Random Matrix Theory and Wireless Communications

Random Matrix Theory and Wireless Communications

Antonia M. Tulino

Dept. Ingegneria Elettronica e delle Telecomunicazioni
Universitá degli Studi di Napoli "Federico II"
Naples 80125, Italy

atulino@ee.princeton.edu

Sergio Verdú

Dept. Electrical Engineering
Princeton University
Princeton, New Jersey 08544, USA

verdu@princeton.edu

now

the essence of knowledge

Boston – Delft

Foundations and Trends® in Communications and Information Theory

Published, sold and distributed by:
now Publishers Inc.
PO Box 1024
Hanover, MA 02339
USA
Tel. +1 781 871 0245
www.nowpublishers.com
sales@nowpublishers.com

Outside North America:
now Publishers Inc.
PO Box 179
2600 AD Delft
The Netherlands
Tel. +31-6-51115274

A Cataloging-in-Publication record is available from the Library of Congress

Printed on acid-free paper

ISBN: 1-933019-00-X; ISSNs: Paper version 1567-2190; Electronic version 1567-2328

Contents

1

Introduction

From its inception, random matrix theory has been heavily influenced by its applications in physics, statistics and engineering. The landmark contributions to the theory of random matrices of Wishart (1928) [311], Wigner (1955) [303], and Marčenko and Pastur (1967) [170] were motivated to a large extent by practical experimental problems. Nowadays, random matrices find applications in fields as diverse as the Riemann hypothesis, stochastic differential equations, condensed matter physics, statistical physics, chaotic systems, numerical linear algebra, neural networks, multivariate statistics, information theory, signal processing, and small-world networks. Despite the widespread applicability of the tools and results in random matrix theory, there is no tutorial reference that gives an accessible overview of the classical theory as well as the recent results, many of which have been obtained under the umbrella of free probability theory.

In the last few years, a considerable body of work has emerged in the communications and information theory literature on the fundamental limits of communication channels that makes substantial use of results in random matrix theory.

The purpose of this monograph is to give a tutorial overview of random matrix theory with particular emphasis on asymptotic theorems on the distribution of eigenvalues and singular values under various assumptions on the joint distribution of the random matrix entries. While results for matrices with fixed dimensions are often cumbersome and offer limited insight, as the matrices grow large with a given aspect ratio (number of columns to number of rows), a number of powerful and appealing theorems ensure convergence of the empirical eigenvalue distributions to deterministic functions.

The organization of this monograph is the following. Section 1.1 introduces the general class of vector channels of interest in wireless communications. These channels are characterized by random matrices that admit various statistical descriptions depending on the actual application. Section 1.2 motivates interest in large random matrix theory by focusing on two performance measures of engineering interest: Shannon capacity and linear minimum mean-square error, which are determined by the distribution of the singular values of the channel matrix. The power of random matrix results in the derivation of asymptotic closed-form expressions is illustrated for channels whose matrices have the simplest statistical structure: independent identically distributed (i.i.d.) entries. Section 1.3 gives a brief historical tour of the main results in random matrix theory, from the work of Wishart on Gaussian matrices with fixed dimension, to the recent results on asymptotic spectra. Chapter 2 gives a tutorial account of random matrix theory. Section 2.1 focuses on the major types of random matrices considered in the literature, as well on the main fixed-dimension theorems. Section 2.2 gives an account of the Stieltjes, η, Shannon, Mellin, R- and S-transforms. These transforms play key roles in describing the spectra of random matrices. Motivated by the intuition drawn from various applications in communications, the η and Shannon transforms turn out to be quite helpful at clarifying the exposition as well as the statement of many results. Considerable emphasis is placed on examples and closed-form expressions. Section 2.3 uses the transforms defined in Section 2.2 to state the main asymptotic distribution theorems. Section 2.4 presents an overview of the application of Voiculescu's free probability theory to random matrices. Recent results on the speed of convergence to the asymptotic limits are reviewed in Section 2.5. Chapter

3 applies the results in Chapter 2 to the fundamental limits of wireless communication channels described by random matrices. Section 3.1 deals with direct-sequence code-division multiple-access (DS-CDMA), with and without fading (both frequency-flat and frequency-selective) and with single and multiple receive antennas. Section 3.2 deals with multi-carrier code-division multiple access (MC-CDMA), which is the time-frequency dual of the model considered in Section 3.1. Channels with multiple receive and transmit antennas are reviewed in Section 3.3 using models that incorporate nonideal effects such as antenna correlation, polarization, and line-of-sight components.

1.1 Wireless Channels

The last decade has witnessed a renaissance in the information theory of wireless communication channels. Two prime reasons for the strong level of activity in this field can be identified. The first is the growing importance of the efficient use of bandwidth and power in view of the ever-increasing demand for wireless services. The second is the fact that some of the main challenges in the study of the capacity of wireless channels have only been successfully tackled recently. Fading, wideband, multiuser and multi-antenna are some of the key features that characterize wireless channels of contemporary interest. Most of the information theoretic literature that studies the effect of those features on channel capacity deals with linear vector memoryless channels of the form

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1.1}$$

where $\mathbf{x}$ is the K-dimensional input vector, $\mathbf{y}$ is the N-dimensional output vector, and the N-dimensional vector $\mathbf{n}$ models the additive circularly symmetric Gaussian noise. All these quantities are, in general, complex-valued. In addition to input constraints, and the degree of knowledge of the channel at receiver and transmitter, (1.1) is characterized by the distribution of the $N \times K$ random *channel matrix* $\mathbf{H}$ whose entries are also complex-valued.

The nature of the K and N dimensions depends on the actual application. For example, in the single-user narrowband channel with n_{T}

and n_{R} antennas at transmitter and receiver, respectively, we identify $K = n_{\mathrm{T}}$ and $N = n_{\mathrm{R}}$; in the DS-CDMA channel, K is the number of users and N is the spreading gain.

In the multi-antenna case, $\mathbf{H}$ models the propagation coefficients between each pair of transmit-receive antennas. In the synchronous DS-CDMA channel, in contrast, the entries of $\mathbf{H}$ depend on the received signature vectors (usually pseudo-noise sequences) and the fading coefficients seen by each user. For a channel with J users each transmitting with n_{T} antennas using spread-spectrum with spreading gain G and a receiver with n_{R} antennas, $K = n_{\mathrm{T}}J$ and $N = n_{\mathrm{R}}G$.

Naturally, the simplest example is the one where $\mathbf{H}$ has i.i.d. entries, which constitutes the canonical model for the single-user narrowband multi-antenna channel. The same model applies to the randomly spread DS-CDMA channel not subject to fading. However, as we will see, in many cases of interest in wireless communications the entries of $\mathbf{H}$ are not i.i.d.

1.2 The Role of the Singular Values

Assuming that the channel matrix $\mathbf{H}$ is completely known at the receiver, the capacity of (1.1) under input power constraints depends on the distribution of the singular values of $\mathbf{H}$. We focus in the simplest setting to illustrate this point as crisply as possible: suppose that the entries of the input vector $\mathbf{x}$ are i.i.d. For example, this is the case in a synchronous DS-CDMA multiaccess channel or for a single-user multi-antenna channel where the transmitter cannot track the channel.

The empirical cumulative distribution function of the eigenvalues (also referred to as the *spectrum* or empirical distribution) of an $n \times n$ Hermitian matrix $\mathbf{A}$ is denoted by $\mathsf{F}^n_{\mathbf{A}}$ defined as[1]

$$\mathsf{F}^n_{\mathbf{A}}(x) = \frac{1}{n}\sum_{i=1}^{n} 1\{\lambda_i(\mathbf{A}) \le x\}, \tag{1.2}$$

where $\lambda_1(\mathbf{A}), \ldots, \lambda_n(\mathbf{A})$ are the eigenvalues of $\mathbf{A}$ and $1\{\cdot\}$ is the indicator function.

[1] If $\mathsf{F}^n_{\mathbf{A}}$ converges as $n \to \infty$, then the corresponding limit (asymptotic empirical distribution or asymptotic spectrum) is simply denoted by $\mathsf{F}_{\mathbf{A}}(x)$.

Now, consider an arbitrary $N \times K$ matrix $\mathbf{H}$. Since the nonzero singular values of $\mathbf{H}$ and $\mathbf{H}^\dagger$ are identical, we can write

$$N\mathsf{F}^N_{\mathbf{HH}^\dagger}(x) - Nu(x) = K\mathsf{F}^K_{\mathbf{H}^\dagger\mathbf{H}}(x) - Ku(x) \tag{1.3}$$

where $u(x)$ is the unit-step function ($u(x) = 0$, $x \le 0$; $u(x) = 1$, $x > 0$).

With an i.i.d. Gaussian input, the normalized input-output mutual information of (1.1) conditioned on $\mathbf{H}$ is[2]

$$\frac{1}{N} I(\mathbf{x};\mathbf{y}|\mathbf{H}) = \frac{1}{N}\log\det\left(\mathbf{I} + \mathsf{SNR}\,\mathbf{HH}^\dagger\right) \tag{1.4}$$

$$= \frac{1}{N}\sum_{i=1}^{N}\log\left(1 + \mathsf{SNR}\,\lambda_i(\mathbf{HH}^\dagger)\right)$$

$$= \int_0^\infty \log\left(1 + \mathsf{SNR}\,x\right) d\mathsf{F}^N_{\mathbf{HH}^\dagger}(x) \tag{1.5}$$

with the transmitted signal-to-noise ratio (SNR)

$$\mathsf{SNR} = \frac{N\mathbb{E}[||\mathbf{x}||^2]}{K\mathbb{E}[||\mathbf{n}||^2]}, \tag{1.6}$$

and with $\lambda_i(\mathbf{HH}^\dagger)$ equal to the ith squared singular value of $\mathbf{H}$.

If the channel is known at the receiver and its variation over time is stationary and ergodic, then the expectation of (1.4) over the distribution of $\mathbf{H}$ is the channel capacity (normalized to the number of receive antennas or the number of degrees of freedom per symbol in the CDMA channel). More generally, the distribution of the random variable (1.4) determines the outage capacity (e.g. [22]).

Another important performance measure for (1.1) is the minimum mean-square-error (MMSE) achieved by a linear receiver, which determines the maximum achievable output signal-to-interference-and-noise

[2] The celebrated log-det formula has a long history: In 1964, Pinsker [204] gave a general log-det formula for the mutual information between jointly Gaussian random vectors but did not particularize it to the linear model (1.1). Verdú [270] in 1986 gave the explicit form (1.4) as the capacity of the synchronous DS-CDMA channel as a function of the signature vectors. The 1991 textbook by Cover and Thomas [47] gives the log-det formula for the capacity of the power constrained vector Gaussian channel with arbitrary noise covariance matrix. In the mid 1990s, Foschini [77] and Telatar [250] gave (1.4) for the multi-antenna channel with i.i.d. Gaussian entries. Even prior to those works, the conventional analyses of Gaussian channels with memory via vector channels (e.g. [260, 31]) used the fact that the capacity can be expressed as the sum of the capacities of independent channels whose signal-to-noise ratios are governed by the singular values of the channel matrix.

ratio (SINR). For an i.i.d. input, the arithmetic mean over the users (or transmit antennas) of the MMSE is given, as function of **H**, by [271]

$$\frac{1}{K}\min_{\mathbf{M}\in C^{K\times N}} \mathbb{E}\left[||\mathbf{x}-\mathbf{M}\mathbf{y}||^2\right] = \frac{1}{K}\mathrm{tr}\left\{\left(\mathbf{I}+\mathsf{SNR}\,\mathbf{H}^\dagger\mathbf{H}\right)^{-1}\right\} \quad (1.7)$$

$$= \frac{1}{K}\sum_{i=1}^{K}\frac{1}{1+\mathsf{SNR}\,\lambda_i(\mathbf{H}^\dagger\mathbf{H})} \quad (1.8)$$

$$= \int_0^\infty \frac{1}{1+\mathsf{SNR}\,x}\,d\mathsf{F}^K_{\mathbf{H}^\dagger\mathbf{H}}(x)$$

$$= \frac{N}{K}\int_0^\infty \frac{1}{1+\mathsf{SNR}\,x}\,d\mathsf{F}^N_{\mathbf{H}\mathbf{H}^\dagger}(x) - \frac{N-K}{K} \quad (1.9)$$

where the expectation in (1.7) is over **x** and **n** while (1.9) follows from (1.3). Note, incidentally, that both performance measures as a function of SNR are coupled through

$$\mathsf{SNR}\,\frac{d}{d\mathsf{SNR}}\log_e\det\left(\mathbf{I}+\mathsf{SNR}\,\mathbf{H}\mathbf{H}^\dagger\right) = K - \mathrm{tr}\left\{\left(\mathbf{I}+\mathsf{SNR}\,\mathbf{H}^\dagger\mathbf{H}\right)^{-1}\right\}.$$

As we see in (1.5) and (1.9), both fundamental performance measures (capacity and MMSE) are dictated by the distribution of the empirical (squared) singular value distribution of the random channel matrix. In the simplest case of **H** having i.i.d. Gaussian entries, the density function corresponding to the expected value of $\mathsf{F}^N_{\mathbf{H}\mathbf{H}^\dagger}$ can be expressed explicitly in terms of the Laguerre polynomials. Although the integrals in (1.5) and (1.9) with respect to such a probability density function (p.d.f.) lead to explicit solutions, limited insight can be drawn from either the solutions or their numerical evaluation. Fortunately, much deeper insights can be obtained using the tools provided by *asymptotic* random matrix theory. Indeed, a rich body of results exists analyzing the asymptotic spectrum of **H** as the number of columns and rows goes to infinity while the aspect ratio of the matrix is kept constant.

Before introducing the asymptotic spectrum results, some justification for their relevance to wireless communication problems is in order. In CDMA, channels with K and N between 32 and 64 would be fairly typical. In multi-antenna systems, arrays of 8 to 16 antennas would be at the forefront of what is envisioned to be feasible in the foreseeable future. Surprisingly, even quite smaller system sizes are large enough for the asymptotic limit to be an excellent approximation. Furthermore, not only do the averages of (1.4) and (1.9) converge to their limits surprisingly fast, but the randomness in those functionals due to the random outcome of **H** disappears extremely quickly. Naturally, such robustness has welcome consequences for the operational significance of the resulting formulas.

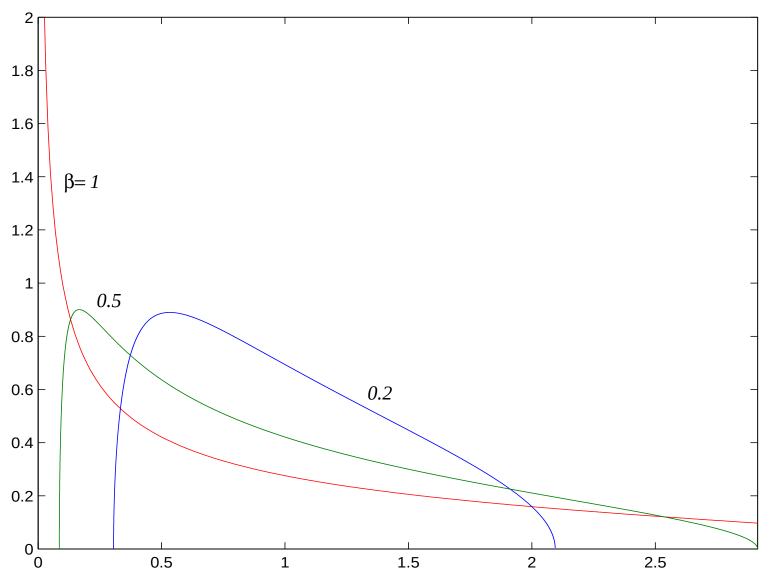

Fig. 1.1 The Marčenko-Pastur density function (1.10) for $\beta = 1, 0.5, 0.2$.

As we will see in Chapter 2, a central result in random matrix theory states that when the entries of **H** are zero-mean i.i.d. with variance $\frac{1}{N}$, the empirical distribution of the eigenvalues of $\mathbf{H}^\dagger\mathbf{H}$ converges almost surely, as $K, N \to \infty$ with $\frac{K}{N} \to \beta$, to the so-called Marčenko-Pastur law whose density function is

$$\mathsf{f}_\beta(x) = \left(1 - \frac{1}{\beta}\right)^+ \delta(x) + \frac{\sqrt{(x-a)^+(b-x)^+}}{2\pi\beta x} \tag{1.10}$$

where $(z)^+ = \max(0, z)$ and

$$a = (1 - \sqrt{\beta})^2 \qquad\qquad b = (1 + \sqrt{\beta})^2. \tag{1.11}$$

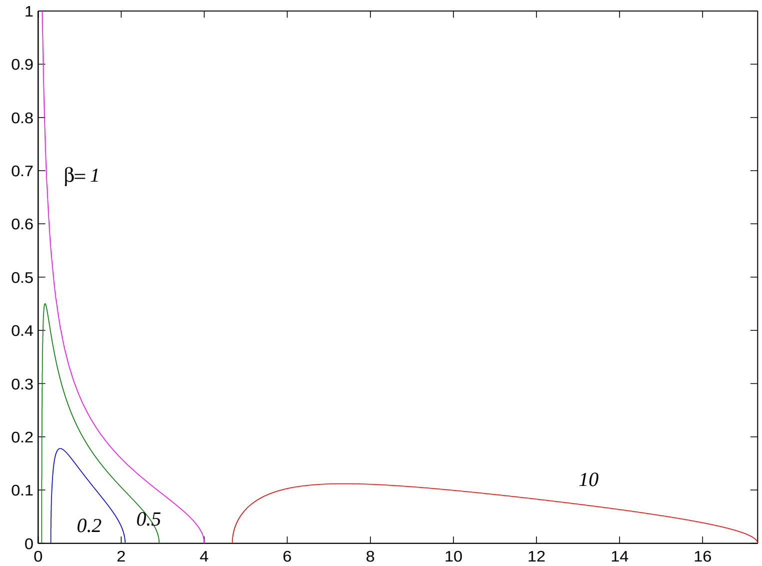

Fig. 1.2 The Marčenko-Pastur density function (1.12) for $\beta = 10, 1, 0.5, 0.2$. Note that the mass points at 0, present in some of them, are not shown.

Analogously, the empirical distribution of the eigenvalues of $\mathbf{H}\mathbf{H}^\dagger$ converges almost surely to a nonrandom limit whose density function is (cf. Fig. 1.2)

$$\begin{aligned} \tilde{\mathsf{f}}_\beta(x) &= (1-\beta)\,\delta(x) + \beta\,\mathsf{f}_\beta(x) \\ &= (1-\beta)^+\,\delta(x) + \frac{\sqrt{(x-a)^+(b-x)^+}}{2\pi x}. \end{aligned} \tag{1.12}$$

Using the asymptotic spectrum, the following closed-form expressions for the limits of (1.4) [275] and (1.7) [271] can be obtained:

$$(1.13)$$

$$\begin{aligned} \frac{1}{N}\log\det\left(\mathbf{I} + \mathsf{SNR}\,\mathbf{H}\mathbf{H}^\dagger\right) &\rightarrow \beta\int_a^b \log(1+\mathsf{SNR}\;x)\mathsf{f}_\beta(x)\;dx \\ &= \beta\log\left(1+\mathsf{SNR} - \frac{1}{4}\mathcal{F}\left(\mathsf{SNR}\,,\beta\right)\right) \\ &+ \log\left(1+\mathsf{SNR}\,\beta - \frac{1}{4}\mathcal{F}\left(\mathsf{SNR}\,,\beta\right)\right) \\ &- \frac{\log e}{4\,\mathsf{SNR}}\mathcal{F}\left(\mathsf{SNR}\,,\beta\right) \end{aligned} \tag{1.14}$$

$$\frac{1}{K}\mathrm{tr}\left\{\left(\mathbf{I}+\mathsf{SNR}\,\mathbf{H}^{\dagger}\mathbf{H}\right)^{-1}\right\} \rightarrow \int_a^b \frac{1}{1+\mathsf{SNR}\;x}\mathsf{f}_{\beta}(x)\,dx \quad (1.15)$$

$$= 1-\frac{\mathcal{F}(\mathsf{SNR}\,,\beta)}{4\,\beta\,\mathsf{SNR}} \quad (1.16)$$

with

$$\mathcal{F}(x,z)=\left(\sqrt{x(1+\sqrt{z})^2+1}-\sqrt{x(1-\sqrt{z})^2+1}\right)^2. \quad (1.17)$$

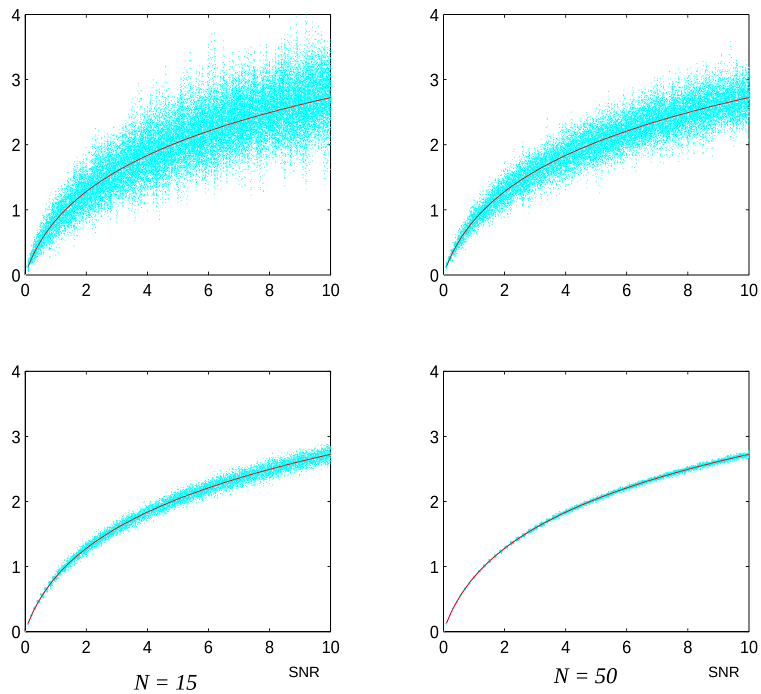

Fig. 1.3 Several realizations of the left-hand side of (1.13) are compared to the asymptotic limit in the right-hand side of (1.13) in the case of $\beta = 1$ for sizes: $N = 3, 5, 15, 50$.

The convergence of the singular values of $\mathbf{H}$ exhibits several key features with engineering significance:

- *Insensitivity* of the asymptotic eigenvalue distribution to the shape of the p.d.f. of the random matrix entries. This property implies, for example, that in the case of a single-user multi-antenna link, the results obtained asymptotically hold for any type of fading statistics. It also implies that restricting the CDMA waveforms to be binary-valued incurs no loss in capacity asymptotically.[3]

- *Ergodic* behavior: it suffices to observe a single matrix realization in order to obtain convergence to a deterministic limit. In other words, the eigenvalue histogram of any matrix realization converges almost surely to the average asymptotic eigenvalue distribution. This "hardening" of the singular values lends operational significance to the capacity formulas even in cases where the random channel parameters do not vary ergodically within the span of a codeword.
- *Fast convergence* of the empirical singular-value distribution to its asymptotic limit. Asymptotic analysis is especially useful when the convergence is so fast that, even for small values of the parameters, the asymptotic results come close to the finite-size results (cf. Fig. 1.3). Recent works have shown that the convergence rate is of the order of the reciprocal of the number of entries in the random matrix [8, 110].

It is crucial for the explicit expressions of asymptotic capacity and MMSE shown in (1.14) and (1.16), respectively, that the channel matrix entries be i.i.d. Outside that model, explicit expressions for the asymptotic singular value distribution such as (1.10) are exceedingly rare. Fortunately, in other random matrix models, the asymptotic singular value distribution can indeed be characterized, albeit not in explicit form, in ways that enable the analysis of capacity and MMSE through the numerical solution of nonlinear equations.

The first applications of random matrix theory to wireless communications were the works of Foschini [77] and Telatar [250] on narrowband multi-antenna capacity; Verdú [271] and Tse-Hanly [256] on the optimum SINR achievable by linear multiuser detectors for CDMA; Verdú [271] on optimum near-far resistance; Grant-Alexander [100],

[3] The spacing between consecutive eigenvalues, when properly normalized, was conjectured in [65, 66] to converge in distribution to a limit that does not depend on the shape of the p.d.f. of the entries. The universality of the level spacing distribution and other *microscopic (local)* spectral characteristics has been extensively discussed in recent theoretical physics and mathematical literature [174, 106, 200, 52, 54].

Verdú-Shamai [275, 217], Rapajic-Popescu [206], and Müller [185] on the capacity of CDMA. Subsequently, a number of works, surveyed in Chapter 3, have successfully applied random matrix theory to a variety of problems in the design and analysis of wireless communication systems.

Not every result of interest in the asymptotic analysis of channels of the form (1.1) has made use of the asymptotic eigenvalue tools that are of central interest in this paper. For example, the analysis of single-user matched filter receivers [275] and the analysis of the optimum asymptotic multiuser efficiency [258] have used various versions of the central-limit theorem; the analysis of the asymptotic uncoded error probability as well as the rates achievable with suboptimal constellations have used tools from statistical physics such as the replica method [249, 103].

1.3 Random Matrices: A Brief Historical Account

In this subsection, we provide a brief introduction to the main developments in the theory of random matrices. A more detailed account of the theory itself, with particular emphasis on the results that are relevant for wireless communications, is given in Chapter 2.

Random matrices have been a part of advanced multivariate statistical analysis since the end of the 1920s with the work of Wishart [311] on fixed-size matrices with Gaussian entries. The first asymptotic results on the limiting spectrum of large random matrices were obtained by Wigner in the 1950s in a series of papers [303, 305, 306] motivated by nuclear physics. Replacing the self-adjoint Hamiltonian operator in an infinite-dimensional Hilbert space by an ensemble of very large Hermitian matrices, Wigner was able to bypass the Schrödinger equation and explain the statistics of experimentally measured atomic energy levels in terms of the limiting spectrum of those random matrices. Since then, research on the limiting spectral analysis of large-dimensional random matrices has continued to attract interest in probability, statistics and physics.

Wigner [303] initially dealt with an $n \times n$ symmetric matrix $\mathbf{A}$ whose diagonal entries are 0 and whose upper-triangle entries are independent and take the values ± 1 with equal probability. Through a combinatorial

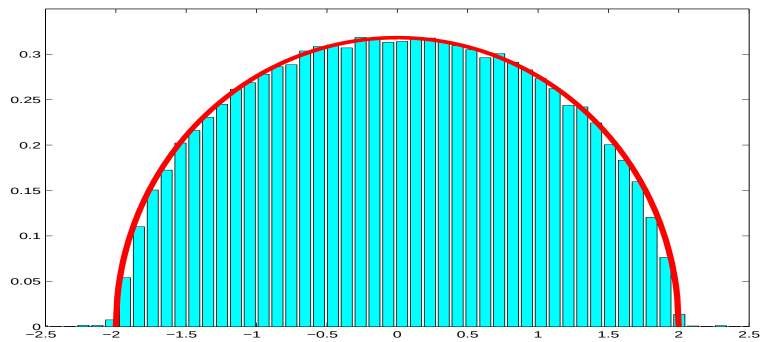

Fig. 1.4 The semicircle law density function (1.18) compared with the histogram of the average of 100 empirical density functions for a Wigner matrix of size $n = 100$.

derivation of the asymptotic eigenvalue moments involving the Catalan numbers, Wigner showed that, as $n \to \infty$, the averaged empirical distribution of the eigenvalues of $\frac{1}{\sqrt{n}}\mathbf{A}$ converges to the *semicircle law* whose density is

$$w(x) = \begin{cases} \frac{1}{2\pi}\sqrt{4-x^2} & \text{if} \quad |x| \leq 2 \\ 0 & \text{if} \quad |x| > 2 \end{cases} \tag{1.18}$$

Later, Wigner [305] realized that the same result would be obtained if the random selection was sampled from a zero-mean (real or complex) Gaussian distribution. In that case, it is even possible to find an exact formula for the joint distribution of the eigenvalues as a function of n [176]. The matrices treated in [303] and [305] are special cases of Wigner matrices, defined as Hermitian matrices whose upper-triangle entries are zero-mean and independent. In [306], Wigner showed that the asymptotic distribution of any Wigner matrix is the semicircle law (1.18) even if only a unit second-moment condition is placed on its entries.

Figure 1.4 compares the semicircle law density function (1.18) with the average of 100 empirical density functions of the eigenvalues of a 10×10 Wigner matrix whose diagonal entries are 0 and whose upper-triangle entries are independent and take the values ± 1 with equal probability.

If no attempt is made to symmetrize the square matrix $\mathbf{A}$ and all its entries are chosen to be i.i.d., then the eigenvalues of $\frac{1}{\sqrt{n}}\mathbf{A}$ are asymptotically uniformly distributed on the unit circle of the complex plane. This is commonly referred to as Girko's *full-circle law*, which is exemplified in Figure 1.5. It has been proved in various degrees of rigor and generality in [173, 197, 85, 68, 9]. If the off-diagonal entries $A_{i,j}$ and $A_{j,i}$ are Gaussian and pairwise correlated with correlation coefficient ρ, then [238] shows that the eigenvalues of $\frac{1}{\sqrt{n}}\mathbf{A}$ are asymptotically uniformly distributed on an ellipse in the complex plane whose axes coincide with the real and imaginary axes and have radius $1+\rho$ and $1-\rho$, respectively. When $\rho = 1$, the projection on the real axis of such *elliptic law* is equal to the semicircle law.

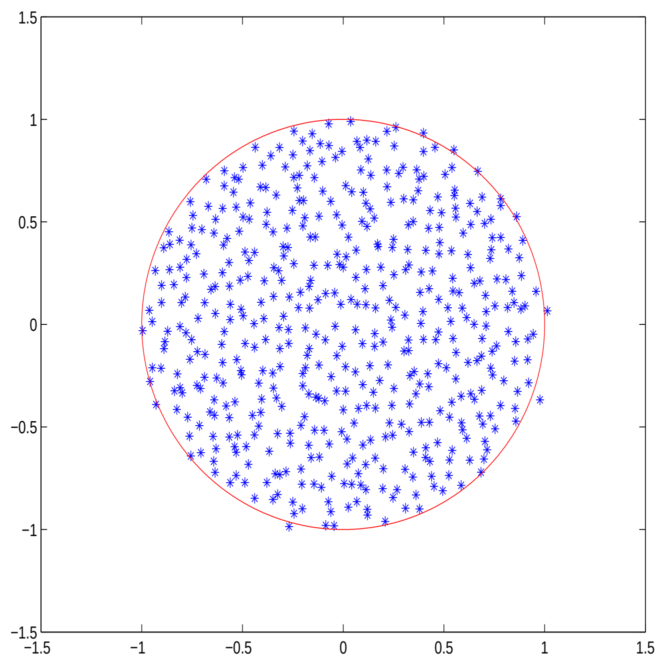

Fig. 1.5 The full-circle law and the eigenvalues of a realization of a matrix of size $n = 500$.

Most of the results surveyed above pertain to the eigenvalues of square matrices with independent entries. However, as we saw in Section 1.2, key problems in wireless communications involve the singular values of rectangular matrices $\mathbf{H}$; even if those matrices have independent entries, the matrices $\mathbf{H}\mathbf{H}^\dagger$ whose eigenvalues are of interest do not have independent entries.

When the entries of $\mathbf{H}$ are zero-mean i.i.d. Gaussian, $\mathbf{H}\mathbf{H}^\dagger$ is commonly referred to as a Wishart matrix. The analysis of the joint distribution of the entries of Wishart matrices is as old as random matrix theory itself [311]. The joint distribution of the eigenvalues of such matrices is known as the Fisher-Hsu-Roy distribution and was discovered simultaneously and independently by Fisher [75], Hsu [120], Girshick [89] and Roy [210]. The corresponding marginal distributions can be expressed in terms of the Laguerre polynomials [125].

The asymptotic theory of singular values of rectangular matrices has concentrated on the case where the matrix aspect ratio converges to a constant

$$\frac{K}{N} \to \beta \tag{1.19}$$

as the size of the matrix grows.

The first success in the quest for the limiting empirical singular value distribution of rectangular random matrices is due to Marčenko and Pastur [170] in 1967. This landmark paper considers matrices of the form

$$\mathbf{W} = \mathbf{W}_0 + \mathbf{H}\mathbf{T}\mathbf{H}^\dagger \tag{1.20}$$

where $\mathbf{T}$ is a real diagonal matrix independent of $\mathbf{H}$, $\mathbf{W}_0$ is a deterministic Hermitian matrix, and the columns of the $N \times K$ matrix $\mathbf{H}$ are i.i.d. random vectors whose distribution satisfies a certain symmetry condition (encompassing the cases of independent entries and uniform distribution on the unit sphere). In the special case where $\mathbf{W}_0 = \mathbf{0}$, $\mathbf{T} = \mathbf{I}$, and $\mathbf{H}$ has i.i.d. entries with variance $\frac{1}{N}$, the limiting spectrum of $\mathbf{W}$ found in [170] is the density in (1.10). In the special case of square $\mathbf{H}$, the asymptotic density function of the singular values, corresponding to the square root of the random variable whose p.d.f. is (1.10) with $\beta = 1$, is equal to the *quarter circle* law:

$$q(x) = \frac{1}{\pi}\sqrt{4 - x^2}, \quad 0 \le x \le 2. \tag{1.21}$$

As we will see in Chapter 2, in general ($\mathbf{W}_0 \neq \mathbf{0}$ or $\mathbf{T} \neq \mathbf{I}$) no closed-form expression is known for the limiting spectrum. Rather, [170] characterized it indirectly through its Stieltjes transform,[4] which uniquely determines the distribution function. Since [170], this transform, which can be viewed as an iterated Laplace transform, has played a fundamental role in the theory of random matrices.

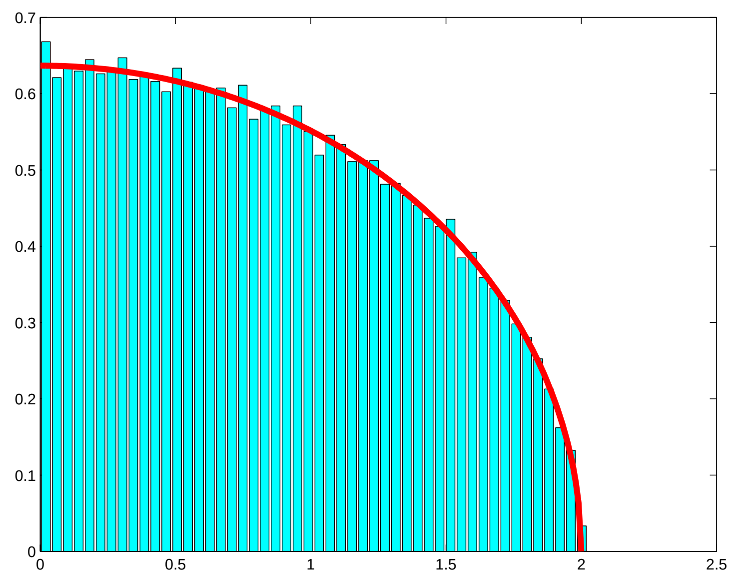

Fig. 1.6 The quarter circle law compared a histogram of the average of 100 empirical singular value density functions of a matrix of size 100×100.

Figure 1.6 compares the quarter circle law density function (1.21) with the average of 100 empirical singular value density functions of a 100×100 square matrix $\mathbf{H}$ with independent zero-mean complex Gaussian entries with variance $\frac{1}{100}$.

Despite the ground-breaking nature of Marčenko and Pastur's contribution, it remained in obscurity for quite some time. For example, in 1977 Grenander and Silverstein [101] rediscovered (1.10) motivated by a neural network problem where the entries of $\mathbf{H}$ take only two values. Also unaware of the in-probability convergence result of [170], in 1978 Wachter [296] arrived at the same solution but in the stronger sense of almost sure convergence under the condition that the entries of $\mathbf{H}$ have

[4] The Stieltjes transform is defined in Section 2.2.1. The Dutch mathematician T. J. Stieltjes (1856-1894) provided the first inversion formula for this transform in [246].

uniformly bounded central moments of order higher than 2 as well as the same means and variances within a row. The almost sure convergence for the model (1.20) considered in [170] was shown in [227]. Even as late as 1991, rediscoveries of the Marčenko-Pastur law can be found in the Physics literature [50].

The case where $\mathbf{W} = \mathbf{0}$ in (1.20), $\mathbf{T}$ is not necessarily diagonal but Hermitian and $\mathbf{H}$ has i.i.d. entries was solved by Silverstein [226] also in terms of the Stieltjes transform.

The special case of (1.20) where $\mathbf{W_0} = \mathbf{0}$, $\mathbf{H}$ has zero-mean i.i.d. Gaussian entries and

$$\mathbf{T} = (\mathbf{Y}\mathbf{Y}^\dagger)^{-1}$$

where the $K \times m$ matrix $\mathbf{Y}$ has also zero-mean i.i.d. Gaussian entries with variance $\frac{1}{m}$, independent of $\mathbf{H}$, is called a (central) multivariate F-matrix. Because of the statistical applications of such matrix, its asymptotic spectrum has received considerable attention culminating in the explicit expression found by Silverstein [223] in 1985.

The speed of convergence to the limiting spectrum is studied in [8]. For our applications it is more important, however, to assess the speed of convergence of the performance measures (e.g. capacity and MMSE) to their asymptotic limits. Note that the sums in the right side of (1.4) involve dependent terms. Thanks to that dependence, the convergence in (1.13) and (1.15) is quite remarkable: the deviations from the respective limits *multiplied by* N converge to Gaussian random variables with fixed mean[5] and variance. This has been established for general continuous functions, not just the logarithmic and rational functions of (1.13) and (1.15), in [15] (see also [131]).

The matrix of eigenvectors of Wishart matrices is known to be uniformly distributed on the manifold of unitary matrices (the so-called *Haar measure*) (e.g. [125, 67]). In the case of $\mathbf{H}\mathbf{H}^\dagger$ where $\mathbf{H}$ has i.i.d. non-Gaussian entries, much less success has been reported in the asymptotic characterization of the eigenvectors [153, 224, 225].

For matrices whose entries are Gaussian and correlated according to a Toeplitz structure, an integral equation is known for the Stielt-

[5] The mean is zero in the interesting special case where $\mathbf{H}$ has i.i.d. complex Gaussian entries [15].

jes transform of the asymptotic spectrum as a function of the Fourier transform of the correlation function [147, 198, 55]. Other results on random matrices with correlated and weakly dependent entries can be found in [170, 196, 146, 53, 199, 145]. Reference [191], in turn, considers a special class of random matrices with dependent entries that falls outside the Marčenko-Pastur framework and that arises in the context of the statistical physics of disordered systems.

Incidentally, another application of the Stieltjes transform approach is the generalization of Wigner's semicircle law to the sum of a Wigner matrix and a deterministic Hermitian matrix. Provided Lindeberg-type conditions are satisfied by the entries of the random component, [147] obtained the *deformed semicircle law*, which is only known in closed-form in the Stieltjes transform domain.

Sometimes, an alternative to the characterization of asymptotic spectra through the Stieltjes transform is used, based on the proof of convergence and evaluation of moments such as $\frac{1}{N}\mathrm{tr}\{(\mathbf{H}\mathbf{H}^\dagger)^k\}$. For most cases of practical interest, the limiting spectrum has bounded support. Thus, the moment convergence theorem can be applied to obtain results on the limiting spectrum through its moments [297, 314, 315, 313].

An important recent development in asymptotic random matrix analysis has been the realization that the non-commutative *free probability theory* introduced by Voiculescu [283, 285] in the mid-1980s is applicable to random matrices. In free probability, the classical notion of independence of random variables is replaced by that of "freeness" or "free independence".

The power of the concept of free random matrices is best illustrated by the following setting. In general, we cannot find the eigenvalues of the sums of random matrices from the eigenvalues of the individual matrices (unless they have the same eigenvectors), and therefore the asymptotic spectrum of the sum cannot be obtained from the individual asymptotic spectra. An obvious exception is the case of independent diagonal matrices in which case the spectrum of the sum is simply the convolution of the spectra. When the random matrices are asymptotically free [287], the asymptotic spectrum of the sum is also obtainable from the individual asymptotic spectra. Instead of convolution (or equivalently, summing the logarithms of the individual Fourier transforms), the "free convolution" is obtained through the sum of the so-called R-transforms introduced by Voiculescu [285]. Examples of asymptotically free random matrices include independent Gaussian random matrices, and $\mathbf{A}$ and $\mathbf{UBU}^*$ where $\mathbf{A}$ and $\mathbf{B}$ are Hermitian and $\mathbf{U}$ is uniformly distributed on the manifold of unitary matrices and independent of $\mathbf{A}$ and $\mathbf{B}$.

In free probability, the role of the Gaussian distribution in classical probability is taken by the semicircle law (1.18) in the sense of the free analog of the central limit theorem [284]: the spectrum of the normalized sum of free random matrices (with given spectrum) converges to the semicircle law (1.18). Analogously, the spectrum of the normalized sum of free random matrices with unit rank converges to the Marčenko-Pastur law (1.10), which then emerges as the free counterpart of the Poisson distribution [239, 295]. In the general context of free random variables, Voiculescu has found an elegant definition of free-entropy [288, 289, 291, 292, 293]. A number of structural properties have been shown for free-entropy in the context of non-commutative probability theory (including the counterpart of the entropy-power inequality [248]). The free counterpart to Fisher's information has been investigated in [289]. However, a free counterpart to the divergence between two distributions is yet to be discovered.

A connection between random matrices and information theory was made by Balian [17] in 1968 considering the inverse problem in which the distribution of the entries of the matrix must be determined while being consistent with certain constraints. Taking a maximum entropy method, the ensemble of Gaussian matrices is the solution to the problem where only a constraint on the energy of the singular values is placed.

References

[1] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions.* Dover Publications, Inc., New York, 1964.
[2] G. Alfano, A. Lozano, A. M. Tulino, and S. Verdú, "Capacity of MIMO channels with one-sided correlation," in *Proc. IEEE Int. Symp. on Spread Spectrum Tech. and Applications (ISSSTA'04)*, Aug. 2004.
[3] G. Alfano, A. Lozano, A. M. Tulino, and S. Verdú, "Mutual information and eigenvalue distribution of MIMO Ricean channels," in *Proc. IEEE Int. Symp. Information Theory & Applications (ISITA'04)*, (Parma, Italy), Oct. 2004.
[4] J. B. Andersen, "Array gain and capacity for known random channels with multiple element arrays at both ends," *IEEE J. on Selected Areas in Communications*, vol. 18, pp. 2172–2178, Nov. 2000.
[5] T. W. Anderson, "The non-central Wishart distribution and certain problems of multivariate statistics," *Annals of Math. Statistics*, vol. 17, no. 4, pp. 409–431, 1946.
[6] M. Andréief, "Note sur une relation entre les intégrals définies des produits des fonctions," *Mém. de la Soc. Sci. de Bordeaux*, vol. 2, pp. 1–14, 1883.
[7] O. Axelsson, *Iterative Solution Methods.* Cambridge University Press, 1994.
[8] Z. D. Bai, "Convergence rate of expected spectral distributions of large random matrices. Part I: Wigner matrices," *Annals of Probability*, vol. 21, no. 2, pp. 625–648, 1993.
[9] Z. D. Bai, "The circle law," *Annals of Probability*, vol. 25, pp. 494–529, 1997.
[10] Z. D. Bai, "Methodologies in spectral analysis of large dimensional random matrices," *Statistica Sinica*, vol. 9, no. 3, pp. 611–661, 1999.

[11] Z. D. Bai, B. Miao, and J. Tsay, "Convergence rates of spectral distributions of large Wigner matrices," *Int. Mathematical Journal*, vol. 1, no. 1, pp. 65–90, 2002.

[12] Z. D. Bai, B. Miao, and J. Yao, "Convergence rates of spectral distributions of large sample covariance matrices," *SIAM J. of Matrix Analysis and Applications*, vol. 25, no. 1, pp. 105–127, 2003.

[13] Z. D. Bai and J. W. Silverstein, "No eigenvalues outside the support of the limiting spectral distribution of large dimensional sample covariance matrices," *Annals of Probability*, vol. 26, pp. 316–345, 1998.

[14] Z. D. Bai and J. W. Silverstein, "Exact separation of eigenvalues of large dimensional sample covariance matrices," *Annals of Probability*, vol. 27, no. 3, pp. 1536–1555, 1999.

[15] Z. D. Bai and J. W. Silverstein, "CLT of linear spectral statistics of large dimensional sample covariance matrices," *Annals of Probability*, vol. 32, no. 1A, pp. 553–605, 2004.

[16] Z. D. Bai and Y. Q. Yin, "Convergence to the semicircle law," *Annals of Probability*, vol. 16, no. 2, pp. 863–875, 1988.

[17] R. Balian, "Random matrices in information theory," *Il Nuovo Cimento*, pp. 183–193, Sep. 1968.

[18] M. S. Barlett, "On the theory of statistical regression," in *Proc. R. Soc. Edinb. 53*, pp. 260–283, 1933.

[19] S. A. Bergmann and H. W. Arnold, "Polarization diversity in portable communications environment," *IEE Electronics Letters*, vol. 22, pp. 609–610, May 1986.

[20] P. Biane, "Minimal factorizations of cycle and central multiplicative functions on the infinite symmetric group," *J. Combinatorial Theory*, vol. A 76, no. 2, pp. 197–212, 1996.

[21] P. Biane, "Free probability for probabilists," *Report MSRI (http://arXiv.org/abs/math/9809193)*, Sep. 1998.

[22] E. Biglieri, J. Proakis, and S. Shamai, "Fading channels: Information-theoretic and communications aspects," *IEEE Trans. on Information Theory*, vol. 44, pp. 2619–2692, Oct. 1998.

[23] E. Biglieri and G. Taricco, "Transmission and reception with multiple antennas: Theoretical foundations," *submitted to Foundations and Trends in Communications and Information Theory*, 2004.

[24] E. Biglieri and G. Taricco, "Large-system analyses of multiple-antenna system capacities," *Journal of Communications and Networks*, vol. 5, No. 2, p. 5764, June 2003.

[25] E. Biglieri, G. Taricco, and A. M. Tulino, "Performance of space-time codes for a large number of antennas," *IEEE Trans. on Information Theory*, vol. 48, pp. 1794–1803, July 2002.

[26] E. Biglieri, A. M. Tulino, and G. Taricco, "How far away is infinity? Using asymptotic analyses in multiple antenna systems," *Proc. IEEE Int. Symp. on Spread Spectrum Techn. and Applications (ISSSTA'02)*, vol. 1, pp. 1–6, Sep. 2002.

[27] G. Birkhoff and S. Gulati, "Isotropic distributions of test matrices," *J. Appl. Math. Physics*, vol. 30, pp. 148–157, 1979.

[28] R. S. Blum, "MIMO capacity with interference," *IEEE J. on Selected Areas in Communications*, vol. 21, pp. 793–801, June 2003.

[29] P. Bougerol and J. Lacroix, *Random Products of Matrices with Applications to Schrödinger Operators.* Basel, Switzerland: Birkhuser, 1985.

[30] J. Boutros and G. Caire, "Iterative multiuser joint decoding: unified framework and asymptotic analysis," *IEEE Trans. on Information Theory*, vol. 48, pp. 1772–1793, July 2002.

[31] L. H. Brandenburg and A. D. Wyner, "Capacity of the Gaussian channel with memory: The multivariate case," *Bell System Technical Journal*, vol. 53, pp. 745–778, May-June 1974.

[32] B. Bronk, "Exponential ensembles for random matrices," *J. of Math. Physics*, vol. 6, pp. 228–237, 1965.

[33] R. Buehrer and B. Woerner, "The asymptotic multiuser efficiency of m-stage interference cancellation receivers," in *Proc. IEEE Int. Symp. on Personal, Indoor and Mobile Radio Communications (PIMRC'97)*, (Helsinki, Finland), pp. 570–574, Sep. 1997.

[34] K. A. Byerly and R. A. Roberts, "Output power based partial adaptive array design," in *Proc. Asilomar Conf. on Signals, Systems and Computers*, (Pacific Grove, CA), pp. 576–580, Oct. 1989.

[35] G. Caire and S. Shamai, "On achievable rates in a multi-antenna broadcast downlink," in *Proc. Allerton Conf. on Communications, Control and Computing*, pp. 1188–1193, Oct. 2000.

[36] G. Caire and S. Shamai, "On the achievable throughput of a multi-antenna Gaussian broadcast channel," *IEEE Trans. on Information Theory*, vol. 49, no. 7, pp. 1691–1706, 2003.

[37] J. M. Chaufray, W. Hachem, and P. Loubaton, "Asymptotic analysis of optimum and sub-optimum CDMA MMSE receivers," *Proc. IEEE Int. Symp. on Information Theory (ISIT'02)*, p. 189, July 2002.

[38] M. Chiani, "Evaluating the capacity distribution of MIMO Rayleigh fading channels," *Proc. IEEE Int. Symp. on Advances in Wireless Communications*, pp. 3–4, Sep. 23-24 2002, Victoria, Canada.

[39] M. Chiani, M. Z. Win, and A. Zanella, "On the capacity of spatially correlated MIMO Rayleigh-fading channels," *IEEE Trans. on Information Theory*, vol. 49, pp. 2363–2371, Oct. 2003.

[40] D. Chizhik, F. R. Farrokhi, J. Ling, and A. Lozano, "Effect of antenna separation on the capacity of BLAST in correlated channels," *IEEE Communications Letters*, vol. 4, pp. 337–339, Nov. 2000.

[41] D. Chizhik, J. Ling, P. Wolniansky, R. A. Valenzuela, N. Costa, and K. Huber, "Multiple-input multiple-output measurements and modelling in Manhattan," *IEEE J. on Selected Areas in Communications*, vol. 21, pp. 321–331, Apr. 2003.

[42] T.-S. Chu and L. J. Greenstein, "A semiempirical representation of antenna diversity gain at cellular and PCS base stations," *IEEE Trans. on Communications*, vol. 45, pp. 644–656, June 1997.

[43] C. Chuah, D. Tse, J. Kahn, and R. Valenzuela, "Capacity scaling in dual-antenna-array wireless systems," *IEEE Trans. on Information Theory*, vol. 48, pp. 637–650, March 2002.

[44] J. Cohen, H. Kesten, and M. Newman, "Oseledec's multiplicative ergodic theorem: a proof," in *Random matrices and their applications*, (J. Cohen, H. Kesten, and M. Newman, eds.), Providence, RI: American Mathematical Society, 1986.

[45] L. Cottatellucci and R. R. Müller, "Asymptotic design and analysis of multistage detectors with unequal powers," in *Proc. IEEE Information Theory Workshop (ITW'02)*, (Bangalore, India), Oct. 2002.

[46] L. Cottatellucci and R. R. Müller, "Asymptotic design and analysis of multistage detectors and multistage channel estimators for multipath fading channels," in *Proc. IEEE Int. Symp. on Information Theory (ISIT'02)*, (Yokohama, Japan), June 2003.

[47] T. M. Cover and J. A. Thomas, *Elements of Information Theory.* John Wiley and Sons, Inc., 1991.

[48] D. Cox, "Universal digital portable radio communications," *Proc. IEEE*, vol. 75, No. 4, pp. 436–477, Apr. 1987.

[49] I. Csiszár and J. Körner, *Information Theory: Coding Theorems for Discrete Memoryless Systems.* New York: Academic, 1981.

[50] Y. L. Cun, I. Kanter, and S. A. Solla, "Eigenvalues of covariance matrices: Application to neural-network learning," *Physical Review Letters*, vol. 66, pp. 2396–2399, 1991.

[51] H. Dai, A. F. Molisch, and H. V. Poor, "Downlink capacity of interference-limited MIMO systems with joint detection," *IEEE Trans. on Wireless Communications*, vol. 3, pp. 442– 453, Mar. 2004.

[52] A. B. de Monvel and A. Khorunzhy, "Asymptotic distribution of smoothed eigenvalue density. I. Gaussian random matrices & II. Wigner random matrices," *Random Operators and Stochastic Equations*, vol. 7, pp. 1–22 & 149–168, 1999.

[53] A. B. de Monvel and A. Khorunzhy, "On the norm and eigenvalue distribution of large random matrices," *Annals of Probability*, vol. 27, no. 2, pp. 913–944, 1999.

[54] A. B. de Monvel and A. Khorunzhy, "On universality of smoothed eigenvalue density of large random matrices," *J. of Physics A: Mathematical and General*, vol. 32, pp. 413–417, 1999.

[55] A. B. de Monvel, A. Khorunzhy, and V. Vasilchuk, "Limiting eigenvalue distribution of random matrices with correlated entries," *Markov Processes and Related Fields*, vol. 2, no. 2, pp. 607–636, 1996.

[56] M. Debbah, W. Hachem, P. Loubaton, and M. de Courville, "MMSE analysis of certain large isometric random precoded systems," *IEEE Trans. on Information Theory*, vol. 49, pp. 1293–1311, May 2003.

[57] H. Dette, "Strong approximation of the eigenvalues of large dimensional Wishart matrices by roots of generalized Laguerre polynomials," *J. of Approximation Theory*, vol. 118, no. 2, pp. 290–304, 2002.

[58] J. A. Díaz-García and J. R. Gutierrez, "Proof of the conjectures of H. Uhlig on the singular multivariate beta and the Jacobian of a certain matrix transformation," *Annals of Statistics*, vol. 25, pp. 2018–2023, 1997.

[59] J. A. Díaz-García, J. R. Gutierrez, and K. V. Mardia, "Wishart and pseudo-Wishart distributions and some applications to shape theory," *J. of Multivariate Analysis*, vol. 63, pp. 73–87, 1997.

[60] S. N. Diggavi, N. Al-Dhahir, A. Stamoulis, and A. R. Calderbank, "Great expectations: The value of spatial diversity in wireless networks," *Proc. IEEE*, vol. 92, pp. 219–270, Feb. 2004.

[61] M. Dohler and H. Aghvami, "A closed form expression of MIMO capacity over ergodic narrowband channels," *preprint*, 2003.

[62] P. Driessen and G. J. Foschini, "On the capacity formula for multiple-input multiple-output channels: A geometric interpretation," *IEEE Trans. Communications*, vol. 47, pp. 173–176, Feb. 1999.

[63] K. J. Dykema, "On certain free product factors via an extended matrix model," *J. of Funct. Analysis*, vol. 112, pp. 31–60, 1993.

[64] K. J. Dykema, "Interpolated free group factors," *Pacific J. Math.*, vol. 163, no. 1, pp. 123–135, 1994.

[65] F. Dyson, "Statistical theory of the energy levels of complex systems," *J. of Math. Physics*, vol. 3, pp. 140–175, 1962.

[66] F. Dyson, "A class of matrix ensembles," *J. of Math. Physics*, vol. 13, p. 90, 1972.

[67] A. Edelman, *Eigenvalues and condition number of random matrices.* PhD thesis, Dept. Mathematics, MIT, Cambridge, MA, 1989.

[68] A. Edelman, "The probability that a random real Gaussian matrix has k real eigenvalues, related distributions, and the circular law," *J. Multivariate Analysis*, vol. 60, pp. 203–232, 1997.

[69] A. Edelman, E. Kostlan, and M. Shub, "How many eigenvalues of a random matrix are real?," *J. Amer. Math. Soc.*, vol. 7, pp. 247–267, 1994.

[70] Y. C. Eldar and A. M. Chan, "On the asymptotic performance of the decorrelator," *IEEE Trans. on Information Theory*, vol. 49, pp. 2309–2313, Sep. 2003.

[71] H. Elders-Boll, A. Busboom, and H. Schotten, "Implementation of linear multiuser detectors for asynchronous CDMA systems by linear interference cancellation algorithms," in *Proc. IEEE Vehicular Technology. Conf. (VTC'98)*, (Ottawa, Canada), pp. 1849–1853, May 1998.

[72] H. Elders-Boll, H. Schotten, and A. Busboom, "Efficient implementation of linear multiuser detectors for asynchronous CDMA systems by linear interference cancellation," *Euro. Trans. Telecommunications*, vol. 9, pp. 427–438, Sep./Oct. 1998.

[73] J. Evans and D. Tse, "Large system performance of linear multiuser receivers in multipath fading channels," *IEEE Trans. on Information Theory*, vol. 46, pp. 2059–2078, Sep. 2000.

[74] F. R. Farrokhi, G. J. Foschini, A. Lozano, and R. A. Valenzuela, "Link-optimal space-time processing with multiple transmit and receive antennas," *IEEE Communications Letters*, vol. 5, pp. 85–87, Mar. 2001.

[75] R. A. Fisher, "The sampling distribution of some statistics obtained from non-linear equations," *Annals of Eugenics*, vol. 9, pp. 238–249, 1939.

[76] G. Foschini and M. Gans, "On limits of wireless communications in fading environment when using multiple antennas," *Wireless Personal Communications*, vol. 6, No. 6, pp. 315–335, Mar. 1998.

[77] G. J. Foschini, "Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas," *Bell Labs Technical Journal*, vol. 1, pp. 41–59, 1996.

[78] D. A. S. Fraser, *The Structure of Inference*. John Wiley and Sons, New York, 1968.

[79] H. Furstenberg and H. Kesten, "Products of random matrices," *Annals of Math. Statistics*, pp. 457–469, 1960.

[80] H. Gao and P. J. Smith, "A determinant representation for the distribution of quadratic forms in complex normal vectors," *J. Multivariate Analysis*, vol. 73, pp. 155–165, May 2000.

[81] M. Gaudin, "Sur la loi limite de l'espacement des valeurs propres d'une matrice aleatoire," *Nuclear Physics*, vol. 25, pp. 447–455, 1961.

[82] D. Gesbert, M. Shafi, D. Shiu, P. J. Smith, and A. Naguib, "From theory to practice: An overview of MIMO Space-Time coded wireless systems," *J. on Selected Areas in Communications*, vol. 21, pp. 281–302, Apr. 2003.

[83] R. Gharavi and V. Anantharam, "An upper bound for the largest Lyapunov exponent of a Markovian product of nonnegative matrices," *submitted to Theoretical Computer Science*, 2004.

[84] V. Ghazi-Moghadam, L. Nelson, and M. Kaveh, "Parallel interference cancellation for CDMA systems," in *Proc. Allerton Conf. on Communication, Control and Computing*, (Monticello, IL), pp. 216–224, Oct. 1995.

[85] V. L. Girko, "Circular law," *Theory Prob. Appl.*, vol. 29, pp. 694–706, 1984.

[86] V. L. Girko, *Theory of Random Determinants*. Dordrecht: Kluwer Academic Publishers, 1990.

[87] V. L. Girko, "A refinement of the central limit theorem for random determinants," *Theory Prob. Appl.*, vol. 42, no. 1, pp. 121–129, 1997.

[88] V. L. Girko, "Convergence rate of the expected spectral functions of symmetric random matrices equal to $o(n^{-1/2})$," *Random Operator and Stochastic Equations*, vol. 6, pp. 359–408, 1998.

[89] M. A. Girshick, "On the sampling theory of roots of determinantal equations," *Annals of Math. Statistics*, vol. 10, pp. 203–204, 1939.

[90] A. Goldsmith, S. A. Jafar, N. Jindal, and S. Vishwanath, "Capacity limits of MIMO channels," *IEEE J. on Selected Areas on Communications*, vol. 21, pp. 684–702, June 2003.

[91] J. S. Goldstein and I. S. Reed, "A new method of Wiener filtering and its application to interference mitigation for communications," in *Proc. IEEE MILCOM*, (Monterey, CA), pp. 1087–1091, Nov. 1997.

[92] J. S. Goldstein and I. S. Reed, "Reduced rank adaptive filtering," *IEEE Trans. Signal Processing*, vol. 45, pp. 492–496, Feb. 1997.

[93] J. S. Goldstein, I. S. Reed, and L. L. Scharf, "A multistage representation of the Wiener filter based on orthogonal projections," *IEEE Trans. on Information Theory*, vol. 44, pp. 2943–2959, Nov. 1998.
[94] C. R. Goodall and K. V. Mardia, "Multivariate aspects of shape theory," *Annals of Statistics*, vol. 21, pp. 848–866, 1993.
[95] F. Gotze and A. N. Tikhomirov, "Rate of convergence to the semicircular law for the Gaussian unitary ensemble," *Theory Prob. Appl.*, vol. 47, no. 2, pp. 323–330, 2003.
[96] P. Graczyk, G. Letac, and H. Massam, "The complex Wishart distribution and the symmetric group," *Annals of Statistics*, vol. 31, no. 1, pp. 287–309, 2003.
[97] S. Gradshteyn and I. Ryzhik, *Table of Integrals, Series and Products.* New York: Academic, 1965.
[98] A. Grant, "Rayleigh fading multi-antenna channels," *EURASIP J. on Applied Signal Processing*, vol. 3, pp. 316–329, 2002.
[99] A. Grant and C. Schlegel, "Convergence of linear interference cancellation multiuser receivers," *IEEE Trans. on Communications*, vol. 10, pp. 1824–1834, Oct. 2001.
[100] A. J. Grant and P. D. Alexander, "Random sequences multisets for synchronous code-division multiple-access channels," *IEEE Trans. on Information Theory*, vol. 44, pp. 2832–2836, Nov. 1998.
[101] U. Grenander and J. W. Silverstein, "Spectral analysis of networks with random topologies," *SIAM J. of Applied Mathematics*, vol. 32, pp. 449–519, 1977.
[102] A. Guionnet and O. Zeitouni, "Concentration of the spectral measure for large matrices," *Electronic Communications in Probability*, vol. 5, pp. 119–136, 2000.
[103] D. Guo and S. Verdú, "Multiuser detection and statistical mechanics," in *Communications Information and Network Security*, (V. Bhargava, H. V. Poor, V. Tarokh, and S. Yoon, eds.), pp. 229–277, Ch. 13, Kluwer Academic Publishers, 2002.
[104] A. K. Gupta and D. K. Nagar, "Matrix variate distributions," in *Monographs and Surveys in pure and applied mathematics*, Boca Raton, FL: Chapman and Hall/CRC, 2000.
[105] W. Hachem, "Low complexity polynomial receivers for downlink CDMA," in *Proc. Asilomar Conf. on Systems, Signals and Computers*, (Pacific Grove, CA), Nov. 2002.
[106] G. Hackenbroich and H. A. Weidenmueller, "Universality of random-matrix results for non-Gaussian ensembles," *Physics Review Letters*, vol. 74, pp. 4118–4121, 1995.
[107] P. R. Halmos, *Measure Theory.* Van Nostrand, Princeton, NJ, 1950.
[108] S. V. Hanly and D. N. C. Tse, "Resource pooling and effective bandwidths in CDMA networks with multiuser receivers and spatial diversity," *IEEE Trans. on Information Theory*, vol. 47, pp. 1328–1351, May 2001.
[109] S. Hara and R. Prasad, "Overview of multicarrier CDMA," *IEEE Communications Magazine*, vol. 35, pp. 126–133, Dec. 1997.

[110] F. Hiai and D. Petz, "Asymptotic freeness almost everywhere for random matrices," *Acta Sci. Math. Szeged*, vol. 66, pp. 801–826, 2000.
[111] F. Hiai and D. Petz, *The Semicircle Law, Free Random Variables and Entropy*. American Mathematical Society, 2000.
[112] B. Hochwald and S. Vishwanath, "Space-time multiple access: Linear growth in the sum-rate," in *Proc. Allerton Conf. on Communication, Control and Computing*, (Monticello, IL), Oct. 2002.
[113] B. M. Hochwald, T. L. Marzetta, and V. Tarokh, "Multi-antenna channel hardening and its implications for rate feedback and scheduling," *IEEE Trans. on Information Theory*, submitted May 2002.
[114] T. Holliday, A. J. Goldsmith, and P. Glynn, "On entropy and Lyapunov exponents for finite state channels," *submitted to IEEE Trans. on Information Theory*, 2004.
[115] M. L. Honig, "Adaptive linear interference suppression for packet DS-CDMA," *Euro. Trans. Telecommunications*, vol. 9, pp. 173–182, Mar./Apr. 1998.
[116] M. L. Honig and W. Xiao, "Performance of reduced-rank linear interference suppression for DS-CDMA," *IEEE Trans. on Information Theory*, vol. 47, pp. 1928–1946, July 2001.
[117] R. Horn and C. Johnson, *Matrix Analysis*. Cambridge University Press, 1985.
[118] D. Hösli and A. Lapidoth, "The capacity of a MIMO Ricean channel is monotonic in the singular values of the mean," *5th Int. ITG Conf. on Source and Channel Coding*, Jan. 2004.
[119] D. C. Hoyle and M. Rattray, "Principal component analysis eigenvalue spectra from data with symmetry breaking structure," *Physical Review E*, vol. 69, 026124, 2004.
[120] P. L. Hsu, "On the distribution of roots of certain determinantal equations," *Annals of Eugenics*, vol. 9, pp. 250–258, 1939.
[121] L. K. Hua, *Harmonic analysis of functions of several complex variables in the classical domains*. Providence, RI: American Mathematical Society, 1963.
[122] P. Jacquet, G. Seroussi, and W. Szpankowski, "On the entropy of a hidden Markov process," in *Proc. Data Compression Conference*, Mar. 23–25 2004.
[123] S. A. Jafar, S. Vishwanath, and A. J. Goldsmith, "Channel capacity and beamforming for multiple transmit and receive antennas with covariance feedback," *Proc. IEEE Int. Conf. on Communications (ICC'01)*, vol. 7, pp. 2266–2270, 2001.
[124] K. Jamal and E. Dahlman, "Multi-stage interference cancellation for DS-CDMA," in *Proc. IEEE Vehicular Technology Conf. (VTC'96)*, (Atlanta, GA), pp. 671–675, Apr. 1996.
[125] A. T. James, "Distributions of matrix variates and latent roots derived from normal samples," *Annals of Math. Statistics*, vol. 35, pp. 475–501, 1964.
[126] R. Janaswamy, "Analytical expressions for the ergodic capacities of certain MIMO systems by the Mellin transform," *Proc. IEEE Global Telecomm. Conf. (GLOBECOM'03)*, vol. 1, pp. 287–291, Dec. 2003.
[127] S. K. Jayaweera and H. V. Poor, "Capacity of multiple-antenna systems with both receiver and transmitter channel state information," *IEEE Trans. on Information Theory*, vol. 49, pp. 2697–2709, Oct. 2003.

[128] N. Jindal, S. Vishwanath, and A. Goldsmith, "On the duality of Gaussian multiple-access and broadcast channels," *IEEE Trans. on Information Theory*, vol. 50, pp. 768–783, May 2004.
[129] K. Johansson, "Universality of the local spacing distribution in certain ensembles of Hermitian Wigner matrices," *Comm. Math. Phys.*, vol. 215, pp. 683–705, 2001.
[130] D. H. Johnson and D. E. Dudgeon, *Array Signal Processing: Concepts and Techniques*. Englewood Cliffs, NJ: Prentice-Hall, 1993.
[131] D. Jonsson, "Some limit theorems for the eigenvalues of a sample covariance matrix," *J. Multivariate Analysis*, vol. 12, pp. 1–38, 1982.
[132] E. Jorswieck and H. Boche, "On transmit diversity with imperfect channel state information," *Proc. IEEE Conf. Acoustics, Speech, and Signal Processing (ICASSP'02)*, vol. 3, pp. 2181–2184, May 2002.
[133] V. Jungnickel, T. Haustein, E. Jorswieck, and C. von Helmolt, "On linear preprocessing in multi-antenna systems," in *Proc. IEEE Global Telecomm. Conf. (GLOBECOM'02)*, pp. 1012–1016, Dec. 2002.
[134] M. Kang and M. S. Alouini, "On the capacity of MIMO Rician channels," in *Proc. 40th Annual Allerton Conference on Communication, Control, and Computing*, (Monticello, IL), Oct. 2002.
[135] M. Kang and M. S. Alouini, "Impact of correlation on the capacity of MIMO channels," in *Proc. IEEE Int. Conf. in Communications (ICC'04)*, pp. 2623–2627, May 2003.
[136] M. Kang and M. S. Alouini, "Largest eigenvalue of complex Wishart matrices and performance analysis of MIMO MRC systems," *IEEE J. on Selected Areas in Communications*, vol. 21, pp. 418–426, Apr. 2003.
[137] M. Kang, L. Yang, and M. S. Alouini, "Capacity of MIMO Rician channels with multiple correlated Rayleigh co-channel interferers," in *Proc. 2003 IEEE Globecom*, (San Francisco, CA), Dec. 2003.
[138] M. Kang, L. Yang, and M. S. Alouini, "Performance analysis of MIMO systems in presence of co-channel interference and additive Gaussian noise," in *Proc. 37th Annual Conf. on Information Sciences and Systems (CISS'2003)*, (Baltimore, MD), Mar. 2003.
[139] H. Kargupta, K. Sivakumar, and S. Ghosh, "Dependency detection in MobiMine and random matrices," *Proc. 6th European Conference on Principles and Practice of Knowledge Discovery in Databases*, pp. 250–262, 2002.
[140] C. G. Khatri, "Distribution of the largest or the smallest characteristic root under null hyperthesis concerning complex multivariate normal populations," *Annals of Math. Statistics*, vol. 35, pp. 1807–1810, Dec. 1964.
[141] C. G. Khatri, "Some results on the non-central multivariate beta distribution and moments of trace of two matrices," *Annals of Math. Statistics*, vol. 36, no. 5, pp. 1511–1520, 1965.
[142] C. G. Khatri, "On certain distribution problems based on positive definite quadratic functions in normal vectors," *Annals of Math. Statistics*, vol. 37, pp. 467–479, Apr. 1966.

[143] C. G. Khatri, "Non-central distributions of i-th largest characteristic roots of three matrices concerning complex multivariate normal populations," *Annals of the Inst. Statist. Math.*, vol. 21, pp. 23–32, 1969.
[144] C. G. Khatri, "On the moments of traces of two matrices in three situations for complex multivariate normal population," *Sankhya: Indian J. of Statistics*, vol. 32, series A, pp. 65–80, 1970.
[145] A. Khorunzhy, "On spectral norm of large band random matrices," *preprint*, Apr. 2004.
[146] A. M. Khorunzhy, B. A. Khoruzhenko, L. A. Pastur, and M. V. Shcherbina, "The large-n limit in statistical mechanics and the spectral theory of disordered systems," *Phase Transitions*, vol. 15, pp. 73–239, 1992.
[147] A. M. Khorunzhy and L. A. Pastur, "Random eigenvalue distributions," *Adv. Soviet Math.*, vol. 19, pp. 97–127, 1994.
[148] M. Kiessling and J. Speidel, "Exact ergodic capacity of MIMO channels in correlated Rayleigh fading environments," in *Proc. Int. Zurich Seminar on Communications (IZS)*, (Zurich, Switzerland), Feb. 2004.
[149] M. Kiessling and J. Speidel, "Mutual information of MIMO channels in correlated Rayleigh fading environments - a general solution," in *Proc. of IEEE Int. Conf. in Communications. (ICC'04)*, (Paris, France), June 2004.
[150] M. Kiessling and J. Speidel, "Unifying analysis of ergodic MIMO capacity in correlated Rayleigh fading environments," in *Fifth European Wireless Conference Mobile and Wireless Systems beyond 3G*, (Barcelona, Catalonia, Spain), Feb. 24-27 2004.
[151] Y.-H. Kim and A. Lapidoth, "On the log-determinant of non-central Wishart matrices," *Proc. 2003 IEEE Int. Symp. on Information Theory*, pp. 54–54, Jul 2003.
[152] Kiran and D. Tse, "Effective interference and effective bandwidth of linear multiuser receivers in asynchronous systems," *IEEE Trans. on Information Theory*, vol. 46, pp. 1426–1447, July 2000.
[153] T. Kollo and H. Neudecker, "Asymptotics of eigenvalues and unit-length eigenvectors of sample variance and correlation matrices," *J. Multivariate Analysis*, vol. 47, pp. 283–300, 1993.
[154] G. Kreweras, "Sur les partitions non-croisees d'un cycle," *Discrete Math.*, vol. 1, pp. 333–350, 1972.
[155] L. Laloux, P. Cizeau, M. Potters, and J. P. Bouchaud, "Random matrix theory and financial correlations," *Int. J. of Theoretical and Applied Finance*, vol. 3, No. 3, pp. 391–397, 2000.
[156] W. C. Y. Lee and Y. S. Yeh, "Polarization diversity system for mobile radio," *IEEE Trans. on Communications*, vol. 20, pp. 912–923, Oct 1972.
[157] O. Lévêque, E. Telatar, and D. Tse, "Upper bounds on the capacity of ad-hoc wireless networks," in *Proc. 2003 Winter School on Coding and Information Theory*, (Monte Veritá, Switzerland), Feb. 24-27, 2003.
[158] L. Li, A. M. Tulino, and S. Verdú, "Asymptotic eigenvalue moments for linear multiuser detection," *Communications in Information and Systems*, vol. 1, pp. 273–304, Sep. 2001.

[159] L. Li, A. M. Tulino, and S. Verdú, “Design of reduced-rank MMSE multiuser detectors using random matrix methods,” *IEEE Trans. on Information Theory*, vol. 50, June 2004.

[160] L. Li, A. M. Tulino, and S. Verdú, “Spectral efficiency of multicarrier CDMA,” *submitted to IEEE Trans. on Information Theory*, 2004.

[161] K. Liu, V. Raghavan, and A. M. Sayeed, “Capacity and spectral efficiency in wideband correlated MIMO channels,” *Proc. 2003 IEEE Int. Symp. on Information Theory (ISIT 2003)*, July, 2003.

[162] P. Loubaton and W. Hachem, “Asymptotic analysis of reduced rank Wiener filter,” *Proc. IEEE Information Theory Workshop (ITW’03)*, pp. 328–332, Paris, France, 2003.

[163] A. Lozano and A. M. Tulino, “Capacity of multiple-transmit multiple-receive antenna architectures,” *IEEE Trans. on Information Theory*, vol. 48, pp. 3117–3128, Dec. 2002.

[164] A. Lozano, A. M. Tulino, and S. Verdú, “Multiple-antenna capacity in the low-power regime,” *IEEE Trans. on Information Theory*, vol. 49, pp. 2527–2544, Oct. 2003.

[165] A. Lozano, A. M. Tulino, and S. Verdú, “High-SNR power offset in multi-antenna communication,” in *Proc. IEEE Int. Symp. on Information Theory (ISIT’04)*, (Chicago, IL), June 2004.

[166] A. Lozano, A. M. Tulino, and S. Verdú, “High-SNR power offset in multi-antenna communication,” *Bell Labs Technical Memorandum*, June 2004.

[167] U. Madhow and M. Honig, “On the average near-far resistance for MMSE detection of direct sequence CDMA signals with random spreading,” *IEEE Trans. on Information Theory*, vol. 45, pp. 2039–2045, Sep. 1999.

[168] R. K. Mallik, “The pseudo-Wishart distribution and its application to MIMO systems,” *IEEE Trans. on Information Theory*, vol. 49, pp. 2761–2769, Oct. 2003.

[169] A. Mantravadi, V. V. Veeravalli, and H. Viswanathan, “Spectral efficiency of MIMO multiaccess systems with single-user decoding,” *IEEE J. on Selected Areas in Communications*, vol. 21, pp. 382–394, Apr. 2003.

[170] V. A. Marčenko and L. A. Pastur, “Distributions of eigenvalues for some sets of random matrices,” *Math. USSR-Sbornik*, vol. 1, pp. 457–483, 1967.

[171] T. Marzetta and B. Hochwald, “Capacity of mobile multiple-antenna communication link in a Rayleigh flat-fading environment,” *IEEE Trans. on Information Theory*, vol. 45, pp. 139–157, Jan. 1999.

[172] M. L. Mehta, “On the statistical properties of the level-spacings in nuclear spectra,” *Nuclear Physics*, vol. 18, p. 395, 1960.

[173] M. L. Mehta, *Random Matrices and the Statistical Theory of Energy Levels.* New York, Academic Press, 1967.

[174] M. L. Mehta, “Power series of level spacing functions of random matrix ensembles,” *Z. Phys. B*, vol. 86, pp. 258–290, 1992.

[175] M. L. Mehta and F. Dyson, “Statistical theory of the energy levels of complex systems,” *J. Math. Physics*, vol. 4, pp. 713–719, 1963.

[176] M. L. Mehta and M. Gaudin, “On the density of the eigenvalues of a random matrix,” *Nuclear Physics*, vol. 18, pp. 420–427, 1960.

[177] X. Mestre, *Space processing and channel estimation: performance analysis and asymptotic results*. PhD thesis, Dept. de Teoria del Senyal i Comunicacions, Universitat Politècnica de Catalunya, Barcelona, Catalonia, Spain, 2002.

[178] X. Mestre, J. R. Fonollosa, and A. Pages-Zamora, "Capacity of MIMO channels: asymptotic evaluation under correlated fading," *IEEE J. on Selected Areas in Communications*, vol. 21, pp. 829– 838, June 2003.

[179] S. Moshavi, E. G. Kanterakis, and D. L. Schilling, "Multistage linear receivers for DS-CDMA systems," *Int. J. of Wireless Information Networks*, vol. 39, no. 1, pp. 1–17, 1996.

[180] A. L. Moustakas and S. H. Simon, "Optimizing multiple-input single-output (MISO) communication systems with general Gaussian channels: nontrivial covariance and nonzero mean," *IEEE Trans. on Information Theory*, vol. 49, pp. 2770–2780, Oct. 2003.

[181] A. L. Moustakas, S. H. Simon, and A. M. Sengupta, "MIMO capacity through correlated channels in the presence of correlated interferers and noise: a (not so) large N analysis," *IEEE Trans. on Information Theory*, vol. 49, pp. 2545–2561, Oct. 2003.

[182] R. J. Muirhead, *Aspects of multivariate statistical theory*. New York, Wiley, 1982.

[183] R. R. Müller, *Power and Bandwidth Efficiency of Multiuser Systems with Random Spreading*. PhD thesis, Universtät Erlangen-Nürnberg, Erlangen, Germany, Nov. 1998.

[184] R. R. Müller, "On the asymptotic eigenvalue distribution of concatenated vector-valued fading channels," *IEEE Trans. on Information Theory*, vol. 48, pp. 2086–2091, July 2002.

[185] R. R. Müller, "Multiuser receivers for randomly spread signals: Fundamental limits with and without decision-feedback," *IEEE Trans. on Information Theory*, vol. 47, no. 1, pp. 268–283, Jan. 2001.

[186] R. R. Müller and W. Gerstacker, "On the capacity loss due to separation of detection and decoding in large CDMA systems," in *IEEE Information Theory Workshop (ITW)*, p. 222, Oct. 2002.

[187] R. R. Müller and S. Verdú, "Design and analysis of low-complexity interference mitigation on vector channels," *IEEE J. on Selected Areas on Communications*, vol. 19, pp. 1429–1441, Aug. 2001.

[188] F. D. Neeser and J. L. Massey, "Proper complex random processes with applications to information theory," *IEEE Trans. on Information Theory*, vol. 39, pp. 1293–1302, July 1993.

[189] A. Nica, *R-transforms in free probability*. Paris, France: Henri Poincare Institute, 1999.

[190] A. Nica and R. Speicher, "On the multiplication of free n-tuples of non-commutative random variables," *American J. Math.*, vol. 118, no. 4, pp. 799–837, 1996.

[191] B. Niederhauser, "Norms of certain random matrices with dependent entries," *Random Operators and Stochastic Equations*, vol. 11, no. 1, pp. 83–101, 2003.

[192] A. Y. Orlov, "New solvable matrix integrals," *Acta Sci. Math*, vol. 63, pp. 383–395, 1997.

[193] V. I. Oseledec, "A multiplicative ergodic theorem. Lyapunov characteristic numbers for dynamical systems," *Trans. Moscow Math. Soc.*, vol. 19, pp. 197–231, 1968.

[194] H. Ozcelik, M. Herdin, W. Weichselberger, G. Wallace, and E. Bonek, "Deficiencies of the Kronecker MIMO channel model," *IEE Electronics Letters*, vol. 39, pp. 209–210, Aug. 2003.

[195] D. P. Palomar, J. M. Cioffi, and M. A. Lagunas, "Uniform power allocation in MIMO channels: A game theoretic approach," *IEEE Trans. on Information Theory*, vol. 49, pp. 1707–1727, July 2003.

[196] L. A. Pastur, "On the spectrum of random matrices," *Teoret. Mat. Fiz. (English translation: Theoretical and Mathematical Physics)*, vol. 10, pp. 67–74, 1972.

[197] L. A. Pastur, "Spectra of random self-adjoint operators," *Uspekhi Mat. Nauk. (Russian Math. Surveys)*, vol. 28, pp. 1–67, 1973.

[198] L. A. Pastur, "Eigenvalue distribution of random matrices: some recent results," *Annals Inst. Henri Poincaré*, vol. 64, pp. 325–337, 1996.

[199] L. A. Pastur, "Random matrices as paradigm," *Mathematical Physics 2000*, pp. 216–265, 2000.

[200] L. A. Pastur and M. Shcherbina, "Universality of the local eigenvalue statistics for a class of unitary invariant ensembles," *J. Stat. Physics*, vol. 86, pp. 109–147, 1997.

[201] M. Peacock, I. Collings, and M. L. Honig, "Asymptotic spectral efficiency of multi-user multi-signature CDMA in frequency-selective channels," *preprint*, 2004.

[202] M. Peacock, I. Collings, and M. L. Honig, "Asymptotic SINR analysis of single-user MC-CDMA in Rayleigh fading," *Proc. IEEE Int. Symp. on Spread Spectrum Systems and Applications (ISSSTA'02)*, pp. 338–342, Sep. 2002.

[203] K. I. Pedersen, J. B. Andersen, J. P. Kermoal, and P. E. Mogensen, "A stochastic multiple-input multiple-output radio channel model for evaluations of space-time coding algorithms," *Proc. IEEE Vehicular Technology Conf. (VTC'2000 Fall)*, pp. 893–897, Sep. 2000.

[204] M. S. Pinsker, *Information and Information Stability of Random Variables and Processes*. San Francisco, CA: Holden-Day, 1964.

[205] G. Raleigh and J. M. Cioffi, "Spatio-temporal coding for wireless communications," *IEEE Trans. on Communications*, vol. 46, pp. 357–366, Mar. 1998.

[206] P. Rapajic and D. Popescu, "Information capacity of a random signature multiple-input multiple-output channel," *IEEE Trans. on Communications*, vol. 48, pp. 1245–1248, Aug. 2000.

[207] L. Rasmussen, A. Johansson, and T. Lim, "One-shot filtering equivalence for linear successive interference cancellation in CDMA," in *Proc. IEEE Vehicular. Technology. Conf. (VTC'97)*, (Phoenix, AZ), pp. 2163–2167, May 1997.

[208] T. Ratnarajah, R. Vaillancourt, and M. Alvo, "Complex random matrices and applications," *Math. Rep. of the Acad. of Sci. of the Royal Soc. of Canada*, vol. 25, pp. 114–120, Dec. 2003.

[209] T. Ratnarajah, R. Vaillancourt, and M. Alvo, "Complex random matrices and Rayleigh channel capacity," *Communications in Information and Systems*, pp. 119–138, Oct. 2003.

[210] S. N. Roy, "p-statistics or some generalizations in the analysis of variance appropriate to multivariate problems," *Sankhya: Indian J. of Statistics*, vol. 4, pp. 381–396, 1939.

[211] O. Ryan, "On the limit distributions of random matrices with independent or free entries," *Communications in Mathematical Physics*, vol. 193, pp. 595–626, 1998.

[212] U. Sacoglu and A. Scaglione, "Asymptotic capacity of space-time coding for arbitrary fading: a closed form expression using Girko's law," in *Proc. Intl. Conf. on Acoust. Speech and Signal Proc., ICASSP(2001)*, (Salt Lake City, UT), May 7-12 2001.

[213] A. Sayeed, "Deconstructing multi-antenna channels," *IEEE Trans. on Signal Processing*, vol. 50, pp. 2563–2579, Oct. 2002.

[214] L. L. Scharf, *Statistical Signal Processing: Detection, Estimation, and Time Series Analysis*. New York, NY: Addison-Wesley, 1990.

[215] L. L. Scharf, "The SVD and reduced rank signal processing," *Signal Processing*, vol. 24, pp. 113–133, Nov. 1991.

[216] A. M. Sengupta and P. P. Mitra, "Capacity of multivariate channels with multiplicative noise: I. Random matrix techniques and large-N expansions for full transfer matrices," *LANL arXiv:physics*, Oct. 2000.

[217] S. Shamai and S. Verdú, "The effect of frequency-flat fading on the spectral efficiency of CDMA," *IEEE Trans. on Information Theory*, vol. 47, pp. 1302–1327, May 2001.

[218] J. Shen, "On the singular values of Gaussian random matrices," *Linear Algebra and its Applications*, vol. 326, no. 1-3, pp. 1–14, 2001.

[219] H. Shin and J. H. Lee, "Capacity of multiple-antenna fading channels: Spatial fading correlation, double scattering and keyhole," *IEEE Trans. on Information Theory*, vol. 49, pp. 2636–2647, Oct. 2003.

[220] D.-S. Shiu, G. J. Foschini, M. J. Gans, and J. M. Kahn, "Fading correlation and its effects on the capacity of multi-element antenna systems," *IEEE Trans. on Communications*, vol. 48, pp. 502–511, Mar. 2000.

[221] D. Shlyankhtenko, "Random Gaussian band matrices and freeness with amalgamation," *Int. Math. Res. Note*, vol. 20, pp. 1013–1025, 1996.

[222] J. Silverstein and S. Choi, "Analysis of the limiting spectral distribution of large dimensional random matrices," *J. of Multivariate Analysis*, vol. 54(2), pp. 295–309, 1995.

[223] J. W. Silverstein, "The limiting eigenvalue distribution of a multivariate F-matrix," *SIAM J. of Math. Analysis*, vol. 16, pp. 641–646, 1985.

[224] J. W. Silverstein, "On the eigenvectors of large dimensional sample covariance matrices," *J. of Multivariate Analysis*, vol. 30, pp. 1–16, 1989.

[225] J. W. Silverstein, "Weak-convergence of random functions defined by the eigenvectors of sample covariance matrices," *Annals of Probability*, vol. 18, pp. 1174–1194, July 1990.

[226] J. W. Silverstein, "Strong convergence of the empirical distribution of eigenvalues of large dimensional random matrices," *J. of Multivariate Analysis*, vol. 55, pp. 331–339, 1995.

[227] J. W. Silverstein and Z. D. Bai, "On the empirical distribution of eigenvalues of a class of large dimensional random matrices," *J. of Multivariate Analysis*, vol. 54, pp. 175–192, 1995.

[228] J. W. Silverstein and P. L. Combettes, "Signal detection via spectral theory of large dimensional random matrices," *IEEE Trans. on Signal Processing*, vol. 40, pp. 2100–2105, Aug. 1992.

[229] S. H. Simon and A. L. Moustakas, "Optimizing MIMO systems with channel covariance feedback," *IEEE J. on Selected Areas in Communications*, vol. 21, pp. 406–417, Apr. 2003.

[230] S. H. Simon and A. L. Moustakas, "Eigenvalue density of correlated complex random Wishart matrices," *Physical Review E*, vol. 69, to appear, 2004.

[231] S. H. Simon, A. L. Moustakas, and L. Marinelli, "Capacity and character expansions: Moment generating function and other exact results for MIMO correlated channels," *Bell Labs Technical Memorandum ITD-04-45211T*, Mar. 2004.

[232] R. Singh and L. Milstein, "Interference suppression for DS-CDMA," *IEEE Trans. on Communications*, vol. 47, pp. 446–453, Mar. 1999.

[233] P. J. Smith and L. M. Garth, "Exact capacity distribution for dual MIMO systems in Ricean fading," *IEEE Communications Letters*, vol. 8, pp. 18–20, Jan. 2004.

[234] P. J. Smith, S. Roy, and M. Shafi, "Capacity of MIMO systems with semi-correlated flat-fading," *IEEE Trans. on Information Theory*, vol. 49, pp. 2781–2788, Oct. 2003.

[235] P. J. Smith and M. Shafi, "On a Gaussian approximation to the capacity of wireless MIMO systems," *Proc. IEEE Int. Conf. in Communications. (ICC'02)*, pp. 406–410, Apr. 2002.

[236] P. Soma, D. S. Baum, V. Erceg, R. Krishnamoorthy, and A. Paulraj, "Analysis and modelling of multiple-input multiple-output (MIMO) radio channel based on outdoor measurements conducted at 2.5 GHz for fixed BWA applications," *Proc. IEEE Int. Conf. on Communications (ICC'02), New York City, NY*, pp. 272–276, 28 Apr.-2 May 2002.

[237] O. Somekh, B. M. Zaidel, and S. Shamai, "Spectral efficiency of joint multiple cell-site processors for randomly spread DS-CDMA systems," in *Proc. IEEE Int. Symp. on Information Theory (ISIT'04)*, (Chicago, IL), June 2004.

[238] H. J. Sommers, A. Crisanti, H. Sompolinsky, and Y. Stein, "Spectrum of large random asymmetric matrices," *Physical Review Letters*, vol. 60, pp. 1895–1899, May 1988.

[239] R. Speicher, "A new example of independence and white noise," *Probability Theory and Related Fields*, vol. 84, pp. 141–159, 1990.

[240] R. Speicher, "Free convolution and the random sum of matrices," *Publ. Res. Inst. Math. Sci.*, vol. 29, pp. 731–744, 1993.

[241] R. Speicher, *Free probability theory and non-crossing partitions*. 39^{e} Séminaire Lotharingien de Combinatoire, 1997.

[242] R. Speicher, “Free calculus,” in *Summer school on Quantum Probability*, (Grenoble, France), 1998.
[243] R. Speicher, *Freie Wahrscheinlichkeitstheorie (WS 97/98)*. Heidelberg University, 1998.
[244] M. S. Srivastava and C. G. Khatri, *An Introduction to Multivariate Statistics*. North-Holland, Amsterdam, 1979.
[245] G. W. Stewart, “The efficient generation of random orthogonal matrices with an application to conditional estimation,” *SIAM J. Numer. Analysis*, vol. 17, pp. 403–409, June 1980.
[246] T. J. Stieltjes, “Recherches sur les fractions continues,” *Annales de la Faculte des Sciences de Toulouse*, vol. 8 (9), no. A (J), pp. 1–47 (1–122), 1894 (1895).
[247] E. G. Ström and S. L. Miller, “Properties of the single-bit single-user MMSE receiver for DS-CDMA system,” *IEEE Trans. on Communications*, vol. 47, pp. 416–425, Mar. 1999.
[248] S. Szarek and D. Voiculescu, “Volumes of restricted Minkowski sums and the free analogue of the entropy power inequality,” *Communications in Mathematical Physics*, vol. 178, pp. 563–570, July 1996.
[249] T. Tanaka, “A statistical-mechanics approach to large-system analysis of CDMA multiuser detectors,” *IEEE Trans. on Information Theory*, vol. 48, pp. 2888–2910, Nov. 2002.
[250] E. Telatar, “Capacity of multi-antenna Gaussian channels,” *Euro. Trans. Telecommunications*, vol. 10, pp. 585–595, Nov.-Dec. 1999.
[251] L. Trichard, I. Collings, and J. Evans, “Parameter selection for multiuser receivers based on iterative methods,” in *Proc. IEEE Vehicular Techn. Conf. (VTC’00)*, (Tokyo, Japan), pp. 926–930, May 2000.
[252] L. Trichard, J. Evans, and I. Collings, “Large system analysis of linear multistage parallel interference cancellation,” *IEEE Trans. on Communications*, vol. 50, pp. 1778–1786, Nov. 2002.
[253] L. Trichard, J. Evans, and I. Collings, “Optimal linear multistage receivers and the recursive large system SIR,” in *Proc. IEEE Int. Symp. on Information Theory (ISIT’02)*, (Lausanne, Switzerland), p. 21, June 2002.
[254] L. Trichard, J. Evans, and I. Collings, “Large system analysis of second-order linear multistage CDMA receivers,” *IEEE Trans. on Wireless Communications*, vol. 2, pp. 591–600, May 2003.
[255] L. Trichard, J. Evans, and I. Collings, “Optimal linear multistage receivers with unequal power users,” in *Proc. IEEE Int. Symp. on Information Theory (ISIT’03)*, (Yokohama, Japan), p. 21, June 2003.
[256] D. Tse and S. Hanly, “Linear multiuser receivers: Effective interference, effective bandwidth and user capacity,” *IEEE Trans. on Information Theory*, vol. 45, pp. 641–657, Mar. 1999.
[257] D. Tse and O. Zeitouni, “Linear multiuser receivers in random environments,” *IEEE Trans. on Information Theory*, vol. 46, pp. 171–188, Jan. 2000.
[258] D. N. Tse and S. Verdú, “Optimum asymptotic multiuser efficiency of randomly spread CDMA,” *IEEE Trans. on Information Theory*, vol. 46, no. 6, pp. 2718–2723, Nov. 2000.

[259] D. N. Tse and P. Viswanath, "On the capacity of the multiple antenna broadcast channel," in *Multiantenna channels: Capacity, Coding and Signal Processing*, (G. Foschini and S. Verdú, eds.), pp. 87–106, American Mathematical Society Press, 2003.

[260] B. S. Tsybakov, "The capacity of a memoryless Gaussian vector channel," *Problems of Information Transmission*, vol. 1, pp. 18–29, 1965.

[261] A. M. Tulino, A. Lozano, and S. Verdú, "Capacity-achieving input covariance for single-user multi-antenna channels," *Bell Labs Tech. Memorandum ITD-04-45193Y (also submitted to IEEE Trans. on Wireless Communications.)*, Sep. 2003.

[262] A. M. Tulino, A. Lozano, and S. Verdú, "Impact of correlation on the capacity of multi-antenna channels," *Bell Labs Technical Memorandum ITD-03-44786F (also submitted to IEEE Trans. on Information Theory)*, Sep. 2003.

[263] A. M. Tulino, A. Lozano, and S. Verdú, "MIMO capacity with channel state information at the transmitter," in *Proc. IEEE Int. Symp. on Spread Spectrum Tech. and Applications (ISSSTA'04)*, Aug. 2004.

[264] A. M. Tulino, A. Lozano, and S. Verdú, "Power allocation in multi-antenna communication with statistical channel information at the transmitter," in *Proc. IEEE Int. Conf. on Personal, Indoor and Mobile Radio Communications. (PIMRC'04)*, (Barcelona, Catalonia, Spain), Sep. 2004.

[265] A. M. Tulino and S. Verdú, "Asymptotic analysis of improved linear receivers for BPSK-CDMA subject to fading," *IEEE J. on Selected Areas in Communications*, vol. 19, pp. 1544–1555, Aug. 2001.

[266] A. M. Tulino, S. Verdú, and A. Lozano, "Capacity of antenna arrays with space, polarization and pattern diversity," *Proc. 2003 IEEE Information Theory Workshop (ITW'03)*, pp. 324–327, Apr. 2003.

[267] H. Uhlig, "On singular Wishart and singular multivariate beta distributions," *Annals of Statistics*, vol. 22, pp. 395–405, 1994.

[268] V. V. Veeravalli, Y. Liang, and A. Sayeed, "Correlated MIMO Rayleigh fading channels: Capacity, optimal signalling and asymptotics," *submitted to IEEE Trans. on Information Theory*, 2003.

[269] S. Venkatesan, S. H. Simon, and R. A. Valenzuela, "Capacity of a Gaussian MIMO channel with nonzero mean," *Proc. 2003 IEEE Vehicular Technology Conf. (VTC'03)*, Oct. 2003.

[270] S. Verdú, "Capacity region of Gaussian CDMA channels: The symbol synchronous case," in *Proc. Allerton Conf. on Communication, Control and Computing*, (Monticello, IL), pp. 1025–1034, Oct. 1986.

[271] S. Verdú, *Multiuser Detection*. Cambridge, UK: Cambridge University Press, 1998.

[272] S. Verdú, "Random matrices in wireless communication, proposal to the National Science Foundation," Feb. 1999.

[273] S. Verdú, "Large random matrices and wireless communications," *2002 MSRI Information Theory Workshop*, Feb 25–Mar 1, 2002.

[274] S. Verdú, "Spectral efficiency in the wideband regime," *IEEE Trans. on Information Theory*, vol. 48, no. 6, pp. 1319–1343, June 2002.

[275] S. Verdú and S. Shamai, "Spectral efficiency of CDMA with random spreading," *IEEE Trans. on Information Theory*, vol. 45, pp. 622–640, Mar. 1999.

[276] S. Vishwanath, N. Jindal, and A. Goldsmith, "On the capacity of multiple input multiple output broadcast channels," in *Proc. IEEE Int. Conf. in Communications (ICC'02)*, pp. 1444–1450, Apr. 2002.

[277] S. Vishwanath, N. Jindal, and A. Goldsmith, "Duality, achievable rates and sum-rate capacity of Gaussian MIMO broadcast channels," *IEEE Trans. on Information Theory*, vol. 49, pp. 2658–2668, Oct. 2003.

[278] S. Vishwanath, G. Kramer, S. Shamai, S. Jafar, and A. Goldsmith, "Capacity bounds for Gaussian vector broadcast channels," in *Multiantenna channels: Capacity, Coding and Signal Processing*, (G. Foschini and S. Verdú, eds.), pp. 107–122, American Mathematical Society Press, 2003.

[279] E. Visotsky and U. Madhow, "Space-time transmit precoding with imperfect feedback," *IEEE Trans. on Information Theory*, vol. 47, pp. 2632–2639, Sep. 2001.

[280] P. Viswanath and D. N. Tse, "Sum capacity of the multiple antenna Gaussian broadcast channel," in *Proc. IEEE Int. Symp. Information Theory (ISIT'02)*, p. 497, June 2002.

[281] P. Viswanath, D. N. Tse, and V. Anantharam, "Asymptotically optimal water-filling in vector multiple-access channels," *IEEE Trans. on Information Theory*, vol. 47, pp. 241–267, Jan. 2001.

[282] H. Viswanathan and S. Venkatesan, "Asymptotics of sum rate for dirty paper coding and beamforming in multiple antenna broadcast channels," in *Proc. Allerton Conf. on Communication, Control and Computing*, (Monticello, IL), Oct. 2003.

[283] D. Voiculescu, "Asymptotically commuting finite rank unitary operators without commuting approximants," *Acta Sci. Math.*, vol. 45, pp. 429–431, 1983.

[284] D. Voiculescu, "Symmetries of some reduced free product c^*-algebra," in *Operator algebras and their connections with topology and ergodic theory, Lecture Notes in Mathematics, vol. 1132*, pp. 556–588, Berlin: Springer, 1985.

[285] D. Voiculescu, "Addition of certain non-commuting random variables," *J. Funct. Analysis*, vol. 66, pp. 323–346, 1986.

[286] D. Voiculescu, "Multiplication of certain non-commuting random variables," *J. Operator Theory*, vol. 18, pp. 223–235, 1987.

[287] D. Voiculescu, "Limit laws for random matrices and free products," *Inventiones Mathematicae*, vol. 104, pp. 201–220, 1991.

[288] D. Voiculescu, "The analogues of entropy and of Fisher's information measure in free probability theory, I," *Communications in Math. Physics*, vol. 155, pp. 71–92, July 1993.

[289] D. Voiculescu, "The analogues of entropy and of Fisher's information measure in free probability theory, II," *Inventiones Mathematicae*, vol. 118, pp. 411–440, Nov. 1994.

[290] D. Voiculescu, "Alternative proofs for the type II free Poisson variables and for the free compression results (appendix to a paper by A. Nica and R. Speicher)," *American J. Math.*, vol. 118, pp. 832–837, 1996.

[291] D. Voiculescu, "The analogues of entropy and of Fisher's information measure in free probability theory III: The absence of Cartan subalgebras," *Geometric and Functional Analysis*, vol. 6, pp. 172–199, 1996.

[292] D. Voiculescu, "The analogues of entropy and of Fisher's information measure in free probability theory, IV: Maximum entropy and freeness," in *Free Probability Theory*, (D. Voiculescu, ed.), pp. 293–302, Fields Inst. Communications, Amer. Math. Soc., 1997.

[293] D. Voiculescu, "The analogues of entropy and of Fisher's information measure in free probability theory V: noncommutative Hilbert transforms," *Inventiones Mathematicae*, vol. 132, pp. 189–227, Apr. 1998.

[294] D. Voiculescu, "A strengthened asymptotic freeness result for random matrices with applications to free entropy," *Int. Math. Res. Notices*, vol. 1, pp. 41–63, 1998.

[295] D. Voiculescu, "Lectures on free probability theory," in *Lectures on Probability theory and Statistics: Ecole d'Ete de Probabilites; Lecture Notes in Mathematics*, pp. 283–349, Springer, 2000.

[296] K. W. Wachter, "The strong limits of random matrix spectra for sample matrices of independent elements," *Annals of Probability*, vol. 6, no. 1, pp. 1–18, 1978.

[297] K. W. Wachter, "The limiting empirical measure of multiple discriminant ratios," *Annals of Statistics*, vol. 8, pp. 937–957, 1980.

[298] X. Wang and H. V. Poor, "Blind multiuser detection: A subspace approach," *IEEE Trans. on Information Theory*, vol. 44, pp. 677–690, Mar. 1998.

[299] Z. Wang and G. Giannakis, "Outage mutual information of space-time MIMO channels," *IEEE Trans. on Information Theory*, vol. 50, pp. 657–663, Apr. 2004.

[300] S. Wei and D. Goeckel, "On the minimax robustness of the uniform transmission power strategy in MIMO systems," *IEEE Communications Letters*, vol. 7, pp. 523–524, Nov. 2003.

[301] W. Weichselberger, M. Herdin, H. Ozcelik, and E. Bonek, "Stochastic MIMO channel model with joint correlation of both link ends," *to appear in IEEE Trans. on Wireless Communications*, 2004.

[302] H. Weingarten, Y. Steinberg, and S. Shamai, "The capacity region of the Gaussian MIMO broadcast channel," *Proc. Conf. on Information Sciences and Systems (CISS'04)*, pp. 7–12, Mar. 2004.

[303] E. Wigner, "Characteristic vectors of bordered matrices with infinite dimensions," *Annals of Mathematics*, vol. 62, pp. 546–564, 1955.

[304] E. Wigner, "Results and theory of resonance absorption," in *Conference on Neutron Physics by Time-of-Flight*, Nov. 1-2 1956. Oak Ridge National Lab. Report ORNL-2309.

[305] E. Wigner, "On the distribution of roots of certain symmetric matrices," *Annals of Mathematics*, vol. 67, pp. 325–327, 1958.

[306] E. Wigner, "Statistical properties of real symmetric matrices with many dimensions," *Proc. 4th Canadian Math. Congress*, pp. 174–176, 1959.

[307] E. Wigner, "Distribution laws for the roots of a random Hermitian matrix," in *Statistical Theories of Spectra: Fluctuations*, (C. E. Porter, ed.), New York: Academic, 1965.

[308] E. Wigner, "Random matrices in physics," *SIAM Review*, vol. 9, pp. 1–123, 1967.

[309] J. H. Winters, "Optimum combining in digital mobile radio with cochannel interference," *IEEE J. on Selected Areas in Communications*, vol. 2, pp. 528–539, July 1984.

[310] J. H. Winters, J. Salz, and R. D. Gitlin, "The impact of antenna diversity on the capacity of wireless communication systems," *IEEE Trans. on Communications*, vol. 42, pp. 1740–1751, Feb./Mar./Apr. 1994.

[311] J. Wishart, "The generalized product moment distribution in samples from a normal multivariate population," *Biometrika*, vol. 20 A, pp. 32–52, 1928.

[312] W. Xiao and M. L. Honig, "Large system convergence analysis of adaptive reduced- and full-rank least squares algorithms," *IEEE Trans. on Information Theory*, 2004, to appear.

[313] Y. Q. Yin, "Limiting spectral distribution for a class of random matrices," *J. of Multivariate Analysis*, vol. 20, pp. 50–68, 1986.

[314] Y. Q. Yin and P. R. Krishnaiah, "A limit theorem for the eigenvalues of product of two random matrices," *J. of Multivariate Analysis*, vol. 13, pp. 489–507, 1984.

[315] Y. Q. Yin and P. R. Krishnaiah, "Limit theorem for the eigenvalues of the sample covariance matrix when the underlying distribution is isotropic," *Theory Prob. Appl.*, vol. 30, pp. 861–867, 1985.

[316] W. Yu and J. Cioffi, "Trellis precoding for the broadcast channel," in *Proc. IEEE Global Telecomm. Conf. (GLOBECOM'01)*, pp. 1344–1348, Oct. 2001.

[317] B. M. Zaidel, S. Shamai, and S. Verdú, "Multicell uplink spectral efficiency of coded DS-CDMA with random signatures," *IEEE Journal on Selected Areas in Communications*, vol. 19, pp. 1556–1569, Aug. 2001.

[318] J. Zhang and X. Wang, "Large-system performance analysis of blind and group-blind multiuser receivers," *IEEE Trans. on Information Theory*, vol. 48, pp. 2507–2523, Sep. 2002.