Geometric Programming for Communication Systems

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Introduction

1.1 Geometric Programming and Applications

Geometric Programming (GP) is a class of nonlinear optimization with many useful theoretical and computational properties. Although GP in standard form is apparently a non-convex optimization problem, it can be readily turned into a convex optimization problem, hence a local optimum is also a global optimum, the duality gap is zero under mild conditions,[1] and a global optimum can be computed very efficiently. Convexity and duality properties of GP are well understood, and large-scale, robust numerical solvers for GP are available. Furthermore, special structures in GP and its Lagrange dual problem lead to computational acceleration, distributed algorithms, and physical interpretations.

GP substantially broadens the scope of Linear Programming (LP) applications, and is naturally suited to model several types of important nonlinear systems in science and engineering. Since its inception

[1] Consider the Lagrange dual problem of a given optimization problem. Duality gap is the difference between the optimized primal objective value and the optimized dual objective value.

in 1960s,[2] GP has found applications in mechanical and civil engineering, chemical engineering, probability and statistics, finance and economics, control theory, circuit design, information theory, coding and signal processing, wireless networking, etc. For areas not related to communication systems, a very small sample of some of the GP application papers include [1, 24, 29, 38, 43, 44, 53, 57, 64, 65, 58, 92, 93, 104, 107, 112, 123, 125, 128]. Detailed discussion of GP can be found in the following books, book chapters, and survey articles: [52, 133, 10, 6, 51, 103, 54, 20]. Most of the applications in the 1960s and 1970s were in mechanical, civil, and chemical engineering. After a relatively quiet period in GP research in the 1980s and early to mid-1990s, GP has generated renewed interest since the late 1990s.

Over the last five years, GP has been applied to study a variety of problems in the analysis and design of communication systems, across many 'layers' in the layered architecture, from information theory and queuing theory to signal processing and network protocols. We also start to appreciate *why*, in addition to *how*, GP can be applied to a surprisingly wide range of problems in communication systems. These applications have in turn spurred new research activities on the theory and algorithms of GP, especially generalizations of GP formulations and distributed algorithms to solve GP in a network. This is a systematic survey of the applications of GP to the study of communication systems. It collects in one place various published results in this area, which are currently scattered in several books and many research papers, as well as a couple of unpublished results.

Although GP theory is already well-developed and very efficient GP algorithms are currently available through user-friendly software packages (e.g., MOSEK [129]), researchers interested in using GP still need to acquire the non-trivial capability of modelling or approximating engineering problems as GP. Therefore, in addition to the focus on the application aspects in the context of communication systems, this survey also provides a rather in-depth tutorial on the theory, algorithms, and modeling methods of GP.

[2] Appendix A briefly describes the history of GP.

1.2 Nonlinear Optimization of Communication Systems

LP and other classical optimization techniques have found important applications in communication systems for several decades (e.g., as surveyed in [15, 56]). Recently, there have been many research activities that utilize the power of recent developments in nonlinear convex optimization to tackle a much wider scope of problems in the analysis and design of communication systems.

These research activities are driven by both new demands in the study of communications and networking, and new tools emerging from optimization theory. In particular, a major breakthrough in optimization over the last two decades has been the development of powerful theoretical tools, as well as highly efficient computational algorithms like the interior-point methods (e.g., [12, 16, 17, 21, 97, 98, 111]), for nonlinear convex optimization, i.e., minimizing a convex function subject to upper bound inequality constraints on other convex functions and affine equality constraints:

$$\begin{array}{ll} \textsf{minimize} & f_0(\mathbf{x}) \\ \textsf{subject to} & f_i(\mathbf{x}) \leq 0, \quad i = 1, 2, \ldots, m \\ & \mathbf{A}\mathbf{x} = \mathbf{c} \\ \textsf{variables} & \mathbf{x} \in \mathbf{R}^n. \end{array} \tag{1.1}$$

The constant parameters are $\mathbf{A} \in \mathbf{R}^{l \times n}$ and $\mathbf{c} \in \mathbf{R}^l$. The objective function f_0 to be minimized and m constraint functions $\{f_i\}$ are convex functions.

From basic results in convex analysis [109], it is well known that for a convex optimization problem, a local minimum is also a global minimum. The Lagrange duality theory is also well developed for convex optimization. For example, the duality gap is zero under constraint qualification conditions, such as Slater's condition [21] that requires the existence of a strictly feasible solution to nonlinear inequality constraints. When put in an appropriate form with the right data structure, a convex optimization problem is also easy to solve numerically by efficient algorithms, such as the primal-dual interior-point methods [21, 97], which has worst-case polynomial-time complexity for a large class of functions and scales gracefully with problem size in practice.

Special cases of convex optimization include convex Quadratic Programming (QP), Second Order Cone Programming (SOCP), and Semidefinite Programming (SDP), as well as seemingly non-convex optimization problems that can be readily transformed into convex problems, such as GP. Some of these are covered in recent books on convex optimization, e.g., [12, 16, 17, 21, 97, 98]. While SDP and its special cases of SOCP and convex QP are now well-known in many engineering disciplines, GP is not yet as widely appreciated. This survey aims at enhancing the awareness of the tools available from GP in the communications research community, so as to further strengthen GP's appreciation–application cycle, where more applications (and the associated theoretical, algorithmic, and software developments) are found by researchers as more people start to appreciate the capabilities of GP in modeling, analyzing, and designing communication systems.

There are three distinctive characteristics in the nonlinear optimization framework for the study of communication systems:

- First, the watershed between efficiently solvable optimization problems and intractable ones is being recognized as 'convexity', instead of 'linearity' as was previously believed.[3] This has opened up opportunities on many nonlinear problems in communications and networking based on more accurate or robust modeling of channels and complex interdependency in networks. Inherently nonlinear problems in information theory may also be tackled.
- Second, the nonlinear optimization framework integrates various protocol layers into a coherent structure, providing a unified view on many disparate problems, ranging from classical Shannon theory on channel capacity and rate distortion [33] to Internet engineering such as inter-operability between TCP Vegas and TCP Reno congestion control [119].
- Third, some of these theoretical insights are being put into practice through field trials and industry adoption. Recent

[3] In some cases, global solutions and systematic relaxation techniques for non-convex optimization have also matured [101, 106].

examples include optimization-theoretic improvements of TCP congestion control [71] and DSL broadband access [118].

The phrase "nonlinear optimization of communication systems" in fact carries three different meanings. In the most straightforward way, an analysis or design problem in a communication system may be formulated as either minimizing a cost or maximizing a utility function over a set of variables confined within a constraint set. In a more subtle and recent approach, a given network protocol may be interpreted as a distributed algorithm solving an implicitly defined global optimization problem. In yet another approach, the underlying theory of a network control method or a communication strategy may be generalized using nonlinear optimization techniques, thus extending the scope of applicability of the theory. In Section 3, we will see that GP applications cover all three categories.

1.3 Overview

There are three main sections in this survey. Section 2 is a tutorial of GP: its basic formulations, convexity and duality properties, various extensions that significantly broaden the scope of applicability of the basic formulations, as well as numerical methods, robust solutions, and distributed algorithms for GP. Although this section does not cover any application topic, it is essential for modeling communication system problems in terms of GP and its generalizations.[4]

Section 3 is the core of this survey, presenting many applications of GP in the analysis and design of communication systems: the information theoretic problems of channel capacity, rate distortion, and error exponent in Subsection 3.1, construction of channel codes, relaxation of source coding problems, and digital signal processing algorithms for physical layer transceiver design in Subsection 3.2, network resource allocation algorithms such as power control in wireless networks in Subsection 3.3, network congestion control protocols in TCP Vegas and its cross-layer extensions in Subsection 3.4, and performance optimization of simple queuing systems in Subsection 3.5.

[4] For another very recent GP tutorial, readers are referred to a recent survey of GP for circuit design problems [20].

These applications generally fall into three categories: analysis (e.g., GP is used to characterize and bound information theoretic limits), forward engineering (e.g., GP is used to control transmit powers in wireless networks), and reverse engineering (e.g., GP is used to model congestion control or Highly Optimized Tolerance systems).

Then Section 4 explains why, rather than just how, GP can be applied to such a variety of problems in communication systems. As shown in Subsection 4.1, for problems based on stochastic models, GP is often applicable because large deviation bounds can be computed by GPs. As shown in Subsection 4.2, for problems based on deterministic models, reasons for applicability of GP is less well understood but may be due to GP's connections with proportional allocation, general market equilibrium, and generalized coding problems.

In the area of GP applications for communication systems, there are three most interesting directions of future research in author's view: distributed algorithms and heuristics for solving GP in a network, a systematic theory of using a nested family of GP relaxations for non-convex, generalized polynomial optimization, and the connections of GP with the theories of large deviation and general market equilibrium. These issues are discussed throughout the survey.

Some subsections in these three sections present unpublished results while most subsections summarize known results. In particular, Subsection 2.1 is partially based on [10, 21, 30, 52, 132], Subsection 2.2 on [6, 7, 10, 20, 51, 52, 103, 133], Subsection 2.3 on [21, 60, 67, 78, 37], Subsection 3.1 on [30, 33, 42, 82, 84, 120, 121, 122], Subsection 3.2 on [25, 30, 69, 75, 91], Subsection 3.3 on [37, 34, 35, 72, 73], Subsection 3.4 on [31, 88], Subsection 3.5 on [36, 68, 76], Subsection 4.1 on [30, 42, 45, 52, 108], and Subsection 4.2 on [28, 49, 70].

A brief historical account of the development of GP is provided in Appendix A and selected proofs are provided in Appendix B.

1.4 Notation

We will use the following notation. Vectors and matrices are denoted in boldface. Given two column vectors $\mathbf{x}$ and $\mathbf{y}$ of length n, we express the sum $\sum_{i=1}^{n} x_i y_i$ as an inner product $\mathbf{x}^T\mathbf{y}$. Componentwise inequalities on a vector $\mathbf{x}$ with n entries are expressed using the $\succeq$ symbol: $\mathbf{x} \succeq 0$ denotes $x_i \geq 0, i = 1, 2, \ldots, n$. A column vector with all entries being 1 is denoted as $\mathbf{1}$. We use $\mathbf{R}^n_+$ and $\mathbf{R}^n_{++}$ to denote the non-negative and strictly positive quadrant of n-dimensional Euclidean space, respectively, and $\mathcal{Z}_+$ to denote the set of non-negative integers.

Sometimes a symbol has different meanings in different sections, because the same symbol is widely accepted as the standard notation representing different quantities in more than one field. For example, $\mathbf{P}$ denotes channel transition matrix in Subsection 3.1.1 on channel capacity, and denotes transmit power vector in Subsections 3.3.1 and 3.4.2 on wireless network power control. Such notational reuse should not cause any confusion since consistency is maintained within any single subsection.

All constrained optimization problems are written in this survey following a common format: objective function, constraints, and optimization variables. Constant parameters are also explicitly stated after the problem statement in cases where confusion may arise.

References

[1] M. Abou-El-Ata, H. Bergany, and M. El-Wakeel, "Probabilistic multi-item inventory model with varying order cost under two restrictions: A geometric programming approach," *International Journal of Production Economics*, vol. 83, no. 3, pp. 223–231, 2003.

[2] R. Albert and A. L. Barabasi, "Statistical mechanics of complex networks," *Review of Modern Physics*, vol. 47, 2002.

[3] S. Arimoto, "An algorithm for computing the capacity of arbitrary discrete memoryless channels," *IEEE Transactions on Information Theory*, vol. 18, pp. 14–20, Jan. 1972.

[4] S. Arimoto, "On the converse to the coding theorem for discrete memoryless channels," *IEEE Transactions on Information Theory*, vol. 19, pp. 357–359, May 1973.

[5] K. Arrow and G. Debreu, "Existence of an equilibrium for a competitive economy," *Econometrica*, vol. 22, pp. 265–290, 1954.

[6] M. Avriel, *Advances in Geometric Programming.* Plenum Press, 1980.

[7] M. Avriel, R. Dembo, and U. Passy, "Solution of generalized geometric programs," *International Journal of Numerical Methods in Engineering*, vol. 9, pp. 149–168, 1975.

[8] M. Avriel, M. J. Rijckaert, and D. J. Wilde, *Optimization and Design.* Prentice Hall, 1973.

[9] P. Bak, *How Nature Works: The Science of Self Organized Criticality.* Copernicus, 1996.

[10] C. S. Beightler and D. T. Philips, *Applied Geometric Programming.* Wiley, 1976.

[11] A. Ben-Tal and A. Nemirovski, "Robust convex optimization," *Mathematics of Operations Research*, vol. 23, no. 4, pp. 769–805, 1998.

[12] A. Ben-Tal and A. Nemirovski, *Lectures on Modern Convex Optimization: Analysis, Algorithms, and Engineering Applications*. SIAM, 2001.

[13] A. Ben-Tal, A. Nemirovski, and C. Ross, "Robust solutions of uncertain quadratic and conic-quadratic problems," *SIAM Journal of Optimization*, vol. 13, no. 2, pp. 535–560, 2002.

[14] T. Berger, *Rate Distortion Theory: A Mathematical Basis for Data Compression*. Prentice Hall, 1971.

[15] D. Bertsekas and R. G. Gallager, *Data Networks*. Prentice Hall, 1991.

[16] D. P. Bertsekas, *Nonlinear Programming, 2nd Ed.* Athena Scientific, 1999.

[17] D. P. Bertsekas, E. Nedic, and A. Ozdaglar, *Convex Analysis and Optimization*. Athena Scientific, 2003.

[18] D. P. Bertsekas and J. N. Tsitsiklis, *Parallel and Distributed Computation*. Prentice Hall, 1989.

[19] R. E. Blahut, "Computation of channel capacity and rate distortion function," *IEEE Transactions on Information Theory*, vol. 18, pp. 450–473, 1972.

[20] S. Boyd, S. J. Kim, L. Vandenberghe, and A. Hassibi, "A tutorial on geometric programming," *Stanford University EE Technical Report*, 2004.

[21] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[22] W. C. Brainard and H. E. Scarf, "How to compute equilibrium prices in 1891," *Cowles Foundation Discussion Paper*, 2000.

[23] L. S. Brakmo and L. L. Peterson, "TCP Vegas: End-to-end congestion avoidance on a global Internet," *IEEE Journal of Selected Areas in Communications*, vol. 13, pp. 1465–1480, October 1995.

[24] D. Bricker, K. Kortanek, and L. Xui, "Maximum likelihood estimates with order restrictions on probabilities and odd ratios: A geometric programming approach," *Journal of Applied Mathematics and Decision Sciences*, vol. 1, no. 1, pp. 53–65, 1997.

[25] L. Campbell, "A coding theorem and Renyi's entropy," *Information and Control*, vol. 8, pp. 423–429, 1965.

[26] B. Y. Cao, *Fuzzy Geometric Programming*. Kluwar Academic Publisher, October 2002.

[27] J. M. Carlson and J. Doyle, "Complexity and robustness," *Proceedings of National Academy of Sciences*, vol. 99, pp. 2538–2545, February 2002.

[28] J. M. Carlson and J. C. Doyle, "Highly Optimized Tolerance: Robustness and design in complex systems," *Physics Review Letters*, vol. 84, no. 11, pp. 2529–2532, 2000.

[29] T. Y. Chen, "Structural optimization using single-term posynomial geometric programming," *Computers and Structures*, vol. 45, pp. 911–918, 1992.

[30] M. Chiang, *Solving Nonlinear Problems in Communication Systems Using Dualities and Geometric Programming*. PhD thesis, Stanford University, 2003.

[31] M. Chiang, "Balancing transport and physical layers in wireless multihop networks: Joingly optimal congestion control and power control," *IEEE Journal of Selected Areas in Communications*, vol. 23, no. 1, pp. 104–116, 2005.

[32] M. Chiang and N. Bambos, "Distributed network control through sum product algorithms on graphs," in *Proceedings of IEEE Infocom*, 2002.

[33] M. Chiang and S. Boyd, "Geometric programming duals of channel capacity and rate distortion," *IEEE Transactions on Information Theory*, vol. 50, pp. 245–258, Feb. 2004.

[34] M. Chiang, D. O'Neill, D. Julian, and S. Boyd, "Resource allocation for QoS provisioning in wireless ad hoc networks.," in *Proceedings of IEEE Infocom*, 2001.

[35] M. Chiang and A. Sutivong, "Efficient nonlinear optimization of resource allocation," in *Proceedings of IEEE Infocom*, 2003.

[36] M. Chiang, A. Sutivong, and S. Boyd, "Efficient nonlinear optimizations of queuing systems," in *Proceedings of IEEE Infocom*, 2002.

[37] M. Chiang, C. W. Tan, D. Palomar, D. O'Neill, and D. Julian, ch. Geometric programming for wireless network power control, *Resource Allocation in Next Generation Wireless Networks*. Nova Science Publisher, 2005.

[38] C. Chu and D. Wong, "VLSI circuit performance optimization by geometric programming," *Annals of Operations Research*, vol. 105, pp. 37–60, 2001.

[39] T. M. Cover and M. Chiang, "Duality between channel capacity and rate distortion with state information," *IEEE Transactions on Information Theory*, vol. 48, pp. 1629–1638, June 2002.

[40] T. M. Cover and J. Thomas, *Elements of Information Theory*. Wiley, 1991.

[41] I. Csiszar, "The method of types," *IEEE Transactions on Information Theory*, vol. 44, no. 6, pp. 2505–2523, 1998.

[42] I. Csiszar and J. Korner, *Information Theory: Coding Theorems for Discrete Memoryless Systems*. Academic Press, 1981.

[43] W. Daems, G. Geilen, and W. Sansen, "Simulation-based generation of posynomial performance models for the sizing of analog integrated circuits," *IEEE Transactions on Computer Aided Design of Integrated Circuits and Systems*, vol. 22, no. 5, pp. 517–534, 2003.

[44] J. Dawson, S. Boyd, M. Hershenson, and T. Lee, "Optimal allocation of local feedback in multistage amplifiers via geometric programming," *IEEE Transactions on Circuits and Systems I*, vol. 48, no. 1, pp. 1–11, 2001.

[45] A. Dembo and D. Zeitouni, *Large Deviations: Techniques and Applications*. Springer Verlag, 1998.

[46] R. Dembo, "Sensitivity analysis in geometric programming," *Journal of Optimization Theory and Applications*, vol. 37, pp. 1–21, 1982.

[47] R. S. Dembo, "A set of geometric programming test problems," *Mathematical Programming*, vol. 10, no. 192-213, 1976.

[48] J. Dinkel, M. Kochenberger, and S. Wong, "Sensitivity analysis procedures for geometric programs: Computational aspects," *ACM Transactions on Mathematical Software*, vol. 4, no. 1, pp. 1–14, 1978.

[49] J. C. Doyle and J. M. Carlson, "Power laws, Highly Optimized Tolerance and generalized source coding," *Physics Review Letters*, vol. 84, no. 24, pp. 5656–5659, 2000.

[50] M. Drmota and W. Szpankowski, "The precise minimax redundancy," in *Proceedings of IEEE International Symposium on Information Theory*, 2002.

[51] R. J. Duffin, "Linearized geometric programs," *SIAM Review*, vol. 12, pp. 211–227, 1970.
[52] R. J. Duffin, E. L. Peterson, and C. Zener, *Geometric Programming: Theory and Applications*. Wiley, 1967.
[53] A. Dutta and D. V. Rama, "An optimization model of communications satellite planning," *IEEE Transactions on Communications*, vol. 40, no. 9, pp. 1463–1473, 1992.
[54] J. Ecker, "Geometric programming: Methods, computations and applications," *SIAM Review*, vol. 22, no. 3, pp. 338–362, 1980.
[55] E. Eisenberg and D. Gale, "Consensus of subjective probabilities: The parimutual method," *Annals of Mathematical Statistics*, vol. 30, pp. 165–168, 1959.
[56] A. Ephremides and S. Verdú, "Control and optimization methods in communication network problems," *IEEE Transactions on Automatic Control*, vol. 9, pp. 930–942, 1989.
[57] S. C. Fang, J. R. Rajasekera, and H. Tsao, *Entropy Optimization and Mathematical Programming*. Kluwer Academic Publishers, 1997.
[58] F. Feigin and U. Passy, "The geometric programming dual to the extinction probability problem in simple branch processes," *The Annals of Probability*, vol. 9, no. 3, pp. 498–503, 1981.
[59] C. A. Floudas, *Deterministic Global Optimization: Theory, Algorithms, and Applications*. Kluwer Academic Publishers, 1999.
[60] G. J. Foschini and Z. Miljanic, "A simple distributed autonomour power control algorithm and its convergence," *IEEE Transactions on Vehicular Technology*, vol. 42, no. 4, 1993.
[61] R. G. Gallager, *Information Theory and Reliable Communication*. Wiley, 1968.
[62] L. E. Ghaoui and H. Lebret, "Robust solutions to least square problems with uncertain data," *SIAM Journal of Matrix Analysis and Applications*, vol. 18, no. 4, pp. 1035–1064, 1997.
[63] L. E. Ghaoui and H. Lebret, "Robust solutions to uncertain semidefinite programs," *SIAM Journal of Optimization*, vol. 9, no. 1, pp. 33–52, 1998.
[64] H. Greenberg, "Mathematical programming models for environmental quality control," *Operations Research*, vol. 43, no. 4, pp. 578–622, 1995.
[65] P. Hajela, "Geometric programming strategies for large scale structural synthesis," *AIAA Journal*, vol. 24, no. 7, pp. 1173–1178, 1986.
[66] M. Hersehnson, S. Boyd, and T. H. Lee, "Optimal design of a CMOS Op-Amp via geometric programming," *IEEE Transactions on Computer Aided Design of Integrated Circuits and Systems*, vol. 20, no. 1, pp. 1–21, 2001.
[67] K. L. Hsiung, S. J. Kim, and S. Boyd, "Robust geometric programming via piecewise linear approximiation," *Mathematical Programming*, 2005.
[68] J. Y. Hui, "Resource allocation for broadband networks," *IEEE Journal of Selected Areas in Communications*, pp. 1598–1608, 1988.
[69] P. A. Humblet, "Generalization of Huffman coding to minimize the probability of buffer overflow," *IEEE Transactions on Information Theory*, vol. 27, pp. 230–232, 1981.

[70] K. Jain, "A polynomial time algorithm for computing the Arrow-Debreu market equilibrium for linear utilities," in *Proceedings of IEEE Foundation of Computer Science*, 2004.

[71] C. Jin, D. X. Wei, and S. H. Low, "TCP FAST: motivation, architecture, algorithms, performance," in *Proceedings of IEEE Infocom*, 2004.

[72] D. Julian, M. Chiang, D. O'Neill, and S. Boyd, "QoS and fairness constrained convex optimization of resource allocation for wireless cellular and ad hoc networks," in *Proceedings of IEEE Infocom*, 2002.

[73] S. Kandukuri and S. Boyd, "Optimal power control in interference limited fading wireless channels with outage probability specifications," *IEEE Transactions on Wireless Communications*, vol. 1, no. 1, Jan. 2002.

[74] J. Karlof, "Permutation codes for the Gaussian channel," *IEEE Transactions on Information Theory*, vol. 35, pp. 726–732, July 1989.

[75] J. K. Karlof and Y. O. Chang, "Optimal permutation codes for the Gaussian channel," *IEEE Transactions on Information Theory*, vol. 43, no. 1, pp. 356–358, 1997.

[76] F. P. Kelly, "Notes on effective bandwidth," *Stochastic Networks: Theory and Applications*, pp. 141–168, 1996.

[77] F. P. Kelly, A. Maulloo, and D. Tan, "Rate control for communication networks: Shadow prices, proportional fairness and stability," *Journal of Operations Research Society*, vol. 49, no. 3, pp. 237–252, 1998.

[78] K. O. Kortanek, X. Xu, and Y. Ye, "An infeasible interior-point algorithm for solving primal and dual geometric programs," *Mathematical Programming*, vol. 76, pp. 155–181, 1996.

[79] S. Kunniyur and R. Srikant, "End-to-end congestion control: Utility functions, random losses and ECN marks," *IEEE/ACM Transactions on Networking*, pp. 689–702, Oct. 2003.

[80] J. Kyparsis, "Sensitivity analysis in geometric programming: Theory and computation," *Annals of Operations Research*, vol. 27, pp. 39–64, 1990.

[81] R. J. La and V. Anantharam, "Utility-based rate control in the Internet for elastic traffic," *IEEE/ACM Transactions on Networking*, vol. 10, no. 2, pp. 272–286, 2002.

[82] A. Lapidoth and N. Miliou, "Duality bounds on the cut-off rate with applications to Ricean fading," *Submitted to IEEE Transactions on Information Theory*, March 2005.

[83] A. Lapidoth and S. M. Moser, "Capacity bounds via duality with applications to multi-antenna systems on flat fading channels," *IEEE Transactions on Information Theory*, vol. 49, no. 10, pp. 2426–2467, 2003.

[84] Y. Li, M. Chiang, and S. Verdu, "Lagrange duality of random coding error exponents," Preprint, Princeton University, 2005.

[85] S. H. Low, "A duality model of TCP and queue management algorithms," *IEEE/ACM Transactions on Networking*, vol. 11, no. 4, pp. 525–536, 2003.

[86] S. H. Low and D. E. Lapsley, "Optimization flow control, I: basic algorithm and convergence," *IEEE/ACM Transactions on Networking*, vol. 7, no. 6, pp. 861–874, 1999.

[87] S. H. Low, F. Paganini, and J. C. Doyle, "Internet congestion control," *IEEE Control Systems Magazine*, vol. 22, pp. 28–43, February 2002.
[88] S. H. Low, L. Peterson, and L. Wang, "Understanding Vegas: A duality model," *Journal of ACM*, vol. 49, no. 2, pp. 207–235, 2002.
[89] S. H. Low and R. Srikant, "A mathematical framework for designing a low-loss, low-delay internet," *Networks and Spatial Economics, special issue on "Crossovers between transportation planning and telecommunications",* E. Altman and L. Wynter, 2003.
[90] D. Luenberger, "A double look at duality," *IEEE Transactions Automatic Control*, 1992.
[91] T. Luo, "Optimal zero forcing transceiver design for multiaccess communications," McMaster University ECE Technical Report, 2001.
[92] C. Maranas and C. Foudas, "Global optimization in generalized geometric programming," *Computers and Chemical Engineering*, vol. 21, no. 4, pp. 351–369, 1997.
[93] M. Mazumdar and T. R. Jefferson, "Maximum likelihood estimates for multinomial probabilities via geometric programming," *Biometrika*, vol. 70, no. 1, pp. 257–261, 1983.
[94] R. McEliece, *Information Theory and Coding*. Wiley, 1976.
[95] J. Mo, R. La, V. Anantharam, and J. Walrand, "Analysis and comparison of TCP Reno and Vegas," in *Proceedings of IEEE Infocom*, 1999.
[96] J. Mo and J. Walrand, "Fair end-to-end window-based congestion control," *IEEE/ACM Transactions on Networking*, vol. 8, no. 5, pp. 556–567, 2000.
[97] Y. Nesterov and A. Nemirovsky, *Interior Point Polynomial Algorithms in Convex Programming*. SIAM, 1994.
[98] J. Nocedal and S. J. Wright, *Numerical Optimization*. Springer Verlag, 1999.
[99] D. O'Neill, "Adaptive congestion control for wireless networks using TCP," in *Proceedings of IEEE International Conference on Communications*, 2003.
[100] A. K. Parekh and R. Gallager, "A Generalized processor sharing approach to flow control in integrated services networks: The single node case," *IEEE/ACM Transactions on Networking*, vol. 1, no. 3, pp. 344–357, 1993.
[101] P. Parrilo, *Structured Semidefinite Programs and Semialgebraic Geometry Methods in Robustness and Optimization*. PhD thesis, Caltech, 2000.
[102] P. Parrilo, "Semidefinite programming relaxations for semialgebraic problems," *Mathematical Programming Series B*, vol. 96, no. 2, pp. 293–320, 2003.
[103] E. L. Perterson, "Geometric programming: A survey," *SIAM Review*, vol. 18, pp. 1–51, 1976.
[104] E. Peterson, "Investigation of path following algorithms for signomial geometric programming problems," *Annals of Operations Research*, vol. 105, pp. 15–19, 2001.
[105] S. Pradhan, J., and K. Ramchandran, "Duality between source coding and channel coding and its extension to the side information caes," *IEEE Transactions on Information Theory*, vol. 49, no. 5, pp. 1181–1203, 2003.
[106] S. Prajna, A. Papachristodoulou, and P. A. Parrilo, "Introducing SOSTOOLS: A general purpose sum of squares programming solver," in *Proceedings of IEEE Conference on Decision and Control*, 2002.

[107] J. Rajasekera and M. Yamada, "Estimating the firm value distribution function by entropy optimization and geometric programming," *Annals of Operations Research*, vol. 105, pp. 61–75, 2001.

[108] F. Reif, *Fundamentals of Statistical and Thermal Physics.* McGraw Hill, 1965.

[109] R. T. Rockafellar, *Convex Analysis.* Princeton University Press, 1970.

[110] R. T. Rockafellar, ch. Saddle-points and convex analysis, *Differential Games and Related Topics.* North-Holland, 1971.

[111] R. T. Rockafellar, "Lagrange multipliers and optimality," *SIAM Review*, vol. 35, pp. 183–283, 1993.

[112] S. Sapatnekar, V. Rao, P. Vaidya, and S. Kang, "An exact solution to the transistor sizing problem for CMOS circuits using convex optimization," *IEEE Transactions on Computer Aided Design of Integrated Circuits and Systems*, vol. 12, no. 11, pp. 1621–1634, 1993.

[113] C. E. Shannon, "A mathematical theory of communications," *Bell System Technical Journal*, pp. 379–423, 623–656, 1948.

[114] C. E. Shannon, "Coding theorems for a discrete source with a fidelity criterion," in *IRE National Convention Record*, 1959.

[115] C. Sims, "Computational methods for permutation groups," in *Computational Problems in Abstract Algebra*, 1970.

[116] D. Slepian, "Group codes for the Gaussian channel," *Bell Systems Technical Journals*, vol. 17, pp. 575–602, 1968.

[117] R. Srikant, *The Mathematics of Internet Congestion Control.* Birkhauser, 2004.

[118] T. Starr, M. Sorbara, J. Cioffi, and P. Silverman, *DSL Advances.* Prentice Hall, 2003.

[119] A. Tang, J. Wang, S. H. Low, and M. Chiang, "Equilibrium of heterogeneous congestion control protocols," in *Proceedings of IEEE Infocom*, 2005.

[120] M. Teboulle and A. Ben-Tal, "Rate distortion theory with generalized infomration measure via convex programming duality," *IEEE Transactions on Information Theory*, vol. 32, pp. 630–641, 1986.

[121] M. Teboulle and A. Ben-Tal, "Extension of some results for capacity using a generalized information measure," *Applied Mathematics and Optimization*, vol. 17, pp. 121–132, 1988.

[122] T. Terlaky, E. Klafszky, and Mayer, "A geometric programming approach to the channel capacity problem," *Engineering Mathematics*, vol. 19, pp. 115–130, 1992.

[123] J. F. Tsai, H. L. Li, and N. Z. Hu, "Global optimization for signomial discrete programming problems in engineering design," *Engineering Optimization*, vol. 34, no. 6, pp. 613–622, 2002.

[124] P. O. Vontobel and D. M. Arnold, "An upper bound on the capacity of channels with memory and constraint input," in *Proceedings of Information Theory on Workshop*, 2001.

[125] T. Wall, D. Greening, and R. Woolsey, "Solving complex chemical equilibria using a geometric programming based technique," *Operations Research*, vol. 34, no. 3, pp. 345–355, 1986.

[126] L. Walras, *Elements of Pure Economics, Or The Theory of Social Wealth.* Lausanne, 1874.

[127] A. Weiss, "An introduction to large deviations for communication networks," *IEEE Journal of Selected Areas in Communications*, vol. 13, no. 6, pp. 938–952, 1995.

[128] D. Wong, "Maximum likelihood, entropy maximization, and the geometric programming approaches to the calibration of trip distribution models," *Transportation Research Part B: Methodological*, vol. 15, no. 5, pp. 329–343, 1981.

[129] www.mosek.com, *MOSEK Optimization Toolbox.* 2002.

[130] A. D. Wyner and J. Ziv, "The rate distortion function for source coding with side information at the decode," *IEEE Transactions on Information Theory*, vol. 22, pp. 1–10, 1976.

[131] L. Xiao, M. Johansson, and S. Boyd, "Simultaneous routing and resource allocation for wireless networks," *IEEE Transactions of Communications*, vol. 52, no. 7, pp. 1136–1144, July 2004.

[132] C. Zener, "A mathematical aid in optimizing engineering design," in *Proceedings of National Academy of Sciences*, pp. 537–539, 1961.

[133] C. Zener, *Engineering Design By Geometric Programming.* Wiley, 1971.