Interference Alignment — A New Look at Signal Dimensions in a Communication Network

# Interference Alignment — A New Look at Signal Dimensions in a Communication Network

Syed A. Jafar

University of California, Irvine, USA syed@uci.edu



the essence of knowledge

Boston – Delft

# Foundations and Trends<sup>®</sup> in Communications and Information Theory

Published, sold and distributed by: now Publishers Inc. PO Box 1024 Hanover, MA 02339 USA Tel. +1-781-985-4510 www.nowpublishers.com sales@nowpublishers.com

Outside North America: now Publishers Inc. PO Box 179 2600 AD Delft The Netherlands Tel. +31-6-51115274

The preferred citation for this publication is S. A. Jafar, Interference Alignment — A New Look at Signal Dimensions in a Communication Network, Foundations and Trends<sup>(R)</sup> in Communications and Information Theory, vol 7, no 1, pp 1–134, 2010

ISBN: 978-1-60198-474-6 © 2011 S. A. Jafar

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, mechanical, photocopying, recording or otherwise, without prior written permission of the publishers.

Photocopying. In the USA: This journal is registered at the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923. Authorization to photocopy items for internal or personal use, or the internal or personal use of specific clients, is granted by now Publishers Inc for users registered with the Copyright Clearance Center (CCC). The 'services' for users can be found on the internet at: www.copyright.com

For those organizations that have been granted a photocopy license, a separate system of payment has been arranged. Authorization does not extend to other kinds of copying, such as that for general distribution, for advertising or promotional purposes, for creating new collective works, or for resale. In the rest of the world: Permission to photocopy must be obtained from the copyright owner. Please apply to now Publishers Inc., PO Box 1024, Hanover, MA 02339, USA; Tel. +1-781-871-0245; www.nowpublishers.com; sales@nowpublishers.com

now Publishers Inc. has an exclusive license to publish this material worldwide. Permission to use this content must be obtained from the copyright license holder. Please apply to now Publishers, PO Box 179, 2600 AD Delft, The Netherlands, www.nowpublishers.com; e-mail: sales@nowpublishers.com

# Foundations and Trends<sup>®</sup> in Communications and Information Theory Volume 7 Issue 1, 2010 Editorial Board

## Editor-in-Chief:

Sergio Verdú Depart of Electrical Engineering Princeton University Princeton, New Jersey 08544

#### Editors

Venkat Anantharam (UC. Berkeley) Ezio Biglieri (U. Torino) Giuseppe Caire (U. Southern California) Roger Cheng (U. Hong Kong) K.C. Chen (Taipei) Daniel Costello (U. Notre Dame) Thomas Cover (Stanford) Anthony Ephremides (U. Maryland) Andrea Goldsmith (Stanford) Dave Forney (MIT) Georgios Giannakis (U. Minnesota) Joachim Hagenauer (TU Munich) Te Sun Han (Tokyo) Babak Hassibi (Caltech) Michael Honig (Northwestern) Johannes Huber (Erlangen) Hideki Imai (Tokyo) Rodney Kennedy (Canberra) Sanjeev Kulkarni (Princeton)

Amos Lapidoth (ETH Zurich) Bob McEliece (Caltech) Neri Merhav (Technion) David Neuhoff (U. Michigan) Alon Orlitsky (UC. San Diego) Vincent Poor (Princeton) Kannan Ramchandran (UC. Berkeley) Bixio Rimoldi (EPFL) Shlomo Shamai (Technion) Amin Shokrollahi (EPFL) Gadiel Seroussi (MSRI) Wojciech Szpankowski (Purdue) Vahid Tarokh (Harvard) David Tse (UC. Berkeley) Ruediger Urbanke (EPFL) Steve Wicker (Cornell) Raymond Yeung (Hong Kong) Bin Yu (UC. Berkeley)

# **Editorial Scope**

Foundations and Trends<sup>®</sup> in Communications and Information Theory will publish survey and tutorial articles in the following topics:

- Coded modulation
- Coding theory and practice
- Communication complexity
- Communication system design
- Cryptology and data security
- Data compression
- Data networks
- Demodulation and Equalization
- Denoising
- Detection and estimation
- Information theory and statistics
- Information theory and computer science
- Joint source/channel coding
- Modulation and signal design

### Information for Librarians

- Multiuser detection
- Multiuser information theory
- Optical communication channels
- Pattern recognition and learning
- Quantization
- Quantum information processing
- Rate-distortion theory
- Shannon theory
- Signal processing for communications
- Source coding
- Storage and recording codes
- Speech and Image Compression
- Wireless Communications
- Foundations and Trends<sup>®</sup> in Communications and Information Theory, 2010, Volume 7, 6 issues. ISSN paper version 1567-2190. ISSN online version 1567-2328. Also available as a combined paper and online subscription.

Foundations and Trends<sup>®</sup> in Communications and Information Theory Vol. 7, No. 1 (2010) 1–134 © 2011 S. A. Jafar DOI: 10.1561/0100000047



# Interference Alignment — A New Look at Signal Dimensions in a Communication Network

Syed A. Jafar

University of California, Irvine, CA 92697, USA, syed@uci.edu

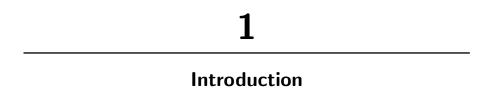
## Abstract

This monograph introduces to the reader the idea of interference alignment, traces its origins, reviews a variety of interference alignment schemes, summarizes the diverse settings where the idea of interference alignment is applicable and highlights the common principles that cut across these diverse applications. The focus is on theoretical aspects.

# Contents

1	Introduction	1
2	Linear Interference Alignment — Concept	3
2.1	Few Observed Equations, Many Unknowns	5
3	Origins of Interference Alignment	9
3.1	Index Coding	9
3.2	X Channel	11
3.3	Interference Channel with $K > 2$ Users	13
4	New Challenges and Solutions	17
4.1	Feasibility of Linear Interference Alignment	18
4.2	Symbol Extensions	25
4.3	Asymmetric Complex Signaling	26
4.4	Channel Variations: Separability	32
4.5	Ergodic Interference Alignment	36
4.6	An Asymptotic Interference Alignment Scheme — [CJ08]	38
4.7	Interference Alignment Based on Separability of	
	Rationally Independent Dimensions	48
4.8	Lattice Alignment	51

58
62
67
68
70
74
78
80
82
85
91
98
101
105
109
113
117
125



Interference alignment is a radical idea that has recently emerged out of the capacity analysis of interference networks. In a relatively short time, this concept has challenged much of the conventional wisdom about the throughput limits of both wired and wireless networks. A representative example is the wireless interference channel with Ktransmitter-receiver pairs where, because of interference alignment, each user is simultaneously able to send at a data rate equal to half of his interference-free channel capacity to his desired receiver, even though the number of users K can be arbitrarily large, thus showing that the interference channel is not fundamentally interferencelimited. While the remarkable benefits of interference alignment have so far been shown mostly under idealized assumptions such as global channel knowledge, bandwidth expansion, unlimited resolution, high signal strengths and significant delays, the idea has garnered rapidly increasing interest in the communication, signal processing, networking and information theory communities and has produced an array of surprising and fundamental insights into the number of accessible signaling dimensions in both wired and wireless communication networks. A diversity of tools from linear algebra, algebraic geometry, diophantine approximation theory as well as coding and

#### 2 Introduction

traditional Shannon theory continue to be the basis for an increasing variety of interference alignment schemes that include spatial alignment, lattice alignment, asymptotic alignment, asymmetric complex signal alignment, opportunistic alignment, ergodic alignment, aligned interference neutralization, blind alignment and retrospective alignment schemes. Applications include wireless interference networks, X networks, cellular networks, two-way communication networks, multicast and compound networks, multihop multiflow networks, tactical communication networks, cognitive radio networks, distributed data storage networks, index coding networks, and wired multiple unicast networks.

The goal of this monograph is to provide both a tutorial and a survey of the state-of-the-art on the topic of interference alignment. The majority of the paper is written to be accessible to a graduate student working in communication, signal processing, networking or information theory. The focus is on theoretical aspects. The presentation style of this monograph is informal, favoring broad intuition over mathematical rigor, and the finer details are mostly left to the references.

This monograph is organized into six sections, starting with this section — the introduction. Section 2 introduces the fundamental idea of interference alignment in its simplest form — in the language of elementary linear algebra. The origins of interference alignment are the topic of Section 3 where the earliest applications of interference alignment are reviewed. Section 4 explores the challenges faced by interference alignment schemes and the ingenious solutions that get around those challenges. Section 5 reviews the diverse settings, including both wireless and wired networks, where interference alignment has been applied while highlighting how many of these diverse problems can be reduced to the same essential problem for which a systematic solution already exists. Each subsection of this section ends with pointers to one or more open problems. Section 6 concludes the paper. An Appendix is included at the end which provides a rudimentary introduction to the Degrees-of-Freedom (DoF) and Generalized Degrees of Freedom (GDoF) metrics that are necessary to appreciate the concepts discussed throughout the paper.

- M. Aldridge, O. Johnson, and R. Piechocki, "Asymptotic sum-capacity of random Gaussian interference networks using interference alignment," in *Proceedings of IEEE International Symposium on Information Theory (ISIT)*, 2010, pp. 410–414, IEEE, 2010.
- [2] V. Annapureddy, A. El Gamal, and V. Veeravalli, "Degrees of freedom of the K-user interference channel with transmitter cooperation," in *IEEE International Symposium on Information Theory Proceedings (ISIT) 2010*, pp. 385– 389, IEEE, 2010.
- [3] A. S. Avestimehr, S. Diggavi, and D. Tse, "Wireless Network Information Flow: A Deterministic Approach," *IEEE Transactions on Information Theory*, vol. 57, pp. 1872–1905, 2011.
- [4] S. Avestimehr, A. Sezgin, and D. Tse, "Capacity of the Two Way Relay Channel within a Constant Gap," *European Transactions on Telecommunications*, vol. 21, pp. 363–374, April 2010.
- [5] O. E. Ayach and R. W. Heath, Jr., "Interference alignment with analog channel state feedback," *CoRR*, vol. abs/1010.2787, [Online]. Available: http://arxiv.org/abs/1010.2787, 2010.
- [6] G. Bagherikaram, A. S. Motahari, and A. K. Khandani, "On the secure degrees-of-freedom of the multiple-access-channel," *CoRR*, vol. abs/1003.0729, [Online]. Available: http://arxiv.org/abs/1003.0729, 2010.
- [7] B. Bandemer, A. E. Gamal, and G. Vazquez-Vilar, "On the sum capacity of a class of cyclically symmetric deterministic interference channels," in *IEEE International Symposium on Information Theory*, *ISIT*, pp. 2622–2626, 2009.
- [8] R. Bassily and S. Ulukus, "Ergodic secret alignment," CoRR, vol. abs/ 1010.6057, [Online]. Available: http://arxiv.org/abs/1010.6057, 2010.

- [9] T. D. Bavirisetti, G. Abhinav, K. Prasad, and B. S. Rajan, "A transform approach to linear network coding for acyclic networks with delay," *CoRR*, vol. abs/1103.3882, [Online]. Available: http://arxiv.org/abs/1103.3882, 2011.
- [10] V. Beresnevich, "A Groshev type theorem for convergence on manifolds," Acta Mathematica Hungarica 94, vol. 1–2, pp. 99–130, 2002.
- [11] S. Berger, T. Unger, M. Kuhn, A. Klein, and A. Wittneben, "Recent advances in amplify-and-forward two-hop relaying," *Communications Magazine*, [Online]. Available: http://www.nari.ee.ethz.ch/wireless/pubs/ p/CommMag08, July 2009.
- [12] I. Bernik, D. Kleinbock, and G. Margulis, "Khintchine-type theorems on manifolds: The convergence case for standard and multiplicative versions," *International Mathematics Research Notices*, vol. 9, pp. 453–386, 2001.
- [13] Y. Birk and T. Kol, "Informed-source coding-on-demand (ISCOD) over broadcast channels," in *Proceedings of the Seventeenth Annual Joint Conference* of the IEEE Computer and Communications Societies, IEEE INFOCOM'98, pp. 1257–1264, 1998.
- [14] Y. Birk and T. Kol, "Coding on demand by an informed source (ISCOD) for efficient broadcast of different supplemental data to caching clients," *IEEE Transactions on Information Theory*, vol. 52, no. 6, pp. 2825–2830, June 2006.
- [15] H. Boelcskei, R. Nabar, O. Oyman, and A. Paulraj, "Capacity scaling laws in MIMO relay networks," *Transactions on Wireless Communications*, vol. 5, no. 6, pp. 1433–1444, June 2006.
- [16] S. Borade, L. Zheng, and R. Gallager, "Maximizing degrees of freedom in wireless networks," in *Proceedings of 40th Annual Allerton Conference on Communication, Control and Computing*, pp. 561–570, October 2003.
- [17] G. Bresler, D. Cartwright, and D. Tse, "Settling the feasibility of interference alignment for the MIMO interference channel: The symmetric square case," *CoRR*, vol. abs/1104.0888, [Online]. Available: http://arxiv.org/abs/ 1104.0888, 2011.
- [18] G. Bresler, A. Parekh, and D. Tse, "The approximate capacity of the manyto-one and one-to-many Gaussian interference channels," *IEEE Transactions* on Information Theory, vol. 56, pp. 4566–4592, September 2010.
- [19] G. Bresler and D. Tse, "The two-user Gaussian interference channel: A deterministic view," *European Transactions in Telecommunications*, vol. 19, no. 4, pp. 333–354, June 2008.
- [20] G. Bresler and D. Tse, "3 user interference channel: Degrees of freedom as a function of channel diversity," in 47th Annual Allerton Conference on Communication, Control, and Computing, pp. 265–271, 2009.
- [21] V. Cadambe, C. Huang, S. A. Jafar, and J. Li, "Optimal repair of MDS codes in distributed storage via subspace interference alignment," *Arxiv preprint arXiv:1106.1250*, [Online]. Available: http://arxiv.org/pdf/ 1106.1250v1, 2011.
- [22] V. Cadambe and S. Jafar, "Interference alignment and the degrees of freedom of the K user interference channel," *IEEE Transactions on Information Theory*, vol. 54, no. 8, pp. 3425–3441, August 2008.

- [23] V. Cadambe and S. Jafar, "Interference alignment and the degrees of freedom of wireless X networks," *IEEE Transactions on Information Theory*, no. 9, pp. 3893–3908, September 2009.
- [24] V. Cadambe and S. Jafar, "Parallel Gaussian interference channels are not always separable," *IEEE Transactions on Information Theory*, vol. 55, no. 9, pp. 3983–3990, 2009.
- [25] V. Cadambe and S. Jafar, "Sum-capacity and the unique separability of the parallel Gaussian MAC-Z-BC network," in *Proceedings of IEEE International* Symposium on Information Theory (ISIT), 2010, pp. 2318–2322, 2010.
- [26] V. Cadambe, S. Jafar, and S. Shamai, "Interference alignment on the deterministic channel and application to Gaussian networks," *IEEE Transactions* on Information Theory, vol. 55, no. 1, pp. 269–274, January 2009.
- [27] V. Cadambe, S. Jafar, and C. Wang, "Interference alignment with asymmetric complex signaling settling the Høst-Madsen–Nosratinia conjecture," *IEEE Transactions on Information Theory*, vol. 56, no. 9, pp. 4552–4565, 2010.
- [28] V. R. Cadambe and S. A. Jafar, "Degrees of freedom of wireless networks with relays, feedback, cooperation and full duplex operation," *IEEE Transactions* on Information Theory, vol. 55, pp. 2334–2344, May 2009.
- [29] V. R. Cadambe, S. A. Jafar, and H. Maleki, "Distributed data storage with minimum storage regenerating codes — exact and functional repair are asymptotically equally efficient," *CoRR*, vol. abs/1004.4299, [Online]. Available: http://arxiv.org/abs/1004.4299, 2010.
- [30] K. Cai, K. Letaief, P. Fan, and R. Feng, "On the solvability of 2-pair unicast networks—A cut-based characterization," Arxiv preprint arXiv:1007.0465, 2010.
- [31] G. Caire and S. Shamai, "On the achievable throughput of a multiantenna Gaussian broadcast channel," *IEEE Transactions on Information Theory*, vol. 49, no. 7, pp. 1691–1706, July 2003.
- [32] Y. Cao and B. Chen, "Capacity bounds for two-hop interference networks," 47th Annual Allerton Conference on Communication, Control, and Computing, vol. abs/0910.1532, [Online]. Available: http://arxiv.org/abs/0910.1532, 2009.
- [33] S. Chen and R. Cheng, "Achieve the degrees of freedom of K-User MIMO interference channel with a MIMO relay," in *IEEE GLOBECOM*, December 2010.
- [34] S. W. Choi and S.-Y. Chung, "On the separability of parallel Gaussian interference channels," *Proceedings of IEEE International Symposium* on Information Theory (ISIT 2009), pp. 2592–2596, [Online]. Available: http://arxiv.org/abs/0905.1537, 2009.
- [35] B. Da and R. Zhang, "Exploiting interference alignment in multi-cell cooperative OFDMA resource allocation," *CoRR*, vol. abs/1103.3093, [Online]. Available: http://arxiv.org/abs/1103.3093, 2011.
- [36] A. K. Das, S. Vishwanath, S. A. Jafar, and A. Markopoulou, "Network coding for multiple unicasts: An interference alignment approach," *CoRR*, vol. abs/1008.0235, [Online]. Available: http://arxiv.org/abs/1008. 0235, 2010.

- [37] A. G. Dimakis, K. Ramchandran, Y. Wu, and C. Suh, "A survey on network codes for distributed storage," *Proceedings of the IEEE*, vol. 99, pp. 476–489, [Online]. Available: http://arxiv.org/abs/1004.4438, 2011.
- [38] C. Esli and A. Wittneben, "A hierarchical AF protocol for distributed orthogonalization in multiuser relay networks," *IEEE Transactions on Vehicular Technology*, vol. 59, pp. 3902–3916, [Online]. Available: http://www.nari.ee.ethz.ch/wireless/pubs/p/esli\_tvt\_2010, August 2010.
- [39] R. Etkin and E. Ordentlich, "The degrees-of-freedom of the K-User Gaussian interference channel is discontinuous at rational channel coefficients," *IEEE Transactions on Information Theory*, vol. 55, pp. 4932–4946, November 2009.
- [40] R. Etkin, D. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," *IEEE Transactions on Information Theory*, vol. 54, no. 12, pp. 5534–5562, 2008.
- [41] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," Wireless Personal Commun. : Kluwer Academic Press, no. 6, pp. 311–335, 1998.
- [42] A. E. Gamal and M. Costa, "The capacity region of a class of deterministic interference channels," *IEEE Transactions on Information Theory*, vol. 2, pp. 343–346, March 1982.
- [43] D. Gesbert, S. Hanly, H. Huang, S. Shitz, O. Simeone, and W. Yu, "Multi-cell MIMO cooperative networks: A new look at interference," *IEEE Journal on Selected Areas in Communications*, vol. 28, no. 9, pp. 1380–1408, December 2010.
- [44] A. Ghasemi, A. Motahari, and A. Khandani, "Interference alignment for the K user MIMO interference channel," in *Proceedings of International Symposium* on Information Theory (ISIT), pp. 360–364, IEEE, 2010.
- [45] A. Goldsmith, S. A. Jafar, N. Jindal, and S. Vishwanath, "Capacity limits of MIMO channels," *IEEE Journal on Selected Areas in Communications*, vol. 21, no. 5, pp. 684–702, June 2003.
- [46] A. Goldsmith and P. Varaiya, "Capacity of fading channels with channel side information," *IEEE Transactions on Information Theory*, vol. 43, pp. 1986– 1992, November 1997.
- [47] S. Gollakota, S. Perli, and D. Katabi, "Interference alignment and cancellation," in *Proceedings of the ACM SIGCOMM 2009 conference on Data communication*, pp. 159–170, ACM, 2009.
- [48] K. Gomadam, V. Cadambe, and S. Jafar, "A distributed numerical approach to interference alignment and applications to wireless interference networks," *IEEE Transactions on Information Theory*, pp. 3309–3322, June 2011.
- [49] T. Gou and S. Jafar, "On the secure degrees of freedom of wireless X networks," in Proceedings of 46th Annual Allerton Conference on Communication, Control and Computing, September 2008.
- [50] T. Gou and S. Jafar, "Degrees of freedom of the K user M × N MIMO interference channel," *IEEE Transactions on Information Theory*, vol. 56, no. 12, pp. 6040–6057, December 2010.
- [51] T. Gou, S. Jafar, S. Jeon, and S. Chung, "Aligned interference neutralization and the degrees of freedom of the  $2 \times 2 \times 2$  interference channel," *Proceedings*

of IEEE International Symposium on Information Theory (ISIT), [Online]. Available: http://arxiv.org/abs/1012.2350, 2011.

- [52] T. Gou, S. Jafar, and C. Wang, "Degrees of freedom of finite state compound wireless networks," To Appear in the IEEE Transactions on Information Theory. Full paper available at arXiv:0909.4203, [Online]. Available: http://arxiv.org/abs/0909.5424, 2009.
- [53] T. Gou and S. A. Jafar, "Capacity of a class of symmetric SIMO Gaussian interference channels within O (1)," *IEEE Transactions on Information Theory*, vol. 57, no. 4, pp. 1932–1958, April 2011.
- [54] T. Gou, C. Wang, and S. A. Jafar, "Aiming perfectly in the dark Blind interference alignment through staggered antenna switching," *IEEE Transactions on Signal Processing*, vol. 59, pp. 2734–2744, June 2011.
- [55] L. Grokop, D. Tse, and R. Yates, "Interference alignment for line-of-sight channels," CORR, vol. 0809.3035, [online]. Available: http://arXiv:cs.IT/ 0809.3035, 2008.
- [56] X. He and A. Yener, "K-user interference channels: Achievable secrecy rate and degrees of freedom," *CoRR*, vol. abs/0905.2643, [Online]. Available: http://arxiv.org/abs/0905.2643, 2009.
- [57] A. Host-Madsen and A. Nosratinia, "The multiplexing gain of wireless networks," in *Proceedings of ISIT*, 2005.
- [58] C. Huang, V. Cadambe, and S. Jafar, "On the capacity and generalized degrees of freedom of the X channel," arxiv:0810.4741, October 2008.
- [59] C. Huang, S. A. Jafar, S. Shamai, and S. Vishwanath, "On degrees of freedom region of MIMO networks without CSIT," *CoRR*, vol. abs/0909.4017, [Online]. Available: http://arxiv.org/abs/0909.4017, 2009.
- [60] H. Huang, V. Lau, Y. Du, and S. Liu, "Robust lattice alignment for K-user MIMO interference channels with imperfect channel knowledge," *IEEE Transactions on Signal Processing*, vol. 59, pp. 3315–3325, 2011.
- [61] S. Jafar, "Too much mobility limits the capacity of wireless ad-hoc networks," *IEEE Transactions on Information Theory*, vol. 52, no. 7, pp. 3954–3965, November 2005.
- [62] S. Jafar, "Degrees of Freedom on the MIMO X channel- optimality of the MMK scheme," CoRR, vol. abs/cs/0607099v2, [Online]. Available: http://arxiv.org/abs/cs/0607099v2, September 2006.
- [63] S. Jafar, "Exploiting channel correlations Simple interference alignment schemes with no CSIT," *Preprint available on Arxiv arXiv:0910.0555*, October 2009.
- [64] S. Jafar and V. Cadambe, "Degrees of freedom of wireless networks what a difference delay makes," in Asilomar Conference on Signals, Systems, and Computers, Pacific Grove, CA, November 2007.
- [65] S. Jafar and M. Fakhereddin, "Degrees of freedom for the MIMO interference channel," *IEEE Transactions on Information Theory*, vol. 53, no. 7, pp. 2637– 2642, July 2007.
- [66] S. Jafar and A. Goldsmith, "Isotropic fading vector broadcast channels: the scalar upperbound and loss in degrees of freedom," *IEEE Transactions on Information Theory*, vol. 51, no. 3, pp. 848–857, March 2005.

- [67] S. Jafar and S. Shamai, "Degrees of freedom region for the MIMO X channel," *IEEE Transactions on Information Theory*, vol. 54, no. 1, pp. 151–170, January 2008.
- [68] S. Jafar and S. Vishwanath, "Generalized degrees of freedom of the symmetric gaussian K user interference channel," *IEEE Transactions on Information Theory*, vol. 56, no. 7, pp. 3297–3303, July 2010.
- [69] S. A. Jafar, "The ergodic capacity of interference networks," CoRR, vol. abs/0902.0838, [Online]. Available: http://arxiv.org/abs/0902.0838, 2009.
- [70] S.-W. Jeon and S.-Y. Chung, "Capacity of a class of multi-source relay networks," *CoRR*, vol. abs/0907.2510, [Online]. Available: http://arxiv.org/abs/0907.2510, 2009.
- [71] S.-W. Jeon, S.-Y. Chung, and S. Jafar, "Degrees of freedom region of a class of multisource Gaussian relay networks," *IEEE Transactions on Information Theory*, vol. 57, pp. 3032–3044, May 2011.
- [72] O. Johnson, M. Aldridge, and R. Piechocki, "Interference alignment-based sum capacity bounds for random dense Gaussian interference networks," *IEEE Transactions on Information Theory*, vol. 57, pp. 282–290, [Online]. Available: http://arxiv.org/abs/1004.0208, January 2011.
- [73] O. Johnson, M. Aldridge, and R. J. Piechocki, "Delay-rate tradeoff in ergodic interference alignment," *CoRR*, vol. abs/1004.0208, [Online]. Available: http:// arxiv.org/abs/1004.0208, 2010.
- [74] S. Karmakar and M. K. Varanasi, "Capacity of the MIMO interference channel to within a constant gap," *CoRR*, vol. abs/1102.0267, [Online]. Available: http://arxiv.org/abs/1102.0267, 2011.
- [75] S. Karmakar and M. K. Varanasi, "The Generalized Degrees of Freedom Region of the MIMO Interference Channel," *CoRR*, vol. abs/1103.2560, [Online]. Available: http://arxiv.org/abs/1103.2560, 2011.
- [76] S. Katti, S. Gollakota, and D. Katabi, "Embracing wireless interference: Analog network coding," ACM SIGCOMM Computer Communication Review, vol. 37, no. 4, pp. 397–408, 2007.
- [77] S. Katti, H. Rahul, W. Hu, D. Katabi, M. Médard, and J. Crowcroft, "XORs in the air: practical wireless network coding," *IEEE/ACM Transactions on Networking (TON)*, vol. 16, no. 3, pp. 497–510, 2008.
- [78] L. Ke, A. Ramamoorthy, Z. Wang, and H. Yin, "Degrees of Freedom Region for an Interference Network with General Message Demands," *CoRR*, vol. abs/1101.3068, [Online]. Available: http://arxiv.org/abs/1101.3068, 2011.
- [79] L. Ke and Z. Wang, "Degrees of Freedom Regions of Two-User MIMO Z and Full Interference Channels: The Benefit of Reconfigurable Antennas," *ArXiv e-prints*, November 2010.
- [80] A. Khisti, "Interference Alignment for the Multi-Antenna Compound Wiretap Channel," *IEEE Transactions on Information Theory*, vol. 57, pp. 2976–2993, May 2011.
- [81] J. C. Koo, W. Wu, and J. T. G. III, "Delay-rate tradeoff for ergodic interference alignment in the Gaussian case," 48th Annual Allerton Conference on Communication, Control, and Computing, vol. abs/1001.2582, pp. 1069–1075, [Online]. Available: http://arxiv.org/abs/1001.2582, 2010.

- [82] J. Korner and K. Marton, "How to encode the modulo-two sum of binary sources," *IEEE Transactions on Information Theory*, vol. 25, pp. 219–221, March 1979.
- [83] O. O. Koyluoglu, H. E. Gamal, L. Lai, and H. V. Poor, "Interference Alignment for Secrecy," *IEEE Transactions on Information Theory*, vol. 57, pp. 3323–3332, June 2011.
- [84] G. Kramer, I. Marić, and R. Yates, "Cooperative communications," Foundations and Trends in Networking, vol. 1, no. 3, pp. 271–425, 2006.
- [85] R. T. Krishnamachari and M. K. Varanasi, "Interference Alignment Under Limited Feedback for MIMO Interference Channels," CoRR, vol. abs/0911.5509, [Online]. Available: http://arxiv.org/abs/0911.5509, 2009.
- [86] A. Lapidoth, S. Shamai, and M. Wigger, "On the capacity of Fading MIMO broadcast channels with imperfect transmitter side-information," in *Proceed*ings of 43rd Annual Allerton Conference on Communications, Control and Computing, pp. 28–30, September 2005.
- [87] N. Lee, J. Lim, and J. Chun, "Degrees of freedom of the MIMO Y channel: signal space alignment for network coding," *IEEE Transactions on Information Theory*, vol. 56, no. 7, pp. 3332–3342, 2010.
- [88] N. Lee, W. Shin, and B. Clerckx, "Interference Alignment with Limited Feedback on Two-cell Interfering Two-User MIMO-MAC," CoRR, vol. abs/1010.0933, [Online]. Available: http://arxiv.org/abs/1010.0933, 2010.
- [89] M. Maddah-Ali, A. Motahari, and A. Khandani, "Communication over X channel: Signaling and Performance Analysis," in *Technical Report. UW-ECE-*2006-27, University of Waterloo, December 2006.
- [90] M. Maddah-Ali, A. Motahari, and A. Khandani, "Communication over X channel: Signalling and multiplexing gain," in *Technical Report. UW-ECE-*2006-12, University of Waterloo, July 2006.
- [91] M. Maddah-Ali, A. Motahari, and A. Khandani, "Signaling over MIMO multibase systems - combination of multi-access and broadcast schemes," in *Proceedings of ISIT*, pp. 2104–2108, July 2006.
- [92] M. Maddah-Ali, A. Motahari, and A. Khandani, "Communication over MIMO X Channels: Interference Alignment, Decomposition, and Performance Analysis," in *IEEE Transactions on Information Theory*, pp. 3457–3470, August 2008.
- [93] M. A. Maddah-Ali, "The Degrees of Freedom of the Compound MIMO Broadcast Channels with Finite States," *CoRR*, vol. abs/0909.5006, [Online]. Available: http://arxiv.org/abs/0909.5006, 2009.
- [94] M. A. Maddah-Ali and D. Tse, "On the Degrees of Freedom of MISO Broadcast Channels with Delayed Feedback," Technical Report UCB/EECS-2010-122, EECS Department, University of California, Berkeley, [Online]. Available: http://www.eecs.berkeley.edu/Pubs/TechRpts/2010/EECS-2010-122.html, September 2010.
- [95] H. Maleki, S. A. Jafar, and S. Shamai, "Retrospective Interference Alignment," CoRR, vol. abs/1009.3593, [Online]. Available: http:// arxiv.org/abs/1009.3593, 2010.

- [96] R. Mathar and M. Zivkovic, "How to position n transmitter-receiver pairs in n-1 dimensions such that each can use half of the channel with zero interference from the others," in *Global Telecommunications Conference*, 2009. *GLOBECOM 2009. IEEE*, pp. 1–4, IEEE, 2010.
- [97] S. Mohajer, S. Diggavi, C. Fragouli, and D. Tse, "Approximate Capacity of a Class of Gaussian Interference-Relay Networks," *IEEE Transactions on Information Theory*, vol. 57, pp. 2837–2864, May 2011.
- [98] V. Morgenshtern and H. Bölcskei, "Crystallization in large wireless networks," *IEEE Transactions on Information Theory*, vol. 53, no. 10, pp. 3319–3349,
  [Online]. Available: http://www.nari.ee.ethz.ch/commth/ pubs/p/transit06,
  October 2007.
- [99] A. Motahari, S. Gharan, M. Maddah-Ali, and A. Khandani, "Real Interference Alignment: Exploiting the Potential of Single Antenna Systems," CoRR, vol. abs/0908.2282, [Online]. Available: http://arxiv.org/abs/0908.2282, 2009.
- [100] B. Nazer and M. Gastpar, "Compute-and-Forward: Harnessing Interference through Structured Codes," CoRR, vol. abs/0908.2119, [Online]. Available: http://arxiv.org/abs/0908.2119, 2009.
- [101] B. Nazer, M. Gastpar, S. Jafar, and S. Vishwanath, "Ergodic Interference Alignment," in *ISIT*, 2009.
- [102] B. Nazer, M. Gastpar, S. Jafar, and S. Vishwanath, "Interference alignment at finite SNR: General message sets," in 47th Annual Allerton Conference on Communication, Control, and Computing, pp. 843–848, IEEE, 2009.
- [103] B. Nazer, A. Sanderovich, M. Gastpar, and S. Shamai, "Structured Superposition for Backhaul Constrained Cellular Uplink," in *IEEE ISIT 2009*, June– July 2009.
- [104] F. Negro, S. Shenoy, D. Slock, and I. Ghauri, "Interference alignment limits for K-user frequency-flat MIMO interference channels," in *Proceedings European Signal Proc. Conf. (Eusipco)*, 2009.
- [105] U. Niesen, "Interference alignment in dense wireless networks," *IEEE Trans-actions on Information Theory*, vol. 57, pp. 2889–2901, [Online]. Available: http://arxiv.org/abs/0912.0868, May 2011.
- [106] U. Niesen and P. Whiting, "The Degrees of Freedom of Computeand-Forward," CoRR, vol. abs/1101.2182, [Online]. Available: http://arxiv.org/abs/1101.2182, 2011.
- [107] H. Ning, C. Ling, and K. Leung, "Relay-aided interference alignment: Feasibility conditions and algorithm," in *IEEE International Symposium on Information Theory Proceedings (ISIT)*, pp. 390–394, IEEE, 2010.
- [108] B. Niu and A. Haimovich, "Interference subspace tracking for network interference alignment in cellular systems," in *Global Telecommunications Conference*, 2009. GLOBECOM 2009. IEEE, pp. 1–5, IEEE, 2010.
- [109] B. Nosrat-Makouei, J. G. Andrews, and R. W. H. Jr., "MIMO Interference Alignment Over Correlated Channels with Imperfect CSI," CoRR, vol. abs/1010.2741, [Online]. Available: http://arxiv.org/abs/1010.2741, 2010.
- [110] B. Nourani, S. Motahari, and A. Khandani, "Relay-aided interference alignment for the quasi-static X channel," in *IEEE International Symposium on Information Theory, ISIT*, pp. 1764–1768, IEEE, 2009.

- [111] B. Nourani, S. Motahari, and A. Khandani, "Relay-aided Interference Alignment for the quasi-static interference channel," in *IEEE International Symposium on Information Theory Proceedings (ISIT)*, pp. 405–409, IEEE, 2010.
- [112] D. Papailiopoulos and A. Dimakis, "Interference alignment as a rank constrained rank minimization," *IEEE GLOBECOM*, December 2010.
- [113] D. Papailiopoulos and A. Dimakis, "Distributed Storage Codes through Hadamard Designs," Arxiv preprint arXiv:1106.1652, [Online]. Available: http://arxiv.org/pdf/1106.1652, 2011.
- [114] J. Park, Y. Sung, and V. Poor, "On Beamformer Design for Multiuser MIMO Channels," *CoRR*, vol. abs/1011.6121, [Online]. Available: http://arxiv.org/abs/1011.6121, November 2010.
- [115] P. Parker, D. Bliss, and V. Tarokh, "On the Degrees-of-Freedom of the MIMO Interference Channel," in 42nd Annual Conference on Information Sciences and Systems (CISS), March 2008.
- [116] S. Peters and R. Heath, "Interference alignment via alternating minimization," in *IEEE International Conference on Acoustics, Speech and Signal Processing*, 2009. ICASSP 2009, pp. 2445–2448, IEEE, 2009.
- [117] S. W. Peters and R. W. Heath, Jr., "User partitioning for less overhead in MIMO interference channels," *CoRR*, vol. abs/1007.0512, [Online]. Available: http://arxiv.org/abs/1007.0512, 2010.
- [118] S. W. Peters and R. W. Heath Jr., "Cooperative Algorithms for MIMO Interference Channels," *IEEE Transactions on Vehicular Technology*, vol. 60, pp. 206–218, January 2011.
- [119] T. Philosof and R. Zamir, "On the loss of single-letter characterization: the dirty multiple access channel," *IEEE Transactions on Information Theory*, vol. 55, no. 6, pp. 2442–2454, 2009.
- [120] A. Ramakrishnan, A. Das, H. Maleki, A. Markopoulou, S. Jafar, and S. Vishwanath, "Network coding for three unicast sessions: Interference alignment approaches," Allerton Conference on Communications, Control and Computing, October 2010.
- [121] B. Rankov and A. Wittneben, "Achievable rate regions for the two-way relay channel," in *Proceedings IEEE International Sympo*sium on Information Theory (ISIT), July 2006. [Online]. Available: http://www.nari.ee.ethz.ch/wireless/pubs/p/isit2006.
- [122] B. Rankov and A. Wittneben, "Spectral efficient protocols for half-duplex fading relay channels," *IEEE Journal on Selected Areas in Communications*, [Online]. Available: http://www.nari.ee.ethz.ch/wireless/ pubs/p/jsac2006, February 2007.
- [123] M. Razaviyayn, G. Lyubeznik, and Z. Luo, "On the degrees of freedom achievable through interference alignment in a MIMO interference channel," CoRR, vol. abs/1104.0992, [Online]. Available: http://arxiv.org/abs/1104.0992, 2011.
- [124] M. Razaviyayn, M. Sanjabi, and Z.-Q. Luo, "Linear Transceiver Design for Interference Alignment: Complexity and Computation," 2010 IEEE Eleventh International Workshop on Signal Processing Advances in

Wireless Communications (SPAWC), vol. abs/1009.3481, [Online]. Available: http://arxiv.org/abs/1009.3481, 2010.

- [125] S. Saha and R. Berry, "On the Sum Capacity of a Class of 3 User Deterministic Interference Channels," in *Forty-Eighth Annual Allerton Conference on Communication, Control and Computing*, September 2010.
- [126] L. Sankar, X. Shang, E. Erkip, and H. Poor, "Ergodic Fading Interference Channels: Sum-Capacity and Separability," *IEEE Transactions on Information Theory*, vol. 57, pp. 2605–2626, May 2011.
- [127] D. Schmidt, C. Shi, R. Berry, M. Honig, and W. Utschick, "Minimum mean squared error interference alignment," in *Conference Record of the Forty-Third Asilomar Conference on Signals, Systems and Computers, 2009*, pp. 1106– 1110, IEEE, 2010.
- [128] D. Schmidt, W. Utschick, and M. Honig, "Large System Performance of Interference Alignment in Single-Beam MIMO Networks," in *IEEE Global Telecommunications Conference (GLOBECOM) 2010*, December 2010.
- [129] A. Sezgin, A. S. Avestimehr, M. A. Khajehnejad, and B. Hassibi, "Divide-and-conquer: Approaching the capacity of the two-pair bidirectional Gaussian relay network," *CoRR*, vol. abs/1001.4271, [Online]. Available: http://arxiv.org/abs/1001.4271, 2010.
- [130] N. B. Shah, K. V. Rashmi, P. V. Kumar, and K. Ramchandran, "Explicit Codes Minimizing Repair Bandwidth for Distributed Storage," *CoRR*, vol. abs/0908.2984, [Online]. Available: http://arxiv.org/abs/0908.2984, 2009.
- [131] H. Shen, B. Li, M. Tao, and Y. Luo, "The new interference alignment scheme for the MIMO interference channel," in Wireless Communications and Networking Conference (WCNC), 2010 IEEE, pp. 1–6, IEEE, 2010.
- [132] S. Shenvi and B. Dey, "A simple necessary and sufficient condition for the double unicast problem," in *IEEE International Conference on Communications* (*ICC*), 2010, pp. 1–5, IEEE, 2010.
- [133] W. Shin, N. Lee, J. B. Lim, C. Shin, and K. Jang, "Interference alignment through user cooperation for two-cell MIMO interfering broadcast channels," *CoRR*, vol. abs/1011.3867, [Online]. Available: http://arxiv.org/ abs/1011.3867, 2010.
- [134] I. Shomorony and S. Avestimehr, "Two-Unicast Wireless Networks: Characterizing the Sum Degrees of Freedom," CoRR, vol. abs/1102.2498, [Online]. Available: http://arxiv.org/abs/1102.2498, 2011.
- [135] O. Simeone, O. Somekh, Y. Bar-Ness, H. V. Poor, and S. Shamai, "Capacity of Linear Two-hop Mesh Networks with Rate Splitting, Decodeand-forward Relaying and Cooperation," roceedings of the 45th Annual Allerton Conference on Communication, Control and Computing, Monticello, IL, vol. abs/0710.2553, [Online]. Available: http://arxiv.org/abs/0710.2553, September 2007.
- [136] S. Sridharan, A. Jafarian, S. Vishwanath, and S. Jafar, "Capacity of Symmetric K-User Gaussian Very Strong Interference Channels," in *Proceedings* of *IEEE GLOBECOM*, December 2008.

- [137] C. Suh and K. Ramchandran, "Exact Regeneration Codes for Distributed Storage Repair Using Interference Alignment," *CoRR*, vol. abs/1001.0107, [Online]. Available: http://arxiv.org/abs/1001.0107, 2010.
- [138] C. Suh and K. Ramchandran, "On the Existence of Optimal Exact-Repair MDS Codes for Distributed Storage," *CoRR*, vol. abs/1004.4663, [Online]. Available: http://arxiv.org/abs/1004.4663, 2010.
- [139] C. Suh and D. Tse, "Interference Alignment for Cellular Networks," in Proceedings of 40th Annual Allerton Conference on Communication, Control and Computing, September 2008.
- [140] I. Tamo, Z. Wang, and J. Bruck, "MDS array codes with optimal rebuilding," in *IEEE ISIT 2011*, July–August 2011.
- [141] R. Tannious and A. Nosratinia, "The interference channel with MIMO relay: Degrees of freedom," in *IEEE International Symposium on Information The*ory, (ISIT) 2008, pp. 1908–1912, IEEE, 2008.
- [142] E. Telatar, "Capacity of Multi-antenna Gaussian Channels," European Transactions on Telecomm. ETT, vol. 10, no. 6, pp. 585–596, November 1999.
- [143] E. Telatar and D. Tse, "Bounds on the capacity region of a class of interference channels," in *Proceedings of IEEE International Symposium on Information Theory (ISIT)*, 2007.
- [144] J. Thukral and H. Boelcskei, "Interference alignment with limited feedback," in Proceedings of the 2009 IEEE International Symposium on Information Theory (ISIT), pp. 1759–1763, 2009.
- [145] M. Torbatian, H. Najafi, and O. Damen, "Asynchronous Interference Channel: Degrees of Freedom and Interference Alignment," CoRR, vol. abs/1101.0275, [Online]. Available: http://arxiv.org/abs/1101.0275, 2011.
- [146] R. Tresch, M. Guillaud, and E. Riegler, "On the achievability of interference alignment in the K-user constant MIMO interference channel," in *IEEE/SP* 15th Workshop on Statistical Signal Processing, 2009. SSP'09., pp. 277–280, IEEE, 2009.
- [147] D. Tse, P. Viswanath, and L. Zheng, "Diversity-multiplexing tradeoff in multiple-access channels," *IEEE Transactions on Information Theory*, vol. 50, no. 9, pp. 1859–1874, Sept 2004.
- [148] C. S. Vaze and M. K. Varanasi, "The Degrees of Freedom Regions of MIMO Broadcast, Interference, and Cognitive Radio Channels with No CSIT," *CoRR*, vol. abs/0909.5424, 2009.
- [149] C. S. Vaze and M. K. Varanasi, "The Degrees of Freedom Region and Interference Alignment for the MIMO Interference Channel with Delayed CSI," *CoRR*, vol. abs/1101.5809, [Online]. Available: http://arxiv.org/abs/1101.5809, 2011.
- [150] C. S. Vaze and M. K. Varanasi, "The Degrees of Freedom Region of the Two-User MIMO Broadcast Channel with Delayed CSI," *CoRR*, vol. abs/1101.0306, [Online]. Available: http://arxiv.org/abs/1101.0306, 2011.
- [151] S. Vishwanath, N. Jindal, and A. Goldsmith, "Duality, Achievable Rates, and Sum-Rate Capacity of MIMO Broadcast Channels," *IEEE Transactions on Information Theory*, pp. 2895–2909, October 2003.

- [152] P. Viswanath and D. Tse, "Sum capacity of the vector Gaussian broadcast channel and uplink-downlink duality," *IEEE Transactions on Information Theory*, pp. 1912–1921, August 2003.
- [153] C. Wang, T. Gou, and S. Jafar, "Interference Alignment through Staggered Antenna Switching for MIMO BC with no CSIT," in Asilomar Conference on Signals, Systems and Computers, November 2010.
- [154] C. Wang, T. Gou, and S. Jafar, "Multiple Unicast Capacity of 2-Source 2-Sink Networks," CoRR, vol. abs/1104.0954, [Online]. Available: http://arxiv.org/abs/1104.0954, 2011.
- [155] C. Wang, S. Jafar, S. Shamai, and M. Wigger, "Interference, Cooperation and Connectivity - A Degrees of Freedom Perspective," *CoRR*, vol. abs/1103.6060, [Online]. Available: http://arxiv.org/abs/1103.6060, 2011.
- [156] C. Wang, H. Papadopoulos, S. Ramprashad, and G. Caire, "Design and Operation of Blind Interference Alignment in Cellular and Cluster-based Systems," in UCSD Information Theory and its Applications (ITA) Workshop 2011, February 2011.
- [157] C. Wang, H. Papadopoulos, S. Ramprashad, and G. Caire, "Improved Blind Interference Alignment in a Cellular Environment using Power Allocation and Cell-Based Clusters," in *IEEE International Conference on Communications* (ICC) 2011, June 2011.
- [158] C. Wang and N. Shroff, "Beyond the butterfly-a graph-theoretic characterization of the feasibility of network coding with two simple unicast sessions," in *IEEE International Symposium on Information Theory, ISIT*, pp. 121–125, IEEE, 2007.
- [159] H. Weingarten, S. Shamai, and G. Kramer, "On the compound MIMO broadcast channel," in *Proceedings of Annual Information Theory and Applications* Workshop UCSD, January 2007.
- [160] H. Weingarten, Y. Steinberg, and S. Shamai, "The capacity region of the Gaussian MIMO broadcast channel," *IEEE Transactions on Information Theory*, vol. 52, pp. 3936–3964, September 2006.
- [161] Y. Wu, S. Shamai, and S. Verdu, "Degrees of Freedom of Interference Channel: a General Formula," in *Proceedings of International Symposium on Informa*tion Theory (ISIT) 2011, 2011.
- [162] C. Yetis, T. Gou, S. Jafar, and A. Kayran, "On feasibility of interference alignment in MIMO interference networks," *IEEE Transactions on Signal Process-ing*, vol. 58, no. 9, pp. 4771–4782, 2010.
- [163] H. Yin, "Comments on Degrees of freedom region for K-user interference channel with M antennas," *CoRR*, vol. abs/1011.3812, [Online]. Available: http://arxiv.org/abs/1011.3812, 2010.
- [164] H. Yu, Y. Sung, H. Kim, and Y. Lee, "Adaptive beam tracking for interference alignment for multiuser time-varying MIMO interference channels," in Acoustics Speech and Signal Processing (ICASSP), 2010 IEEE International Conference on, pp. 3086–3089, IEEE, 2010.
- [165] W. Yu and J. Cioffi, "Sum capacity of Gaussian vector broadcast channels," *IEEE Transactions on Information Theory*, vol. 50, no. 9, pp. 1875–1892, Sept 2004.

- [166] Y. Zhu and D. Guo, "Isotropic MIMO interference channels without CSIT: The loss of degrees of freedom," 47th Annual Allerton Conference on Communication, Control, and Computing, vol. abs/0910.2961, [Online]. Available: http://arxiv.org/abs/0910.2961, 2009.
- [167] Y. Zhu and D. Guo, "The degrees of freedom of MIMO interference channels without state information at transmitters," *CoRR*, vol. abs/1008.5196, [Online]. Available: http://arxiv.org/abs/1008.5196, 2010.