. A. Ohlson (1996), 'Uncertainty resolution and the theory of depreciation'. *Journal of Accounting Research* **34**(2), 209–234.

Feltham, G. A. and J. A. Ohlson (1999), 'Residual earnings valuation with risk and stochastic interest rates'. *The Accounting Review* **74**(2), 165–183.

Gamba, A., G. A. Sick, and C. A. León (2008), 'Investment under uncertainty, debt and taxes'. *Economic Notes by Banca Monte dei Paschi di Siena SpA* **37**(1), 31–58.

Gebhardt, W., C. Lee, and B. Swaminathan (2001), 'Toward an implied cost of capital'. *Journal of Accounting Research* **39**(1), 135–176.

Grinblatt, M. and S. Titman (2002), *Financial Markets and Corporate Strategy*. New York: McGraw-Hill/Irwin, second edition.

Hakansson, N. H. (1970), 'Optimal investment and consumption strategies under risk for a class of utility functions'. *Econometrica* **38**(5), 587–607.

Hamada, R. (1972), 'The effect of the firm's capital structure on the systematic risk of common stocks'. *The Journal of Finance* **27**(2), 435–452.

Hughes, J., J. Liu, and J. Liu (2009), 'On the relation between expected returns and implied cost of capital'. *Review of Accounting Studies* **14**(2–3), 246–259.

Jensen, B. A. (2009), 'Valuation before and after tax in the discrete time, finite state no arbitrage model'. *Annals of Finance* **5**(1), 91–123.

Kemsley, D. and D. Nissim (2002), 'Valuation of the debt tax shield'. *The Journal of Finance* **57**(5), 2045–2073.

Kraus, A. and J. S. Sagi (2006), 'Asset pricing with unforeseen contingencies'. *Journal of Financial Economics* **82**(2), 417–453.

Lettau, M. and S. Ludvigson (2001), 'Consumption, aggregate wealth, and expected returns'. *The Journal of Finance* **56**(3), 815–849.

Lundholm, R. J. and R. G. Sloan (2004), *Equity Valuation and Analysis*. New York: McGraw-Hill.

Mehra, R. (2006), 'The equity premium puzzle: A review'. *Foundations and Trends in Finance* **2**(1), 1–81.

Miles, J. and R. Ezzel (1980), 'The weighted average cost of capital, perfect capital markets and project life: A clarification'. *Journal of Financial and Quantitative Analysis* **15**(3), 719–730.

Miller, M. H. (1977), 'Debt and taxes'. *The Journal of Finance* **32**(2), 261–276.

Nekrasov, A. and P. K. Shroff (2009), 'Fundamentals-based risk measurement in valuation'. Forthcoming in *The Accounting Review* **84**(6).

Nelson, C. R. and A. F. Siegel (1987), 'Parsimonious modeling of yield curves'. *The Journal of Business* **60**(4), 473–489.

Nielsen, L. T. and M. Vassalou (2006), 'The instantaneous capital market line'. *Economic Theory* **28**(3), 651–664.

Ohlson, J. A. and Z. Gao (2006), 'Earnings, earnings growth and value'. *Foundations and Trends in Accounting* **1**(1), 1–70.

Ohlson, J. A. and B. Juettner-Nauroth (2005), 'Expected EPS and EPS growth as determinants of value'. *Review of Accounting Studies* **10**(2–3), 349–365.

Penman, S. H. (2007), *Financial Statement Analysis and Security Valuation*. New York: McGraw-Hill, third edition.

Rubinstein, M. (1976), 'The valuation of uncertain income streams and the pricing of options'. *Bell Journal of Economics* **7**(2), 407–425.

Vissing-Jorgensen, A. (2002), 'Limited asset market participation and the elasticity of intertemporal substitution'. *Journal of Political Economy* **110**(4), 825–853.

Wachter, J. A. (2006), 'A consumption-based model of the term structure of interest rates'. *Journal of Financial Economics* **79**(2), 365–399.

Wilson, R. (1968), 'The theory of syndicates'. *Econometrica* **36**(1), 119–132.