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A Tutorial

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Line Drawings from 3D Models

A Tutorial

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Line Drawings from 3D Models

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ABSTRACT

This tutorial describes the geometry and algorithms for generating line drawings from 3D models, focusing on occluding contours.

The geometry of occluding contours on meshes and on smooth surfaces is described in detail, together with algorithms for extracting contours, computing their visibility, and creating stylized renderings and animations. Exact methods and hardware-accelerated fast methods are both described, and the trade-offs between different methods are discussed. The tutorial brings together and organizes material that, at present, is scattered throughout the literature. It also includes some novel explanations, and implementation tips.

A thorough survey of the field of non-photorealistic 3D rendering is also included, covering other kinds of line drawings and artistic shading.

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1

Introduction

Humans have been drawing pictures since the days of prehistoric cave painting. Various forms of line drawing have been developed since then, including Egyptian hieroglyphs, medieval etching, and industrial-era printmaking. Nowadays, line drawing and outline drawing methods are used throughout cartoons and comics, architectural rendering, instructional tutorials, and many other settings. Drawing is the starting point for many kinds of tasks, for everyone from children making pictures to professional architects sketching ideas. Drawing seems to be fundamentally connected to how we represent the world visually.

While most computer graphics focuses on realistic visual simulation, over the past few decades, line drawing algorithms have also matured. We now have the ability to automatically create reasonable line drawings from 3D geometry, much like photorealistic rendering. These algorithms provide deep insight into the geometry and topology of line drawings, which can be surprisingly subtle, given how simple line drawing might seem. Versions of these algorithms have been used throughout art, entertainment, and visualization. User evaluation has shown that these algorithms, indeed, accurately describe important aspects of how artists draw lines. This shows that these algorithms can contribute to a scientific understanding of art.

This tutorial provides a detailed guide to the mathematical theory and computer algorithms for line drawing of 3D objects. We focus on the curves known as *occluding contours* or, simply, *contours*. These are the most important curves for line drawing of 3D surfaces. They have a rich theory around them, and, once one understands this theory, understanding how other curves operate is much simpler. We describe the different algorithms required to compute and render these curves, together with references to the literature. We also explain boundary curves and surface-surface intersection curves, since these are straightforward to include and often important. We also discuss open research problems in contour rendering.

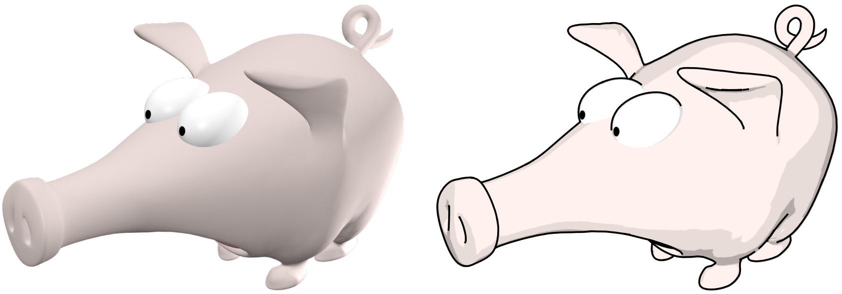
In addition, we survey of other topics in 3D non-photorealistic rendering, with extensive pointers to the literature, including: other types of curves, stroke rendering, and non-photorealistic shading. We do not cover the complementary topic of image-based non-photorealistic rendering; for a survey of image-based methods, we refer the reader to the book by Rosin and Collomosse (2013).

The theory of line drawing is currently scattered about and incomplete in research papers. The algorithms for line drawing include many subtleties that are not described in the literature, and many pitfalls await the coder attempting them. There remain some important open problems, but these gaps are not obvious from the literature. This tutorial is meant to address these issues.

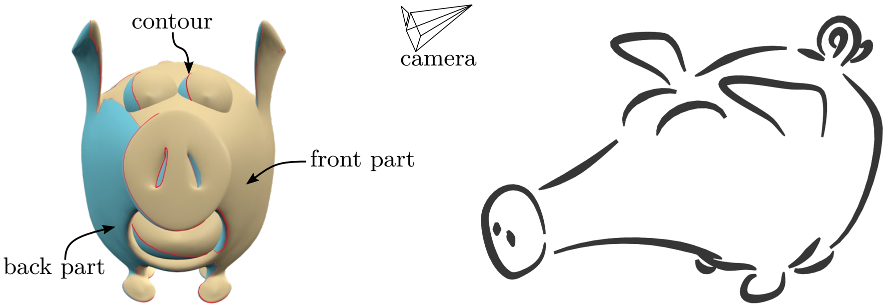
We believe that these topics ought to be known by anyone interested in understanding the curves in visual representational art. It is one where computer graphics can make a unique contribution. Arguably, the algorithmic simulation of line drawing is a crucial step in understanding visual art.

1.1 Occluding contours

This tutorial focuses on the curves known as *occluding contours* or, simply *contours*. In some computer graphics research, these have been



(a) 3D object with diffuse shading **(b)** Contours with toon shading



(c) Side view of the scene **(d)** Stylized curves

Figure 1.1: Occluding contours — The occluding contours of the 3D model "Origins of the Pig" © Keenan Crane, shown in **(a)** with diffuse shading, are depicted in **(b)** composited with toon shading to produce a cel-like drawing. As illustrated in **(c)** from a side view, they delineate the frontier between the front and back parts of the surface when seen from the camera. These contour curves can be further process to produce stylized imagery, such as the calligraphic brush strokes in **(d)**.

called *silhouettes*, though the *silhouettes* are technically a separate set of curves, as we will describe below.

The occluding contours of a simple 3D object are shown in Figure 1.1. As a first definition, suppose we have a 3D object that we wish to render from a specific viewpoint. The occluding contours are *surface curves that separate visible parts from invisible parts*. By rendering the visible portions of these 3D curves together with the object, we get a basic line drawing (Figure 1.1b).

There are many different contour detection and rendering algorithms, and some significant tradeoffs between them. The most important tradeoff is between simple algorithms that produce approximate results, and more complex algorithms that give more precision, control, and stylization capabilities. Just rendering reasonable-looking contours as solid black lines is very straightforward for a graphics programmer. These most basic algorithms can be implemented in a few additional lines of code in an existing renderer, and have been implemented in many real-time applications, including many popular video games (one of the earliest was *Jet Set Radio* in 2000). However, if we wish to stylize the curves, for example, by rendering the curves with sketchy or calligraphic strokes (Figure 1.1d), things become more difficult. With a bit of perseverance, renderings with distinctive and lovely styles can be created. At times, these renderings may still contain topological artifacts that are not suitable for very high-end production. High-quality algorithms that remove these artifacts are more complex; in the extreme, no provably-correct algorithm exists for this problem. However, there are a number of partial solutions that are good enough to be used in many circumstances, and we discuss in detail the issues involved.

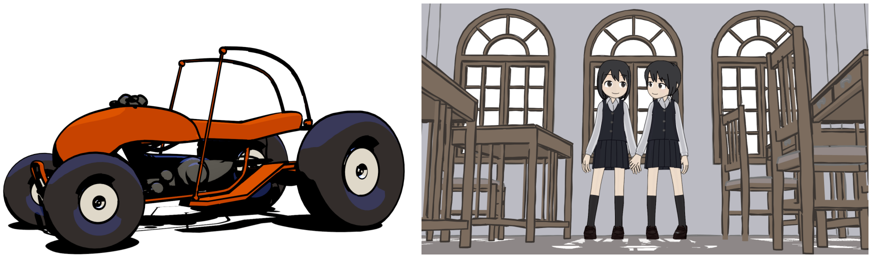
Note that, formally, the occluding contour and occluding contour generator are separate curves in 2D and 3D. However, we will frequently use the term “contour” to refer to each of them, since the correct terms are very cumbersome, and the meaning of “contour” is usually obvious from context.

1.2 How to use this tutorial

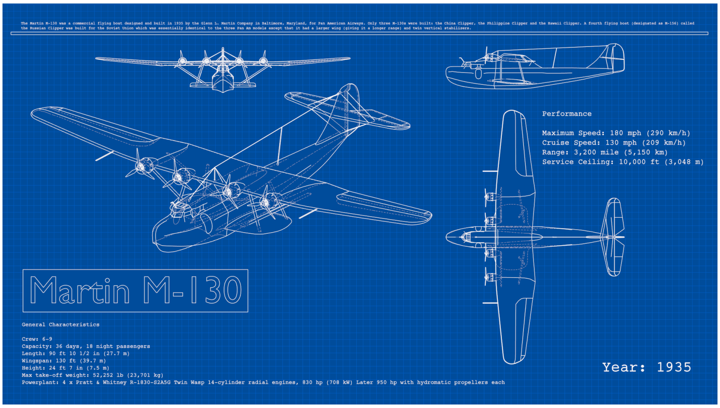
This tutorial is two things: a detailed tutorial of the core contour algorithms, and a high-level survey of nearly all of 3D non-photorealistic rendering. We cover some core topics more thoroughly than any previous publication, and, for other topics, we mainly provide pointers to the literature.

Hence, reading the tutorial directly will give a good overview of the field, but one may skim through survey sections. Alternatively, a practitioner may wish to jump directly to the algorithms relevant to their task.

Generally speaking, real-time image-based methods, especially based on graphics shaders, offer the best real-time performance and have been



(a) Buggy by Rylan Wright 🅭🅯 **(b)** Anime by mato.sus304 🅭🅯🄎



The Martin M-130 was a commercial flying boat designed and built in 1935 by the Glenn L. Martin Company in Baltimore, Maryland, for Pan American Airways. Only three M-130s were built: the China Clipper, the Philippine Clipper and the Hawaii Clipper. A fourth flying boat (designated as M-156) called the Russian Clipper was built for the Soviet Union which was essentially identical to the three Pan Am models except that it had a larger wing (giving it a longer range) and twin vertical stabilizers.

Performance

Maximum Speed: 180 mph (290 km/h)
Cruise Speed: 130 mph (209 km/h)
Range: 3,200 mile (5,150 km)
Service Ceiling: 10,000 ft (3,048 m)

General Characteristics

Crew: 6-9
Capacity: 36 days, 18 night passengers
Length: 90 ft 10 1/2 in (27.7 m)
Wingspan: 130 ft (39.7 m)
Height: 24 ft 7 in (7.5 m)
Max take-off weight: 52,252 lb (23,701 kg)
Powerplant: 4 x Pratt & Whitney R-1830-S2A5G Twin Wasp 14-cylinder radial engines, 830 hp (708 kW) Later 950 hp with hydromatic propellers each

Year: 1935

(c) Martin M-130 blueprint by LightBWK 🅭🄍

(d) Ryner by Lucas Gogol 🅭🄍

Figure 1.2: Artworks created by artists using Blender Freestyle — Each of these is a non-photorealistic rendering, using the techniques described in this tutorial in different ways.

used in many games. These are described in the next Chapter, and pointers to further reading are provided there. This chapter can also help build intuitions for all readers.

The core chapters of the tutorial focus on contour detection and visibility on 3D models. We start with 3D mesh representations, and then apply the same ideas to different smooth surface representations in the subsequent chapters.

We then cover the core topic of detecting contours on meshes (Chapter 3) and computing their visibility (Chapter 4). Contour detection and visibility on meshes is the most basic and well-understood problem, and we go into the most detail in algorithms here. We describe fast, approximate hardware based visibility in Chapter 6.

While it may be tempting to use mesh algorithms for smooth surfaces, in Chapter 5, we explain some of the problems with doing so. We then describe a method called Interpolated Contours that provides a compromise position, being almost as simple as mesh contours to implement, with relatively few inconsistencies.

We then discuss true contours on parametric surface representations (Chapter 7). Understanding these curves involves some differential geometry (reviewed in Appendix A), and the resulting mathematics and theory is rather elegant. We describe detection and visibility algorithms, which are adapted from the mesh algorithms. We describe the different strategies that have been applied to this problem and how they compare. In the following chapter, we then discuss these algorithms as applied to implicit smooth surface representations (Chapter 8).

Finally, we discuss stylized rendering and animations algorithms (Chapter 9), and conclude with a discussion of the state of research and applications in 3D non-photorealistic rendering (Chapter 10).

1.3 The importance of visualizations

Although we have done our best to explain contours in text, they can take some time to wrap your head around. Understanding how the 2D curves and 3D curves relate in an image like Figure 1.1 can be challenging. It is worthwhile spending time with these figures, perhaps starting with simpler examples like different views of a torus, to understand how the

2D and 3D shapes relate, what the curves look like at singular points, and so on.

We provide an interactive viewer at https://benardp.github.io/contours_viewer/. Experimenting with this viewer can help give intuitions on contours.

Even better is to use, or build, a 3D visualization. If you implement a 3D contour rendering system, it is essential to also implement visualizations that let you zoom into the 2D drawing and rotate around the 3D model. In each view, you should be able to render the different types of curves and singularities and their attributes. These visualizations are essential for deep understanding of these curves, as well as for debugging and algorithm development. You can start with simple 3D drawings, e.g., rendering contour edges on the 3D model, and coloring mesh faces according to facing direction as in Figure 1.1c. As your system becomes more sophisticated, you may eventually have visualizations like those in Figure 4.6.

These visualizations are also useful in making certain design choices. As we discuss, there is no current foolproof system for smooth contour rendering, and so there are some choices to be made, e.g., selecting heuristics. Good visualizations can also be helpful in understanding how different heuristics behave.

1.4 The science and perception of art

The algorithms described in this tutorial provide a new level of insight and understanding into the science of art (Hertzmann, 2010). For centuries, artists, historians, philosophers, and scientists have sought a formal understanding of visual art: how do we make it, and how do we perceive it? One of the first generative tools in art was the development of linear perspective during the Italian Renaissance. The theory of occluding contours, which is the main subject of this tutorial, originated in perceptual psychology and computer vision, and was developed into the sophisticated algorithms we described here by computer graphics researchers.

As perceptual psychology has developed, so have perceptual theories of art. For example, one of the most influential modern writers on visual art is Ernst Gombrich (1961), who argued that artists created artistic styles of depiction over the centuries. Nelson Goodman (1968) took this further to argue that all artistic style functions purely as a denotational system of symbols, like characters on a written page, and, presumably, purely learned as a product of culture. Rudolf Arnheim (1974) attempted to formulate Gestalt-like perceptual rules to drawings. Sayim and Cavanagh (2011) attempt to apply modern neuroscience to understanding the perception of art.

In attempt to formalize the description of styles, John Willats (1997) created a denotational semantics to describe different kinds of realistic styles — expanded by Willats and Durand (2005) to include insights from computer graphics.

Non-photorealistic rendering provides a generative theory for how artists create representational art. Like any theory, it does not cover every case or describe every phenomenon accurately, nor does it say anything about cultural, psychological, or other outside factors in the work. However, this generative theory provides considerable potential insight into how art is made.

We can compare the generative theory to the world before and after Newtonian mechanics. Before Newton, philosophers like Aristotle could make qualitative observations about how objects move (e.g., “heavy objects like to fall”) but could make no real predictions. Newtonian mechanics is predictive, it generates insights, and leads to real understanding. Likewise, understanding the generative model of representational art provides a potentially compact way to understand many phenomena.

Two landmark studies validate and justify the use of line drawing algorithms developed in the non-photorealistic rendering literature. Cole *et al.* (2008) undertook a careful study of how artists depict 3D objects. They asked a collection of art students to illustrate several 3D models with line drawings, and compared how the artists’ drawings related to the line drawing algorithms in this section (Figure 1.3). They showed that roughly 80% of a typical drawing could be explained by existing algorithms. This study helps show which of these algorithms are most useful, while also highlighting gaps in the literature. In a follow-up

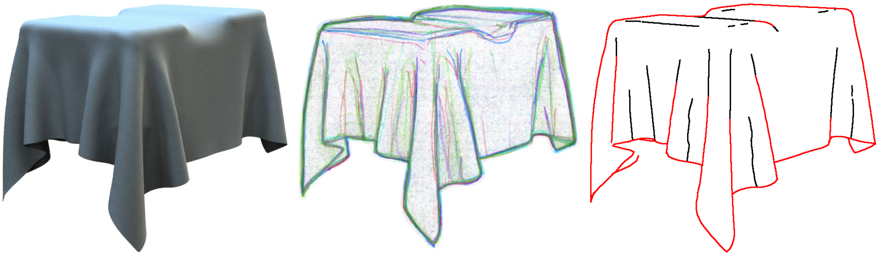


Figure 1.3: Correlation between hand-drawn lines and contours — A 3D model rendered from a given viewpoint and illumination (left) has been hand-drawn by ten artists (center). Observe how consistent the drawings are, especially near the contours of the shape. The contours (in red) and suggestive contours (in black) extracted from the 3D model are depicted in the right. Images taken from the "Javascript Drawing Viewer"[1] of Cole *et al.* (2008).

paper, Cole *et al.* (2009) showed that line drawing algorithms are also very effective at conveying 3D shape.

1.5 Survey of feature curves

This section surveys other important types of surface curves for line drawing, together with pointers to the relevant literature. The remaining chapters focus solely on contour, boundary, and surface-intersection curves.

Most of these curves have been developed both for artistic use and for visualization purposes (Lawonn *et al.*, 2018). However, some types of curves, such as ridges and valleys, seem useful for visualization without mimicking conventional artist curves as well.

Visibility-indicating curves. *Contours* indicate where parts of the surface become visible and invisible, and also indicate where visibility changes. There are a few other important curves that are important for similar reasons.

Boundary curves are simply the boundaries of the surface. Closed surfaces do not have boundaries. These curves are usually rendered when visible. Boundary curves can indicate change of visibility for curves that

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[1] http://gfx.cs.princeton.edu/proj/ld3d/lineset/viewer/index.html

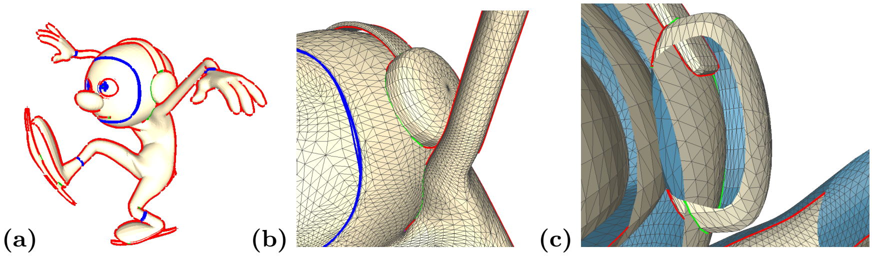

Figure 1.4: Surface-surface intersection curves (from Bénard *et al.* (2014)) — Professionally-modeled surfaces include many intersections between surface, such as this ice-skating character. Surface intersection curves are shown in green, occluding contours in red, and boundaries in blue. Observe how the ear muffs intersects the headband and the hoodie; the shoulder also happens to intersect the hoodie in this animation frame. **(a,b)** Original surface. **(c)** Cross-section from a different viewpoint. Red © Pixar ("Red" character created at Pixar by Andrew Schmidt, Brian Tindall, Bernhard Haux and Paul Aichele, based on the original design of Teddy Newton.)

they intersect in image space, so they are important to handle, and we include them in the discussions of our algorithms in this tutorial.

Surface intersection curves occur when two different sections of surface intersect. These do not occur in the clean models often used in computer graphics research. However, in professional 3D computer animation applications, modelers frequent connect different object parts this way (Figure 1.4). These curves can be detected with standard computer graphics algorithms, and are important to extract since they can indicate changes of visibility with curves that they intersect on the surface. We do not discuss them any further in this tutorial.

All other curves are essentially surface "decorations"; computing them is optional for visibility computations. They typically visualize the surface curvature rather than its outlines.

Contour generalizations. Perhaps the next most significant set of curves are those that generalize contours. These curves were first introduced by DeCarlo *et al.* (2003; 2004), who described a mathematically-elegant generalization of contours and the algorithms needed to render them. Several other variants inspired by this idea were proposed, including apparent ridges (Judd *et al.*, 2007).

The abstracted shading method (Lee *et al.*, 2007) demonstrated how these and lighting-based variants could be computed in image-space. Other variants based on image-space processing include Laplacian Lines (Zhang *et al.*, 2009) and DoG lines (Zhang *et al.* 2012; 2014). In addition to speed, image-space lines have the advantage that they automatically remove clutter as a function of image-space line density, although, like all image-based methods, they potentially lose some fine-scale precision and control.

Figure 1.5 shows some examples of these contour generalizations. Including some form of these curves seems essential for capturing how artists depict surfaces; these curves were essential in the study of Cole *et al.* (2008).

These curves have also been generalized to include highlights that illustrate shading on an object (DeCarlo and Rusinkiewicz, 2007). DeCarlo (2012) provides a thorough survey and comparison of these different types of contour generalizations.

Surface features/properties. Some intrinsic properties of the surface can be drawn, such as sharp creases on smooth surfaces (Saito and Takahashi, 1990), as well as changes in shading (Xie *et al.*, 2007). When objects have assigned texture and materials, one may wish to draw the material boundaries or the texture itself.

Hatching. Hatching strokes illustrate surface shape in line drawings. Winkenbach and Salesin (1994) use manually-authored hatching textures and orientations. For more automation, one can use the iso-parametric curves of parametric surfaces (Elber, 1995a; Winkenbach and Salesin, 1996). However, these lines depend on how the shape was authored, and do not generalize to other types of surfaces. Elber (1998) explored many different possible hatching directions, including principal curvature directions, texture gradients, and illumination gradients. Principal curvature-based hatching is supported by perceptual studies suggesting that human perceive hatching strokes as curvature directions (Mamassian and Landy, 1998). Hertzmann and Zorin (2000) refine principal curvature hatching for umbilic regions (Figure 1.6). Singh and Schaefer

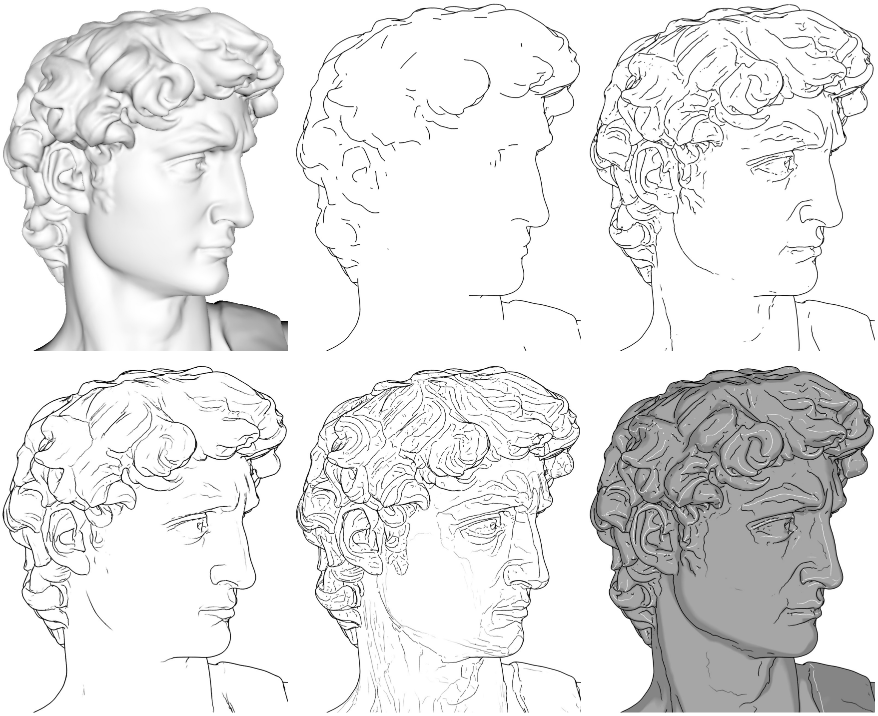

Figure 1.5: Feature curve examples — From left to right, top to bottom: diffuse rendering of the 3D scanned David model by "Scan The World" (http://mmf.io/o/2052), occluding contours (OC), OC + suggestive contours (SC) (DeCarlo *et al.*, 2004), OC + apparent ridges (Judd *et al.*, 2007), OC + ridges & valleys (Rusinkiewicz, 2004), and OC + SC + principal highlights + toon shading (DeCarlo and Rusinkiewicz, 2007). Images generated with "qrtsc" (Cole *et al.*, 2011).

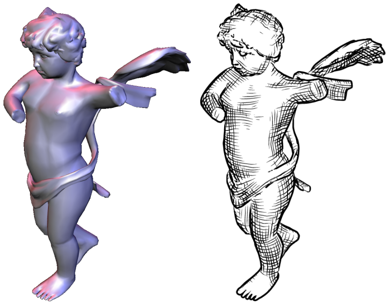

Figure 1.6: Hatching — 3D Cupid model and hatching result obtain with the method of Hertzmann and Zorin (2000).

(2010) describe hatching strokes that follow shading gradients. Since artists draw different types of hatching curves in different situations, Kalogerakis *et al.* (2012) combine these ideas, describing a machine learning system for learning hatching directions, identifying which hatching rules are used in which parts of a 3D surface. Gerl and Isenberg (2013) additionally offer interactive tools to let the user dynamically control the placement and orientation of hatches.

Surface extrema. Extremal curves, such as ridges and valleys, generalize the notion of ridges and valleys in terrain maps, identifying curves of locally maximal or minimal curvature. These types of curves are a visualization technique that can be useful in understanding surface shape; they do not typically correspond to artist-drawn curves otherwise.

Numerous algorithms have been developed to extract ridges and valleys from various types of geometric models (Interrante *et al.*, 1995; Thirion and Gourdon, 1996; Pauly *et al.*, 2003; Rusinkiewicz, 2004; Ohtake *et al.*, 2004; Yoshizawa *et al.*, 2007; Vergne *et al.*, 2011). A variant, called Demarcating Curves (Kolomenkin *et al.*, 2008; DeCarlo, 2012) can help visualize shapes of different regions on a surface.

1.6 Brief history of 3D Non-Photorealistic Rendering

The earliest 3D computer graphics algorithms were hidden-line rendering algorithms (Roberts, 1963), including methods that we discuss in this tutorial (Appel, 1967; Weiss, 1966). While the mainstream of computer graphics focused on photorealistic imagery, a few works aimed at adding artistic stroke textures to architectural drawings and technical illustrations[2], e.g., (Dooley and Cohen, 1990; Yessios, 1979); meanwhile a number of 2D computer paint programs were developed as well. Many of these papers argued for the potential virtues of hand-drawn styles in technical illustration.

In 1990, the flagship computer graphics conference SIGGRAPH held a session entitled "Non Photo Realistic Rendering," which seems to be the first usage of this term. In this session, two significant papers for the

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[2] Many works are being omitted from this history. A much more comprehensive bibliography, up to 2011, can be found here: https://www.npcglib.org.

field were presented. Saito and Takahashi (1990) introduced depth-buffer based line enhancements (Chapter 2), which started to create cartoon-like renderings of smooth objects by emphasizing contours and other feature curves. Haeberli (1990) introduced a range of artistic 2D image-processing effects; these papers together demonstrated a significant step forward in the quality and generality of non-photorealistic effects.

Winkenbach and Salesin (1994) demonstrated the first complete line-drawing algorithm from 3D models, including contours and hatching. Their work was seminal in that their method automatically produced beautiful results from 3D models; one could, for the first time, be fooled into thinking that these images were really drawn by hand. Perhaps even more importantly, their work provided a model for one could develop algorithms by careful study of artistic techniques in textbooks and illustrations.

Meier (1996) demonstrated the first research paper focusing on 3D non-photorealistic animation, describing the problem of temporal coherence for animation. Between the beautiful images of Winkenbach and Salesin (1994; 1996) and beautiful animations of Meier (1996), non-photorealistic rendering was firmly established as an important research direction.

Research activity at SIGGRAPH increased significantly, and the inaugural NPAR symposium on Non-Photorealistic Animation and Rendering met in 2000, sponsored by the Annecy Animation Festival in France and chaired by David Salesin and Jean-Daniel Fekete. Through the following decade, many improvements and extensions to the basic ideas were published, and, occasionally, techniques like toon shading and contour edges appeared in video games and movies. DeCarlo *et al.* (2003) described Suggestive Contours, which substantially improved the quality of line renderings, while making deep connections to perception and differential geometry, notably the work of Koenderink (1984). Several systems were created to help artists design artistic rendering styles, such as WYSIWYG NPR (Kalnins *et al.*, 2002) and a procedural NPR system called Freestyle (Grabli *et al.*, 2010). Cole *et al.* (2008; 2009) performed the scientific studies described in Section 1.5 demonstrating that line drawing algorithms were quite good at capturing how artists draw lines.

Since then, research in 3D non-photorealistic rendering has tapered off, despite the presence of several significant open problems. In contrast, interest in image stylization has recently exploded, due to developments in machine learning. Still, 3D non-photorealistic rendering continues to appear in a few games and movies here and there. This tutorial aims, in part, to summarize the field and highlight open problems, to help researchers and practitioners make progress in this field in order to enable them to be more widely used. We discuss future prospects for the field in the Conclusion (Chapter 10).

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