Bilateral Filtering:
Theory and Applications

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Bilateral Filtering: Theory and Applications

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Abstract

The bilateral filter is a non-linear technique that can blur an image while respecting strong edges. Its ability to decompose an image into different scales without causing haloes after modification has made it ubiquitous in computational photography applications such as tone mapping, style transfer, relighting, and denoising. This text provides a graphical, intuitive introduction to bilateral filtering, a practical guide for efficient implementation and an overview of its numerous applications, as well as mathematical analysis.

Contents

1

Introduction

Bilateral filtering is a technique to smooth images while preserving edges. It can be traced back to 1995 with the work of Aurich and Weule [4] on nonlinear Gaussian filters. It was later rediscovered by Smith and Brady [59] as part of their SUSAN framework, and Tomasi and Manduchi [63] who gave it its current name. Since then, the use of bilateral filtering has grown rapidly and is now ubiquitous in image-processing applications Figure 1.1. It has been used in various contexts such as denoising [1, 10, 41], texture editing and relighting [48], tone management [5, 10, 21, 22, 24, 53], demosaicking [56], stylization [72], and optical-flow estimation [57, 74]. The bilateral filter has several qualities that explain its success:

- Its formulation is simple: each pixel is replaced by a weighted average of its neighbors. This aspect is important because it makes it easy to acquire intuition about its behavior, to adapt it to application-specific requirements, and to implement it.
- It depends only on two parameters that indicate the size and contrast of the features to preserve.
- It can be used in a non-iterative manner. This makes the parameters easy to set since their effect is not cumulative over several iterations.

Fig. 1.1 The bilateral filter converts any input image (a)to a smoothed version (b). It removes most texture, noise, and fine details, but preserves large sharp edges without blurring.

- It can be computed at interactive speed even on large images, thanks to efficient numerical schemes [21, 23, 55, 54, 50, 71], and even in real time if graphics hardware is available [16].

In parallel to applications, a wealth of theoretical studies [6, 7, 13, 21, 23, 46, 50, 60, 65, 66] explain and characterize the bilateral filter's behavior. The strengths and limitations of bilateral filtering are now fairly well understood. As a consequence, several extensions have been proposed [14, 19, 23].

This paper is organized as follows. Section 2 presents linear Gaussian filtering and the nonlinear extension to the bilateral filter. Section 3 revisits several recent, novel and challenging applications of bilateral filtering. Section 4 compares different ways to implement the bilateral filter efficiently. Section 5 presents several links of bilateral filtering with other frameworks and also different ways to interpret it. Section 6 exposes extensions and variants of the bilateral filter. We also provide a website with code and relevant pointers (http://people.csail.mit.edu/sparis/bf_survey/).

References

[1] M. Aleksic, M. Smirnov, and S. Goma, "Novel bilateral filter approach: Image noise reduction with sharpening," in *Proceedings of the Digital Photography II Conference*, volume 6069, SPIE, 2006.

[2] G. Aubert and P. Kornprobst, *Mathematical Problems in Image Processing: Partial Differential Equations and the Calculus of Variations (Second edition)*. volume 147 *of Applied Mathematical Sciences*. Springer-Verlag, 2006.

[3] J. Aujol, G. Aubert, L. Blanc-Féraud, and A. Chambolle, "Image decomposition into a bounded variation component and an oscillating component," *Journal of Mathematical Imaging and Vision*, vol. 22, no. 1, January 2005.

[4] V. Aurich and J. Weule, "Non-linear gaussian filters performing edge preserving diffusion," in *Proceedings of the DAGM Symposium*, pp. 538–545, 1995.

[5] S. Bae, S. Paris, and F. Durand, "Two-scale tone management for photographic look," *ACM Transactions on Graphics*, vol. 25, no. 3, pp. 637–645, Proceedings of the ACM SIGGRAPH conference, 2006.

[6] D. Barash, "A fundamental relationship between bilateral filtering, adaptive smoothing and the nonlinear diffusion equation," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 24, no. 6, pp. 844–847, 2002.

[7] D. Barash and D. Comaniciu, "A Common framework for nonlinear diffusion, adaptive smoothing, bilateral filtering and mean shift," *Image and Video Computing*, vol. 22, no. 1, pp. 73–81, 2004.

[8] B. E. Bayer, "Color imaging array," US Patent 3971065, 1976.

[9] E. P. Bennett, J. L. Mason, and L. McMillan, "Multispectral bilateral video fusion," *IEEE Transactions on Image Processing*, vol. 16, no. 5, pp. 1185–1194, May 2007.

[10] E. P. Bennett and L. McMillan, "Video enhancement using per-pixel virtual exposures," *ACM Transactions on Graphics*, vol. 24, no. 3, pp. 845–852, Proceedings of the ACM SIGGRAPH conference, July, 2005.

[11] M. J. Black, G. Sapiro, D. H. Marimont, and D. Heeger, "Robust anisotropic diffusion," *IEEE Transactions on Image Processing*, vol. 7, no. 3, pp. 421–432, March 1998.

[12] A. Buades, "Image and film denoising by non-local means," PhD thesis, Universitat de les Illes Balears, 2006.

[13] A. Buades, B. Coll, and J.-M. Morel, "Neighborhood filters and PDE's," *Numerische Mathematik*, vol. 105, no. 1, pp. 1–34, November 2006.

[14] A. Buades, B. Coll, and J.-M. Morel, "The staircasing effect in neighborhood filters and its solution," *IEEE Transactions on Image Processing*, vol. 15, no. 6, pp. 1499–1505, 2006.

[15] F. Catté, P.-L. Lions, J.-M. Morel, and T. Coll, "Image selective smoothing and edge detection by nonlinear diffusion," *SIAM Journal of Numerical Analysis*, vol. 29, no. 1, pp. 182–193, February 1992.

[16] J. Chen, S. Paris, and F. Durand, "Real-time edge-aware image processing with the bilateral grid," *ACM Transactions on Graphics*, vol. 26, no. 3, p. 103, Proceedings of the ACM SIGGRAPH conference, 2007.

[17] K. Chiu, M. Herf, P. Shirley, S. Swamy, C. Wang, and K. Zimmerman, "Spatially nonuniform scaling functions for high contrast images," in *Proceedings of Graphics Interface '93*, pp. 245–254, May 1993.

[18] H. Chong, S. Gortler, and T. Zickler, "A perception-based color space for illumination-invariant image processing," *ACM Transactions on Graphics*, vol. 27, no. 3, pp. 1–7, Proceedings of the ACM SIGGRAPH conference, 2008.

[19] P. Choudhury and J. Tumblin, "The trilateral filter for high contrast images and meshes," in *Proceedings of the Eurographics Symposium on Rendering*, pp. 1–11, 2003.

[20] D. DeCarlo and A. Santella, "Stylization and abstraction of photographs," in *Proceedings of the ACM SIGGRAPH conference*, pp. 769–776, 2002.

[21] F. Durand and J. Dorsey, "Fast bilateral filtering for the display of high-dynamic-range images," *ACM Transactions on Graphics*, vol. 21, no. 3, pp. 257–266, Proceedings of the ACM SIGGRAPH conference, 2002.

[22] E. Eisemann and F. Durand, "Flash photography enhancement via intrinsic relighting," *ACM Transactions on Graphics*, vol. 23, no. 3, pp. 673–678, Proceedings of the ACM SIGGRAPH conference, July, 2004.

[23] M. Elad, "On the bilateral filter and ways to improve it," *IEEE Transactions on Image Processing*, vol. 11, no. 10, pp. 1141–1151, October 2002.

[24] M. Elad, "Retinex by two bilateral filters," in *Proceedings of the Scale-Space conference*, pp. 217–229, 2005.

[25] R. Fattal, M. Agrawala, and S. Rusinkiewicz, "Multiscale shape and detail enhancement from multi-light image collections," *ACM Transactions on Graphics*, vol. 26, no. 3, p. 51, Proceedings of the ACM SIGGRAPH conference, 2007.

[26] S. Fleishman, I. Drori, and D. Cohen-Or, "Bilateral mesh denoising," *ACM Transactions on Graphics*, vol. 22, no. 3, pp. 950–953, Proceedings of the ACM SIGGRAPH conference, July, 2003.

[27] S. Geman and D. Geman, "Stochastic relaxation, Gibbs distributions, and the Bayesian restoration of images," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 6, no. 6, pp. 721–741, 1984.

[28] G. Gimel'farb, *Image Textures and Gibbs Random Fields.* Kluwer Academic Publishers, 1999. ISBN 0792359615.

[29] F. R. Hampel, E. M. Ronchetti, P. M. Rousseeuw, and W. A. Stahel, *Robust Statistics — The Approach Based on Influence Functions.* Wiley Interscience, 1986. ISBN 0-471-73577-9.

[30] P. J. Huber, *Robust Statistics. Probability and Statistics.* Wiley-Interscience, February 1981. ISBN: 9780471418054.

[31] L. Itti and C. Koch, "Computational modeling of visual attention," *Nature Reviews Neuroscience*, vol. 2, no. 3, pp. 194–203, 2001.

[32] D. J. Jobson, Z. Rahman, G. A. Woodell, N. Center, and V. A. Hampton, "A multiscale Retinex for bridging the gap between color images and the human observation of scenes," *IEEE Transactions on Image Processing*, vol. 6, no. 7, pp. 965–976, 1997.

[33] T. Jones, F. Durand, and M. Zwicker, "Normal improvement for point rendering," *IEEE Computer Graphics & Applications*, vol. 24, no. 4, pp. 53–56, 2004.

[34] T. R. Jones, F. Durand, and M. Desbrun, "Non-iterative, feature-preserving mesh smoothing," *ACM Transactions on Graphics*, vol. 22, no. 3, Proceedings of the ACM SIGGRAPH conference, July, 2003.

[35] E. A. Khan, E. Reinhard, R. Fleming, and H. Buelthoff, "Image-based material editing," *ACM Transactions on Graphics*, vol. 25, no. 3, pp. 654–663, Proceedings of the ACM SIGGRAPH conference, 2006.

[36] J. J. Koenderink and A. J. Van Doorn, "The structure of locally orderless images," *International Journal of Computer Vision*, vol. 31, no. 2–3, pp. 159–168, 1999.

[37] J. Kopf, M. Uyttendaele, O. Deussen, and M. Cohen, "Capturing and viewing gigapixel images," *ACM Transactions on Graphics*, vol. 26, no. 3, p. 93, Proceedings of the ACM SIGGRAPH conference, 2007.

[38] E. H. Land and J. J. McCann, "Lightness and Retinex theory," *Journal of the Optical Society of America*, vol. 61, no. 1, pp. 1–11, 1971.

[39] H. Land, Edwin, "The Retinex," *American Scientist*, vol. 52, pp. 247–264, 1964.

[40] S. Li, *Markov Random Field Modeling in Computer Vision.* Springer-Verlag, 1995. ISBN 4-431-70145-1.

[41] C. Liu, W. T. Freeman, R. Szeliski, and S. Kang, "Noise estimation from a single image," in *Proceedings of the Conference on IEEE Computer Vision and Pattern Recognition,* volume 1, pp. 901–908, 2006.

[42] B. D. Lucas and T. Kanade, "An iterative image registration technique with an application to stereo vision," in *Proceedings of the International Joint Conference on Artificial Intelligence,* volume 81, pp. 674–679, 1981.

[43] S. Mallat, *A Wavelet Tour of Signal Processing.* Academic Press, 1999. ISBN: 0-12-466606-X.

[44] Y. Meyer, *Oscillating Patterns in Image Processing and Nonlinear Evolution Equations,* volume 22 of *University Lecture Series.* American Mathematical Society, 2001.

[45] A. Miropolsky and A. Fischer, "Reconstruction with 3D geometric bilateral filter," in *Proceedings of the ACM Symposium on Solid Modeling and Applications*, pp. 225–229, 2004.
[46] P. Mrázek, J. Weickert, and A. Bruhn, *Geometric Properties from Incomplete Data*. On Robust Estimation and Smoothing with Spatial and Tonal Kernels. Springer, 2006. ISBN: 978-1-4020-3857-0.
[47] D. A. Murio, *The Mollification Method and the Numerical Solution of Ill-Posed Problems*. Wiley-Interscience, 1993. ISBN: 0471594083.
[48] B. M. Oh, M. Chen, J. Dorsey, and F. Durand, "Image-based modeling and photo editing," in *Proceedings of the ACM SIGGRAPH Conference*, pp. 433–442, 2001.
[49] S. Paris, H. Briceño, and F. Sillion, "Capture of hair geometry from multiple images," *ACM Transactions on Graphics*, vol. 23, no. 3, pp. 712–719, Proceedings of the ACM SIGGRAPH conference, 2004.
[50] S. Paris and F. Durand, "A fast approximation of the bilateral filter using a signal processing approach," *International Journal of Computer Vision*, vol. 81, no. 1, pp. 24–52, 2009.
[51] S. N. Pattanaik, J. A. Ferwerda, M. D. Fairchild, and D. P. Greenberg, "A multiscale model of adaptation and spatial vision for realistic image display," in *Proceedings of the ACM SIGGRAPH conference*, pp. 287–298, 1998.
[52] P. Perona and J. Malik, "Scale-space and edge detection using anisotropic diffusion," *IEEE Transactions Pattern Analysis Machine Intelligence*, vol. 12, no. 7, pp. 629–639, July 1990.
[53] G. Petschnigg, M. Agrawala, H. Hoppe, R. Szeliski, M. Cohen, and K. Toyama, "Digital photography with flash and no-flash image pairs," *ACM Transactions on Graphics*, vol. 23, no. 3, pp. 664–672, Proceedings of the ACM SIGGRAPH Conference, 2004.
[54] T. Q. Pham, "Spatiotonal adaptivity in super-resolution of undersampled image sequences," PhD thesis, Delft University of Technology, 2006.
[55] T. Q. Pham and L. J. van Vliet, "Separable bilateral filtering for fast video preprocessing," in *Proceedings of the IEEE International Conference on Multimedia and Expo*, 2005.
[56] R. Ramanath and W. E. Snyder, "Adaptive demosaicking," *Journal of Electronic Imaging*, vol. 12, no. 4, pp. 633–642, 2003.
[57] P. Sand and S. Teller, "Particle video: Long-range motion estimation using point trajectories," *International Journal of Computer Vision*, vol. 80, no. 1, pp. 72–91, 2008.
[58] C. Schlick, "Quantization techniques for visualization of high dynamic range pictures," in *Proceedings of the Eurographics Rendering Workshop*, pp. 7–20, 1994.
[59] S. M. Smith and J. M. Brady, "SUSAN — A new approach to low level image processing," *International Journal of Computer Vision*, vol. 23, no. 1, pp. 45–78, May 1997.
[60] N. Sochen, R. Kimmel, and A. M. Bruckstein, "Diffusions and confusions in signal and image processing," *Journal of Mathematical Imaging and Vision*, vol. 14, no. 3, pp. 237–244, 2001.

[61] N. Sochen, R. Kimmel, and R. Malladi, “A general framework for low level vision,” *IEEE Transactions in Image Processing*, vol. 7, pp. 310–318, 1998.

[62] T. G. Stockham, “Image processing in the context of a visual model,” *Proceedings of the IEEE*, vol. 60, no. 7, pp. 828–842, 1972.

[63] C. Tomasi and R. Manduchi, “Bilateral filtering for gray and color images,” in *Proceedings of the IEEE International Conference on Computer Vision*, pp. 839–846, 1998.

[64] J. Tumblin and G. Turk, “Low curvature image simplifiers (LCIS): A boundary hierarchy for detail-preserving contrast reduction,” in *Proceedings of the ACM SIGGRAPH Conference*, pp. 83–90, 1999.

[65] J. van de Weijer and R. van den Boomgaard, “Local mode filtering,” in *Proceedings of the conference on IEEE Computer Vision and Pattern Recognition*, pp. 428–433, 2001.

[66] J. van de Weijer and R. van den Boomgaard, “On the equivalence of local-mode finding, robust estimation and mean-shift analysis as used in early vision tasks,” in *Proceedings of the International Conference on Pattern Recognition*, pp. 927–930, 2002.

[67] L. Vese and S. Osher, “Modeling textures with total variation minimization and oscillating patterns in image processing, Journal of Scientific Computing,” *Journal of Scientific Computing*, vol. 19, pp. 553–572, 2003.

[68] C. C. Wang, “Bilateral recovering of sharp edges on feature-insensitive sampled meshes,” *IEEE Transactions on Visualization and Computer Graphics*, vol. 12, no. 4, pp. 629–639, 2006.

[69] L. Wang, L.-Y. Wei, K. Zhou, B. Guo, and H.-Y. Shum, “High dynamic range image hallucination,” in *Proceedings of the Eurographics Symposium on Rendering*, pp. 321–326, 2007.

[70] G. S. Watson, *Statistics on Spheres.* John Wiley and Sons, 1983.

[71] B. Weiss, “Fast median and bilateral filtering,” *ACM Transactions on Graphics*, vol. 25, no. 3, pp. 519–526, Proceedings of the ACM SIGGRAPH conference, 2006.

[72] H. Winnemöller, S. C. Olsen, and B. Gooch, “Real-time video abstraction,” *ACM Transactions on Graphics*, vol. 25, no. 3, pp. 1221–1226, Proceedings of the ACM SIGGRAPH conference, 2006.

[73] W. C. K. Wong, A. C. S. Chung, and S. C. H. Yu, “Trilateral filtering for biomedical images,” in *Proceedings of the IEEE International Symposium on Biomedical Imaging*, pp. 820–823, 2004.

[74] J. Xiao, H. Cheng, H. Sawhney, C. Rao, and M. Isnardi, “Bilateral filtering-based optical flow estimation with occlusion detection,” in *Proceedings of the European Conference on Computer Vision*, pp. 211–224, 2006.

[75] Q. Yáng, R. Yang, J. Davis, and D. Nistér, “Spatial-depth super resolution for range images,” in *Proceedings of the conference on IEEE Computer Vision and Pattern Recognition*, pp. 1–8, 2007.

[76] Q. Yáng, R. Yang, H. Stewénius, and D. Nistér, “Stereo matching with color-weighted correlation, hierarchical belief propagation and occlusion handling,” in *Proceedings of the Conference on IEEE Computer Vision and Pattern Recognition*, pp. 2347–2354, 2006.

[77] L. P. Yaroslavsky, *Digital Picture Processing. An Introduction.* Springer Verlag, 1985.

[78] K. Yoon and I. Kweon, "Adaptive support-weight approach for correspondence search," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 28, no. 4, pp. 650–656, 2006.