# Random-Set Theory and Wireless Communications

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# Random-Set Theory and Wireless Communications

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## Abstract

This monograph is devoted to random-set theory, which allows unordered collections of random elements, drawn from an arbitrary space, to be handled. After illustrating its foundations, we focus on Random Finite Sets, i.e., unordered collections of random cardinality of points from an arbitrary space, and show how this theory can be applied to a number of problems arising in wireless communication systems. Three of these problems are: (1) neighbor discovery in wireless networks, (2) multiuser detection in which the number of active users is unknown and time-varying, and (3) estimation of multipath channels where the number of paths is not known *a priori* and which are possibly time-varying. Standard solutions to these problems are intrinsically suboptimum as they proceed either by assuming a fixed number of vector components, or by first estimating this number and then the values taken on by the components. It is shown how random-set theory provides optimum solutions to all these problems. The complexity issue is also examined, and suboptimum solutions are presented and discussed.

# Contents

# 1

# Solving Estimation Problems Where You Do Not Know the Number of Things You Do Not Know

In this section we briefly glance at random-set theory (RST) and the motivations for its application to problems in digital communication, which will be described in detail in subsequent sections.

Roughly speaking, random sets are random entities whose realizations are subsets of a given space. In RST, the outcome of a random experiment is a set, or a vector whose number of components is unknown. RST can be applied to modeling observed phenomena which are sets rather than vectors, hence it generalizes the concept of random vectors. For a simple example, consider the description of random polygons on a plane. Each polygon can be described by a vector whose components are the coordinates of its vertices. Hence, if the number of vertices can vary from 3 to a finite number $N$, a random polygon is described by a vector whose randomness is in its components as well as in the number of its components. The first systematic exposition of random-set theory was developed in 1975 by Matheron [68]. More recent books investigating RST from a mathematical viewpoint are [72, 73].

The first engineering application of RST was found in the multisensor–multitarget data-fusion area. Multisensor–multitarget systems include “randomly varying numbers of randomly varying objects of various kinds: randomly varying collections of targets, randomly varying collections of sensors and sensor-carrying platforms, and randomly varying observation sets collected by those sensors.” [60, 66, p. 8] while data fusion is “the process of directing the right data sources on the right platform to the right targets at the right times, with the goal of detecting, localizing, identifying, and determining the threat potential of as many targets of interest as possible.” [60]. While a rigorous mathematical tool for solving stochastic multiobject problems is point process theory (see, e.g., [51] and Appendix B, *infra*), a more “engineering-friendly” framework has been advocated by Ronald Mahler in the form of a statistical theory directly based on RST and called *finite-set statistics* (FISST) [66]. The basic idea of FISST is “to transform [a] multisource–multitarget problem into a mathematically equivalent single-sensor, single-target problem. All the sensors are mathematically bundled into a single ‘meta-sensor’ that retains all of the characteristics of the original sensors. (. . . ) The targets are likewise bundled into a single ‘meta-target’ that retains all of the characteristics of the individual targets.”[60] Quoting again from [60], FISST

> is engineering-friendly in that it is geometric (i.e., treats multitarget systems as visualizable images); and directly generalizes the Bayes “Statistics 101” formalism that most signal processing engineers already understand — including formal Bayes-statistical modeling methods.

Since its introduction in 1994, FISST has attracted much interest from a number of research areas. Its first applications to digital communication problems were examined in [5, 6, 22].

The main focus of this monograph is on estimation problems where the quantities to be estimated are in a random number. In addition, a model of their evolution may be available with time. A typical problem is that one observes a superposition of a random number $n$ of random signals in additive noise and one wants to estimate this number as well as certain parameters of the individual signals. In the "classical" framework (see, e.g., [90]), the problem is solved in two steps: (1), determine whether or not signals exist and, if so, to what number, and (2) estimate the parameters of the signals under the assumption that their true number is its estimate $\hat{n}$. As we shall see in Section 2.2, this approach may not provide the best solution, which depends on the cost function selected for the estimation problem.

The monograph has been organized in six sections and three appendixes. More precisely, Section 2 focuses on the statistical characterization of Random Sets, and in particular of Random Finite Sets (RFS): first the concept of integrating "finite-set functions", i.e., functions whose arguments are finite sets, as well as the inverse operation, the so-called "set derivative" amounting to differentiating a set function with respect to a finite set, are introduced and interpreted in an engineering-friendly way. The remainder of Section 2 is concerned with the definition of RFS probability density functions (pdfs), and with the extensions of such concepts as Bayesian recursions and statistical inference to the point where the object of interest is an RFS.

Section 3 is devoted to the problem of reduced-complexity implementation of RFS estimators, and summarizes the major techniques proposed so far in order to scale down the computational burden from combinatorial to algebraic: special attention is paid to Probability Hypothesis Density (PHD) filtering and to its "Cardinalized" form (CPHD), which have been the object of great interest in multiobject tracking for more than a decade now.

Sections 4 and 5 illustrate the application of RFS theory to two relevant problems of Communication Theory, i.e., multiuser detection and channel estimation in dynamic environments. In particular, Section 4 considers the situation where the set of active users varies over time according to a known transition pdf, while the object of interest in Section 5 is the set of active paths in a frequency-selective wireless channel. Finally, Section 6 contains some concluding remarks and a brief list of the topics concerning RST that were not included in this monograph.

Appendix A complements Section 2 by illustrating the mathematical foundations of Random-Set Theory in a more formal way: it is however important to underline that Section 2 is self-contained, and therefore a thorough understanding of Appendix A is not strictly necessary in order to “operate” with RFSs. Likewise, Appendices B and C are reserved to readers who want to have a deeper understanding of the connections of RFS to well-established “classical” theories, such as Point Processes (Appendix B) and Dempster–Shafer Theory (Appendix C).

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