Mesh Parameterization Methods and Their Applications

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Full text available at: http://dx.doi.org/10.1561/0600000011

Foundations and Trends® in Computer Graphics and Vision

Published, sold and distributed by:
now Publishers Inc.
PO Box 1024
Hanover, MA 02339
USA
Tel. +1-781-985-4510
www.nowpublishers.com
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Outside North America:
now Publishers Inc.
PO Box 179
2600 AD Delft
The Netherlands
Tel. +31-6-51115274

Library of Congress Control Number: 2006939152

The preferred citation for this publication is A. Sheffer, E. Praun and K. Rose, Mesh Parameterization Methods and Their Applications, Foundation and Trends® in Computer Graphics and Vision, vol 2, no 2, pp 105–171, 2006

Printed on acid-free paper

ISBN: 1-933019-43-3
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Foundations and Trends® in Computer Graphics and Vision

Volume 2 Issue 2, 2006

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Foundations and Trends® in
Computer Graphics and Vision
Vol. 2, No 2 (2006) 105–171

DOI: 10.1561/0600000011

Mesh Parameterization Methods and Their Applications

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Abstract

We present a survey of recent methods for creating piecewise linear mappings between triangulations in 3D and simpler domains such as planar regions, simplicial complexes, and spheres. We also discuss emerging tools such as global parameterization, inter-surface mapping, and parameterization with constraints. We start by describing the wide range of applications where parameterization tools have been used in recent years. We then briefly review the pertinent mathematical background and terminology, before proceeding to survey the existing parameterization techniques. Our survey summarizes the main ideas of each technique and discusses its main properties, comparing it to other methods available. Thus it aims to provide guidance to researchers and developers when assessing the suitability of different methods for various applications. This survey focuses on the practical aspects of the methods available, such as time complexity and robustness and shows multiple examples of parameterizations generated using different methods, allowing the reader to visually evaluate and compare the results.

Full text available at: http://dx.doi.org/10.1561/0600000011

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1

Introduction

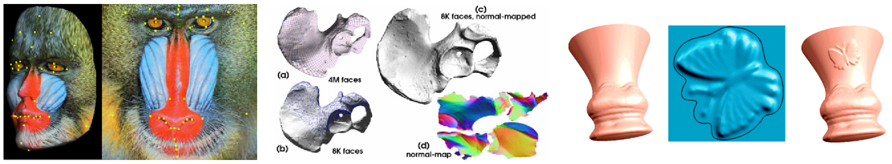
Given any two surfaces with similar topology, it is possible to compute a one-to-one and onto mapping between them. If one of these surfaces is represented by a triangular mesh, the problem of computing such a mapping is referred to as mesh parameterization [7, 35]. The surface that the mesh is mapped to is typically referred to as the parameter domain. Parameterizations between surface meshes and a variety of domains have numerous applications in computer graphics and geometry processing as described below. In recent years numerous methods for parameterizing meshes were developed, targeting diverse parameter domains and focusing on different parameterization properties. This survey reviews the various parameterization methods, summarizing the main ideas of each technique and focusing on the practical aspects of the methods. It also provides examples of the results generated by many of the more popular methods. When several methods address the same parameterization problem, the survey strives to provide an objective comparison between them based on criteria such as parameterization quality, efficiency, and robustness.

We start by surveying the applications which can benefit from parameterization in Section 1.1 and then in Section 2 briefly review the terminology commonly used in the parameterization literature. The rest of the survey describes the different techniques available, classifying them based on the parameter domain used. Section 3 describes techniques for planar parameterization. Section 4 reviews methods for pre-processing meshes for planar parameterization by cutting them into one or more charts. Section 5 examines parameterization methods for alternative domains such as a sphere or a base mesh as well as methods for cross-parameterization between mesh surfaces. Section 6 discusses ways to introduce constraints into a parameterization. Finally, Section 7 summarizes the paper and discusses potential open problems in mesh parameterization.

1.1 Applications

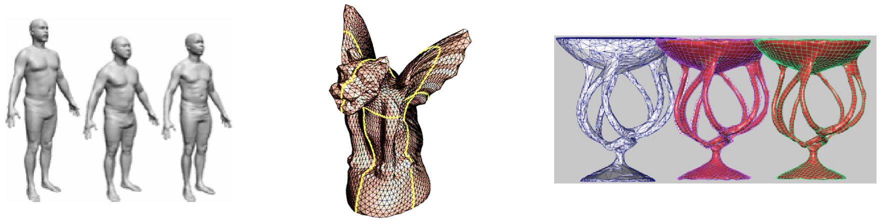
Surface parameterization was introduced to computer graphics as a method for mapping textures onto surfaces [7,84]. Over the last decade, it has gradually become a ubiquitous tool, useful for many mesh processing applications, discussed below (Figure 1.1).

Detail Mapping Detailed objects can be efficiently represented by a coarse geometric shape (polygonal mesh or subdivision surface) with the details corresponding to each triangle stored in a separate 2D array. In traditional texture mapping, the detail is the local albedo of a Lambertian surface. Texture maps alone can enrich the appearance of a surface in a static picture, but since neighboring pixels will have similar shadowing, objects may still look flat in animations with varying lighting conditions. Bump mapping stores small deviations of the pointwise normal from that of the smooth underlying surface and uses the perturbed version during shading [13]. Normal mapping [130, 118] is a similar technique that replaces the normals directly rather than storing a perturbation. As the light direction changes, the shading variations produced by the normal perturbations simulate the shadows caused by small pits and dimples in the surface. Since the actual geometry of the object is not modified, the silhouettes still look polygonal or smooth. Displacement mapping addresses this problem by storing small local deformations of the surface, typically in the direction of the normal.



Texture Mapping [76] Normal Mapping [118] Detail Transfer [8]

Morphing [71] Mesh Completion [114] Editing [77]



Databases [3] Remeshing [98] Surface Fitting [80]

Fig. 1.1 Parameterization applications.

Recent techniques [75,93,96] model a thick region of space in the neighborhood of the surface by using a volumetric texture, rather than a 2D one. Such techniques are needed in order to model detail with complicated topology or detail that cannot be easily approximated locally by a height field, such as sparsely interwoven structures or animal fur. The natural way to map details to surfaces is using planar parameterization (Section 3).

Detail Synthesis While the goal of texture mapping is to *represent* the complicated appearance of 3D objects, several methods make use of mesh parameterization to *create* the local detail necessary for a rich appearance. Such techniques can use as input flat patches with sample detail [92, 97, 129, 127, 131, 119]; parametric or procedural models; or direct user input and editing [57, 17]. The type of detail can be quite varied and the intermediate representations used to create it parallel the final representations used to store it.

Morphing and Detail Transfer A map between the surfaces of two objects allows the transfer of detail from one object to another [81, 121, 99], or the interpolation between the shape and appearance of several objects [71, 2, 66, 63, 109]. By varying the interpolation ratios over time, one can produce morphing animations. In spatially varying and frequency-varying morphs, the rate of change can be different for different parts of the objects, or different frequency bands (coarseness of the features being transformed) [71,66,63]. Such a map can either be computed directly or, as more commonly done, computed by mapping both object surfaces to a common domain (Sections 5 and 6).

In addition to transferring the static appearance of surfaces, inter-surface parameterizations allow the transfer of animation data between shapes, either by transferring the local surface influence from bones of an animation rig, or by directly transferring the local affine transformation of each triangle in the mesh [122].

Mesh Completion Meshes from range scans often contain holes and multiple components. Lévy [77] uses planar parameterization to obtain the natural shape for hole boundaries and to triangulate those. In many cases, prior knowledge about the overall shape of the scanned models exists. For instance, for human scans, templates of a generic human shape are readily available. Allen *et al.* [3], and Anguelov *et al.* [6] use this prior knowledge to facilitate completion of scans by computing a mapping between the scan and a template human model. Kraevoy and Sheffer [67] develop a more generic and robust template-based approach for completion of any type of scans. The techniques typically use an inter-surface parameterization between the template and the scan (Sections 5 and 6).

Mesh Editing Editing operations often benefit from a local parameterization between pairs of models. Biermann *et al.* [8] use local parameterization to facilitate cut-and-paste transfer of details between models.

They locally parameterize the regions of interest on the two models in 2D and overlap the two parameterizations. They use the parameterization to transfer shape properties from one model to the other. Sorkine *et al.* [121] and Lévy [77] use local parameterization for mesh composition in a similar manner. They compute an overlapping planar parameterization of the regions near the composition boundary on the input models and use it to extract and smoothly blend shape information from the two models.

Creation of Object Databases Once a large number of models are parameterized on a common domain (Sections 5 and 6), one can perform an analysis determining the common factors between objects and their distinguishing traits. For example on a database of human shapes [3], the distinguishing traits may be gender, height, and weight, while a database of human faces may add facial expressions [12, 85, 10, 11]. Objects can be compared against the database and scored against each of these dimensions, and the database can be used to create new plausible object instances by interpolation or extrapolation of existing ones.

Remeshing There are many possible triangulations that represent the same shape with similar levels of accuracy. Some triangulation may be more desirable than others for different applications. For example, for numerical simulations on surfaces, triangles with a good aspect ratio (that are not too small or too "skinny") are important for convergence and numerical accuracy. One common way to remesh surfaces, or to replace one triangulation by another, is to parameterize the surface, then map a desirable, well-understood, and easy to create triangulation of the domain back to the original surface. For example, Gu *et al.* [41] use a regular grid sampling of a planar square domain, while subdivision based methods [49, 72, 63] use regular subdivision (usually one-to-four triangle splits) on the faces of a simplicial domain. Such locally regular meshes can usually support the creation of smooth surfaces as the limit process of applying subdivision rules. To generate high quality triangulations Desbrun *et al.* [26] parameterize the input mesh in the plane and then use planar Delaunay triangulation to obtain a high quality remeshing of the surface. One problem these methods face is the appearance of visible discontinuities along the cuts created to facilitate the parameterization.

Surazhsky and Gotsman [123] avoid global parameterization, and instead use local parameterization to move vertices along the mesh as part of an explicit remeshing scheme. Ray *et al.* [102] use global periodic parameterization to generate a predominantly quadrilateral mesh directly on the 3D surface. Dong *et al.* [26] use a parameterization induced by the Morse complex to generate a quad only mesh of the surface.

More details on the use of parameterization for remeshing can be found in a recent survey by Alliez *et al.* [5].

Mesh Compression Mesh compression is used to compactly store or transmit geometric models [4]. As with other data, compression rates are inversely proportional to the data entropy. Thus higher compression rates can be obtained when models are represented by meshes that are as regular as possible, both topologically and geometrically. Topological regularity refers to meshes where almost all vertices have the same degree. Geometric regularity implies that triangles are similar to each other in terms of shape and size, and vertices are close to the centroid of their neighbors. Such meshes can be obtained by parameterizing the original objects and then remeshing with regular sampling patterns [41, 52]. The quality of the parameterization directly impacts the compression efficiency.

Surface Fitting One of the earlier applications of mesh parameterization is surface fitting [32,51,54,80,82]. Many applications in geometry processing require a smooth analytical surface to be constructed from an input mesh. A parameterization of the mesh over a base domain significantly simplifies this task. Earlier methods either parameterized the entire mesh in the plane [32] or segmented it and parameterized each patch independently (Sections 3 and 4). More recent methods [80,82,51] focus on constructing smooth global parameterizations (Section 5.1) and use those for fitting, achieving global continuity of the constructed surfaces.

Modeling from Material Sheets While computer graphics focuses on virtual models, geometry processing has numerous real-world engineering applications. Particularly, planar mesh parameterization is an important tool when modeling 3D objects from sheets of material, ranging from garment modeling to metal forming or forging [7, 88, 86, 60]. All of these applications require the computation of planar patterns to form the desired 3D shapes. Typically, models are first segmented into nearly developable charts (Section 4), and these charts are then parameterized in the plane (Section 3).

Medical Visualization Complex geometric structures are often better visualized and analyzed by mapping the surface normal-map, color, and other properties to a simpler, canonical domain. One of the structures for which such mapping is particularly useful is the human brain [42, 50, 56]. Most methods for brain mapping use the fact that the brain has genus zero, and visualize it through spherical [42, 50] (Section 5.2) or planar [56] (Section 3) parameterization.

Given the vast range of processing techniques that have benefited from parameterization, we expect that many more applications can utilize it as a powerful processing tool.

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