Majorization and Matrix-Monotone Functions in Wireless Communications

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Abstract

This short tutorial presents two mathematical techniques namely *Majorization Theory* and *Matrix-Monotone Functions*, reviews their basic definitions and describes their concepts clearly with many illustrative examples. In addition to this tutorial, new results are presented with respect to Schur-convex functions and regarding the properties of matrix-monotone functions.

The techniques are applied to solve communication and information theoretic problems in wireless communications. The impact of spatial correlation in multiple antenna systems is characterized for many important performance measures, e.g., average mutual information, outage probability, error performance, minimum $\frac{E_b}{N_0}$ and wideband slope, zero-outage capacity, and capacity region. The impact of user distribution in cellular systems is characterized for different scenarios including perfectly informed transmitters and receivers, regarding, e.g., the average sum rate, the outage sum rate, maximum throughput. Finally, a unified framework for the performance analysis of multiple antenna systems is developed based on matrix-monotone functions. The optimization of transmit strategies for multiple antennas is carried out by optimization of matrix-monotone functions. The results within this framework resemble and complement the various results on optimal transmit strategies in single-user and multiple-user multiple-antenna systems.

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1

Introduction

This short tutorial presents two mathematical techniques namely *Majorization Theory* and *Matrix-Monotone Functions* which are applied to solve communication and information theoretic problems in wireless communications.

1.1 Majorization Theory

Inequalities have been always a major mathematical research area beginning with Gauß, Cauchy, and others. Pure and applied mathematical analysis needs inequalities, e.g., absolute inequalities, triangle inequalities, integral or differential inequalities, and so on. The building blocks of Majorization are contained in the book [48]. The complete theory including many applications is presented in [92]. The theory is about the question how to order vectors with nonnegative real components and its order-preserving functions, i.e., functions f which satisfy that for $\boldsymbol{x} \succeq \boldsymbol{y}$ it follows $f(\boldsymbol{x}) \geq f(\boldsymbol{y})$. The characterization of this class of functions is important to exploit the properties of this monotony.

In the wireless communication context, those functions arise naturally in resource allocation for multiple user systems or multiple antenna systems, e.g., sum rate of the multiple access channel (MAC) with K users and channels $\alpha_1,\ldots,\alpha_K$ as a function of the power allocation $p_1,\ldots,p_K$ with inverse noise power ρ

$$C(\boldsymbol{p}) = \log\left(1 + \rho\sum_{k=1}^{K} p_k\alpha_k\right).$$

Assume that the sum power is constraint to K, i.e., $\sum_{k=1}^{K} p_k = K$. Order the components $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_K \geq 0$ and $p_1 \geq p_2 \geq \cdots \geq p_K \geq 0$. The function C turns out to be Schur-convex with respect to $\boldsymbol{p}$, i.e., monotonic decreasing with respect to the Majorization order. If $\boldsymbol{p} \succeq \boldsymbol{q}$ then $C(\boldsymbol{p}) \geq C(\boldsymbol{q})$. Therefore, the maximum value is attained for a power allocation vector with elements, i.e., $C([K,0,\ldots,0]) \geq C(\boldsymbol{p}) \geq C(\mathbf{1})$.

This monotony behavior is illustrated for $K = 2$ with power allocation $\boldsymbol{p} = [2-p,p]$ in Figure 1.1. This result implies that TDMA is optimal, because the complete transmit power is optimally allocated to one user [80].

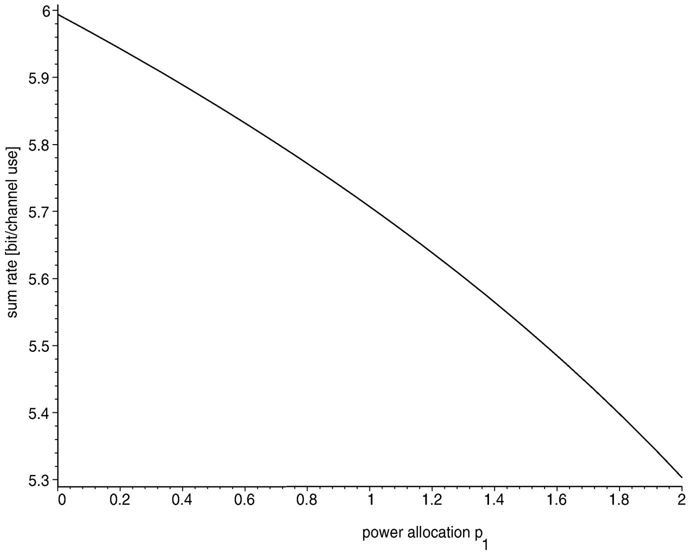

Fig. 1.1 Sum rate of MAC with channels $\alpha_1 = 2$, $\alpha_2 = 1$ as a function of the power allocation $\boldsymbol{p} = [2-p,p]$.

Most of the basic definitions and basic properties can be found in the text books [8, 48, 50, 51, 92]. Majorization theory is a valuable tool and it is successfully applied in many research areas, e.g., in optimization [39, 168], signal processing and mobile communications [59, 105], and quantum information theory [101].

1.2 Matrix-Monotone Functions

More than 70 years have passed since Löwner [88] proposed the notion of matrix-monotone functions. A real, continuous function $f:\mathcal{I}\to\mathbb{R}$ defined on a nontrivial interval $\mathcal{I}$ is said to be matrix monotone of order n if

$$\boldsymbol{X} \geq \boldsymbol{Y} \Rightarrow f(\boldsymbol{X}) \geq f(\boldsymbol{Y})$$

for any pair of self-adjoint $n \times n$ matrices $\boldsymbol{X}$ and $\boldsymbol{Y}$ with eigenvalues in $\mathcal{I}$. Löwner characterized the set of matrix-monotone functions of order n in terms of the positivity of certain determinants (the so-called Löwner determinants and the related Pick determinants), and proved that a function is matrix monotone if and only if it allows an analytic continuation to a Pick function; that is, an analytic function defined in the complex upper half-plane, with nonnegative imaginary part. A function is called matrix monotone if it is matrix monotone for all orders n.

A representation theorem was proven for the class of matrix-monotone functions [34, 83, 88, 156]. Every matrix-monotone function f can be expressed as

$$f(t) = a + bt + \int_0^\infty \frac{st}{s+t} d\mu(s) \tag{1.1}$$

with a positive measure $\mu \in [0,\infty)$ and real constants $a,b \geq 0$.

Representatives of the class of matrix-monotone functions arise naturally in the context of multiple antenna systems in the single- as well as in the multiuser context. The two most important examples are the mutual information and the minimum mean square error (MMSE) in multiple-input multiple-output (MIMO) systems. Consider the mutual information[1] for the vector model $\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}$ between $\boldsymbol{x}$ and $\boldsymbol{y}$ for independently complex zero-mean Gaussian distributed $\boldsymbol{x}$ and $\boldsymbol{n}$ with covariances $\boldsymbol{Q}$ and $\boldsymbol{I}$

$$f(\boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^H) = I(\boldsymbol{x};\boldsymbol{y}) = \log\det\left(\boldsymbol{I} + \boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^H\right).$$

The mutual information denoted as the function $f(\boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^H) = \operatorname{tr}\log\left(\boldsymbol{I} + \boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^H\right)$ can be represented by the matrix-monotone function $f(t) = \log(1+t)$ which has the integral representation

$$f(t) = \int_1^\infty \frac{t}{s+t}\frac{1}{s}ds.$$

Hence, all results that hold for matrix-monotone function also hold for the mutual information and (as we will show later) for the MMSE. This approach allows to unify many recent results and it is possible to extract the main principles and concepts.

Finally, matrix-monotone functions are applied in many other areas, e.g., in optimization [25] and signal processing for communications [71].

1.3 Classification and Organization

1.3.1 Classification and Differences to Related Literature

The well-established book [92] contains more results on Majorization than this short tutorial. The main difference is that this tutorial focusses on a subset of topics from [92], especially results regarding averages and distributions of weighted random variables, as well as averages of trace functions. These topics are treated in more detail, new results are added (from subsection 2.2.3 until subsection 2.2.7), and the connection to the application in communication theory is always kept in mind. Furthermore, the first two tutorial chapters are rigorous in the sense that they contain all necessary definitions and results but additionally contain also many remarks and examples which help the reader to understand the concepts.

There exist approaches in the literature that propose a unified framework for analysis and optimization of MIMO systems. First, the

---

[1] Without any operational meaning for the moment.

PhD thesis [104] provides a framework for optimization of linear MIMO systems also by using Majorization theory. The tutorial [107] extends these results to nonlinear decision feedback MIMO systems. Interestingly, the application of Majorization in the other tutorial [107] is not for analysis of impact of fading parameters on system performance but for the optimization of single-user transmit strategies under various objective criteria. Another difference to the tutorial [107] is that the article at hand offers two own full chapters with a tutorial of the mathematical techniques used. Therefore, both tutorial complement one another well.

Another related tutorial is [122] which studies the active field of interference function calculus. An interesting overview presentation is given in the plenary lecture at the workshop on signal processing advances in wireless communications in June 2007 [12].

Furthermore, a unified analytical description of MIMO systems was studied in the PhD thesis [79]. The main focus in [79] is to derive a framework for analytically computing closed-form expressions of MIMO transceiver performances which are then used for optimization. Finally, the connection between the capacity and mean-square-error (MSE) from an estimation and information theoretic point of view was analyzed in the PhD thesis [42]. The thesis contains one part that clearly shows the connection between the capacity and MMSE for various channel models, e.g., discrete, continuous, scalar, and vector channels and different input signals. In subsection 5.1.2 three different relationships between the mutual information and the MMSE are described.

1.3.2 Organization

The first two chapters present the definitions, properties, and many examples to explain the foundations and concepts of the two techniques. The three main topics discussed are

(a) the partial order on vectors and matrices,
(b) the characterization of order preserving functions,
(c) the optimization of Schur-convex and matrix-monotone functions.

The main goal of these two chapters is to make the reader familiar with the basic concepts and to enable her to apply these techniques to problems in his or her respective research area. The various examples illustrate the theoretical concepts and reconnect to practical problem statements. In "Majorization Theory," we present novel results with respect to Schur-convexity and Schur-concavity for the most general classes of functions and constraints. Later in "Application of Majorization in Wireless Communications," these functions obtain their operational meaning in the context of communication theory. In "Matrix-Monotone Functions," we present novel results in terms of bounds for matrix-monotone functions, optimization of matrix-monotone functions, and discuss the connection to matrix norms as well as to connections and means.

In "Application of Majorization in Wireless Communications" and "Application of Matrix-Monotone Functions in Wireless Communications," we apply the learned techniques to concrete problem statements from wireless communications. The four main application areas are

(a) spatial correlation in multiple antenna systems,
(b) user distributions in cellular systems,
(c) development of a unified performance measure,
(d) optimization of MIMO system performance.

The main goal of these two chapters is to show under which conditions and assumptions both techniques can be used. Furthermore, it is shown how to interpret the results carefully to gain engineering insights into the design of wireless communication systems. In "Application of Majorization in Wireless Communications," a measure for spatial correlation in multiple antenna communications is developed. This measure is exploited for various performance measures and in many scenarios to analyze the impact of spatial correlation. A measure for the user distribution in cellular systems is developed and the sum performance of up- and downlink communication as a function of the user distribution is analyzed. In "Application of Matrix-Monotone Functions in Wireless Communications," we develop a generalized performance measure which unifies mutual information and MMSE criteria. Finally, the results from "Matrix-Monotone Functions" are used to optimize the single-user and multi-user MIMO system performance.

The appendix contains two sections with basic results from Linear Algebra and Convex Optimization. These results are used extensively throughout the book.

1.4 Notation

Vectors are denoted by boldface small letters $\boldsymbol{a},\boldsymbol{b}$, and matrices by boldface capital letters $\boldsymbol{A},\boldsymbol{B}$. $\boldsymbol{A}^T$, $\boldsymbol{A}^H$, and $\boldsymbol{A}^{-1}$ are the transpose, the conjugate transpose, and the inverse matrix operation, respectively. The identity matrix is $\boldsymbol{I}$, and $\mathbf{1}$ is the vector with all ones. $\circ$ is the Schur-product and $\otimes$ is the Kronecker product. $\mathrm{diag}(\boldsymbol{X})$ is a vector with the entries of $\boldsymbol{X}$ on the diagonal. $\mathrm{Diag}(\boldsymbol{x})$ is a diagonal matrix with the entries of the vector $\boldsymbol{x}$ on its diagonal. $\mathrm{Diag}(\boldsymbol{A},\boldsymbol{B})$ is a block-diagonal matrix with matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ on the diagonal. $\boldsymbol{A}^{1/2}$ is the square root matrix of $\boldsymbol{A}$ and $[\boldsymbol{A}]_{j,k}$ denotes the entry in the jth row and the kth column of $\boldsymbol{A}$.

The set of real numbers is denoted by $\mathbb{R}$ and the set of complex numbers by $\mathbb{C}$. The set of positive integers is $\mathbb{N}^+$. Denote the set of all $n \times n$ positive semi-definite matrices by $\mathcal{H}_n$. The multivariate complex Gaussian distribution with mean $\boldsymbol{m}$ and covariance matrix $\boldsymbol{Q}$ is denoted by $\mathcal{CN}(\boldsymbol{m},\boldsymbol{Q})$. The expectation is denoted by $\mathbb{E}$. The partial order for vectors $\boldsymbol{x} \succeq \boldsymbol{y}$ says $\boldsymbol{x}$ majorizes $\boldsymbol{y}$, or equivalently $\boldsymbol{x} \preceq \boldsymbol{y}$ means $\boldsymbol{x}$ is majorized by $\boldsymbol{y}$. For matrices the order $\boldsymbol{A} \geq \boldsymbol{B}$ means that $\boldsymbol{A} - \boldsymbol{B}$ is positive semi-definite. The strict versions of these orders for vectors and matrices are denoted by $\succ$, $\prec$, $>$, and $<$. $[a]^+$ denotes the maximum of a and 0.

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