Two-User Gaussian Interference Channels: An Information Theoretic Point of View

Xiaohu Shang
Bell Labs, Alcatel-Lucent
xiaohu.shang@alcatel-lucent.com

Biao Chen
Syracuse University
bichen@syr.edu

the essence of knowledge
Boston — Delft

Foundations and Trends® in Communications and Information Theory

Published, sold and distributed by:
now Publishers Inc.
PO Box 1024
Hanover, MA 02339
United States
Tel. +1-781-985-4510
www.nowpublishers.com
sales@nowpublishers.com

Outside North America:
now Publishers Inc.
PO Box 179
2600 AD Delft
The Netherlands
Tel. +31-6-51115274

The preferred citation for this publication is

X. Shang and B. Chen. *Two-User Gaussian Interference Channels: An Information Theoretic Point of View*. Foundations and Trends® in Communications and Information Theory, vol. 10, no. 3, pp. 247–378, 2013.

This Foundations and Trends® issue was typeset in LaTeX using a class file designed by Neal Parikh. Printed on acid-free paper.

ISBN: 978-1-60198-733-4

Foundations and Trends® in Communications and Information Theory
Volume 10, Issue 3, 2013

Editorial Scope

Topics

Foundations and Trends® in Communications and Information Theory publishes survey and tutorial articles in the following topics:

- Coded modulation
- Coding theory and practice
- Communication complexity
- Communication system design
- Cryptology and data security
- Data compression
- Data networks
- Demodulation and Equalization
- Denoising
- Detection and estimation
- Information theory and statistics
- Information theory and computer science
- Joint source/channel coding
- Modulation and signal design
- Multiuser detection
- Multiuser information theory
- Optical communication channels
- Pattern recognition and learning
- Quantization
- Quantum information processing
- Rate-distortion theory
- Shannon theory
- Signal processing for communications
- Source coding
- Storage and recording codes
- Speech and Image Compression
- Wireless Communications

Information for Librarians

Foundations and Trends® in Communications and Information Theory, 2013, Volume 10, 4 issues. ISSN paper version 1567-2190. ISSN online version 1567-2328. Also available as a combined paper and online subscription.

Foundations and Trends® in Communications and Information Theory
Vol. 10, No. 3 (2013) 247–378

DOI: 10.1561/0100000071

Two-User Gaussian Interference Channels: An Information Theoretic Point of View

Xiaohu Shang
Bell Labs, Alcatel-Lucent
xiaohu.shang@alcatel-lucent.com

Biao Chen
Syracuse University
bichen@syr.edu

Contents

Abstract

The purpose of this monograph is to introduce up-to-date capacity theorems for the two-user Gaussian interference channel, including both single-antenna and multiple-antenna cases.

The monograph starts with the single antenna case and introduces the Han and Kobayashi achievable rate region and its various subregions. Several capacity outer bounds are then presented; these outer bounds, together with the achievable rate region, yield several capacity results for the single-antenna Gaussian interference channel. They include the capacity region for strong interference and the sum-rate capacity for Z interference, noisy interference, mixed interference, and degraded interference.

For the more complex multiple-antenna case, the interference state is no longer determined solely by the interference strength, as is the case for the single-antenna Gaussian interference channel. Instead, the structure of the interference in different multi-dimensional subspaces plays an equally important role. As a result of this multiple-dimensional signaling, new interference states, including generally strong, generally noisy, and generally mixed interference, are introduced to obtain capacity theorems that generalize those for the single-antenna case.

X. Shang and B. Chen. *Two-User Gaussian Interference Channels: An Information Theoretic Point of View*. Foundations and Trends® in Communications and Information Theory, vol. 10, no. 3, pp. 247–378, 2013.

DOI: 10.1561/0100000071.

Notations and Acronyms

Scalars, vectors and matrices

X	scalar X
$\boldsymbol{x}$	vector $\boldsymbol{x}$
$\mathbf{X}$	matrices $\mathbf{X}$
$\boldsymbol{x}^n$	$\boldsymbol{x}^n = \{\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_n\}$, a sequence of vectors $\boldsymbol{x}_i$, $i = 1, \ldots, n$
$\mathbf{I}$	identity matrix
$\mathbf{I}_{(k)}$	$k \times k$ identity matrix
$\mathbf{0}$	all-zero vector or matrix
$\mathbf{0}_{(k)}$	$k \times k$ all-zero matrix
$\mathbf{0}_{k\times r}$	a $k \times r$ all-zero matrix
$\boldsymbol{x}^{\mathrm{T}}$ or $\mathbf{X}^{\mathrm{T}}$	transpose of vector $\boldsymbol{x}$ or matrix $\mathbf{X}$
$\boldsymbol{x}^{\mathrm{H}}$ or $\mathbf{X}^{\mathrm{H}}$	conjugate transpose of vector $\boldsymbol{x}$ or matrix $\mathbf{X}$
$\mathbf{X}^{-1}$	inverse of matrix $\mathbf{X}$
$\\|\boldsymbol{x}\\|$	Euclidean vector norm of $\boldsymbol{x}$, i.e., $\\|\boldsymbol{x}\\|^2 = \boldsymbol{x}^{\mathrm{T}}\boldsymbol{x}$
$\lvert\mathbf{X}\rvert$	determinant of matrix $\mathbf{X}$
$\mathrm{tr}(\mathbf{X})$	trace of matrix $\mathbf{X}$
$\mathrm{rank}(\mathbf{X})$	rank of matrix $\mathbf{X}$
$\mathrm{Vec}(\mathbf{X})$	vectorization of matrix $\mathbf{X}$
$\mathrm{radius}(\mathbf{X})$	numerical radius of matrix $\mathbf{X}$
$\mathbf{A} \otimes \mathbf{B}$	Kronecker product of matrices $\mathbf{A}$ and $\mathbf{B}$
$\mathbf{A} \succeq \mathbf{B}$	matrices $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{A} - \mathbf{B}$ are all symmetric and

	positive semi-definite.
Probability	
$p(x)$	probability density function of random variable X
$p(x\|y)$	conditional density function of X given Y
$\boldsymbol{x} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma})$	$\boldsymbol{x}$ has a Gaussian distribution with zero mean and covariance matrix $\boldsymbol{\Sigma}$
$E[X]$	expectation of random variable X
$\mathrm{Var}(X)$	variance of random variable X
$\mathrm{Cov}(\boldsymbol{x})$	covariance matrix of random vector $\boldsymbol{x}$
$\mathrm{Cov}(\boldsymbol{x}, \boldsymbol{y})$	cross covariance matrix of random vectors $\boldsymbol{x}$ and $\boldsymbol{y}$
Information theory	
$H(X)$	entropy of discrete random variable X
$h(X)$	differential entropy of random variable X
$I(X;Y)$	mutual information between X and Y
Other	
$\mathrm{abs}(X)$	absolute value of X
$\mathrm{atan}(x)$	arctangent function
$\log(x)$	$\log_e(x)$
$\mathrm{sign}(x)$	$\mathrm{sign}(x) = 1$ if $x \geq 0$ and -1 if $x < 0$

1

Introduction

Interference commonly exists in multi-user wireless networks in which all the users share the same communication medium. While sending information to its intended receiver, each transmitter generates interference to all other receivers. The existence of interference may heavily degrade the overall performance of the system. Therefore, each user has to choose prudently its transmission strategy in order to optimize the performance of the entire network. Substantial effort has been devoted to understanding the impact of interference and to looking for capacity-achieving schemes of dealing with interference.

The scope of this monograph is limited to the study of a two-user Gaussian interference channel with a static setting, i.e., the channel state is assumed fixed and known to both the transmitters and the receivers. There have been significant efforts in studying more complex interference channel models, including that of multiple users and/or with fading channels. Our focus on the simple two-user Gaussian interference channel allows us to explore some of the subtle yet potentially fundamental aspects of interference in more details. It is not unreasonable to state that our understanding for such a simple channel model is still rather limited especially in some important parameter regimes that are of practical significance.

1.1 Interference Channel Model

The two-user interference channel (IC) was first introduced by Shannon [55]. This model consists of two pairs of transceivers, in which each transmitter communicates with its intended receiver while generating interference at the other receiver. The channel model and its related encoding decoding functions are defined as follows, for user i, $i = 1, 2$:

$$\begin{aligned}
\text{Message set} &: \quad \mathcal{M}_i = \{1, 2, \cdots, 2^{nR_i}\} \\
\text{Encoding function} &: \quad En_i : k_i \to X_i^n(k_i), \quad k_i \in \mathcal{M}_i \\
\text{Channel statistics} &: \quad p(y_1^n y_2^n | x_1^n x_2^n) = \prod_{j=1}^{n} p\left(y_{1j} y_{2j} | x_{1j} x_{2j}\right) \\
\text{Decoding function} &: \quad De_i : Y_i^n \to \hat{k}_i \\
\text{Error probability} &: \quad \frac{1}{2^{n(R_1+R_2)}} \sum_{k_1=1}^{2^{nR_1}} \sum_{k_2=1}^{2^{nR_2}} \Pr\left\{ \left(\hat{k}_1, \hat{k}_2\right) \neq (k_1, k_2) \right| \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \left. (k_1, k_2) \text{ sent} \right\}
\end{aligned}$$

where n is the block length, $X_i^n = \{X_{i1}, \cdots, X_{in}\}$ and $Y_i^n = \{Y_{i1}, \cdots, Y_{in}\}$ are respectively the transmitted and received signal sequences, k_i is the message index and $\hat{k}_i$ is its estimate, and R_i is the transmission rate.

If for a given (R_1, R_2) pair, there exist encoding and decoding functions such that the error probability is arbitrarily small when $n \to \infty$, then the rate pair (R_1, R_2) is achievable. The closure of the collection of all achievable rate pairs is the capacity region of this IC.

1.1.1 Capacity and Channel Transition Probabilities

Though the channel statistics of an IC is characterized by joint channel transition probability $p(y_1 y_2 | x_1 x_2)$, the capacity is often determined by marginal channel transition probabilities $p(y_1 | x_1 x_2)$ and $p(y_2 | x_1 x_2)$. The reason is that decoding is performed individually at each receiver. Therefore, dependence between Y_1 and Y_2 for a given pair of X_1 and X_2

does not have impact on the decoding performance. For a GIC, dependence between Y_1 and Y_2 given X_1 and X_2 can be introduced through, and controlled by, the correlation between the additive noises at the two receivers. Any two GICs with different transition probabilities still have the same capacity as long as their respective marginal transition probabilities are identical.

1.1.2 Single-Antenna Gaussian Interference Channel

Specializing to the Gaussian case, the received signals can be written as

$$Y_1 = \mathsf{h}_1 X_1 + \mathsf{f}_2 X_2 + \sigma_{z1} Z_1 \tag{1.1a}$$
$$Y_2 = \mathsf{h}_2 X_2 + \mathsf{f}_1 X_1 + \sigma_{z2} Z_2 \tag{1.1b}$$

where for $i = 1, 2$, h_i and f_i are channel coefficients known at both transmitters and receivers, Z_i is zero-mean Gaussian noise with unit variance, σ_{zi} is a positive constant, and the transmitted signal X_i is subject to an average power constraint:

$$\sum_{j=1}^{n} E\left(X_{ij}^2\right) \leq n\bar{P}_i. \tag{1.2}$$

It is easy to show that the capacity region of the Gaussian interference channel (GIC) defined in (1.1) is equivalent to that of the GIC in the standard form [11]:

$$Y_1 = X_1 + a_2 X_2 + Z_1 \tag{1.3a}$$
$$Y_2 = X_2 + a_1 X_1 + Z_2 \tag{1.3b}$$

where

$$a_1 = \frac{\mathsf{f}_2 \sigma_{z2}}{\mathsf{h}_2 \sigma_{z1}} \tag{1.4a}$$
$$a_2 = \frac{\mathsf{f}_1 \sigma_{z1}}{\mathsf{h}_1 \sigma_{z2}} \tag{1.4b}$$
$$P_i = \frac{\mathsf{h}_i^2}{\sigma_{zi}^2} \bar{P}_i. \tag{1.4c}$$

The GIC in the standard form is shown in Figure 1.1 where the transmit signals are subject to respective power constraints P_1 and P_2.

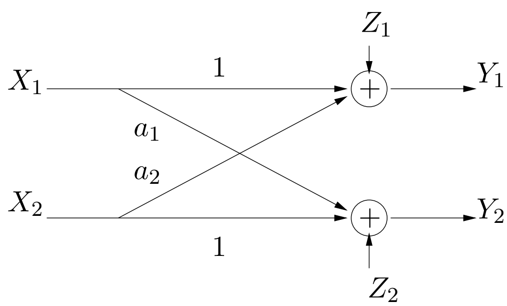

Figure 1.1: The two-user GIC model.

1.2 Existing Results for Gaussian Interference Channels

The IC model was first introduced by Shannon [55]. Alshwede [2] derived a limit expression for the capacity region:

$$\bigcup \left\{ (R_1, R_2) \left| \begin{aligned} R_1 &\le \lim_{n\to\infty} \frac{1}{n} I\left(X_1^n; Y_1^n\right) \\ R_2 &\le \lim_{n\to\infty} \frac{1}{n} I\left(X_2^n; Y_2^n\right) \end{aligned} \right. \right\}. \tag{1.5}$$

However, the practical significance of the above limit expression for the capacity region is not clear. Attempting to directly evaluate the limit expression of channel capacity is known to be a fruitless exercise [13]. Even for the simple case of the Gaussian multiple access channel, calculating the capacity using the limit expression was shown to yield a rate region that is strictly smaller than the capacity region when the input is limited to multivariate Gaussian distributions. This is despite the fact that Gaussian input achieves the single-letter expressed capacity region of the Gaussian multiple access channel. For two-user GIC, except for the noisy-interference case, multivariate Gaussian input distribution was also shown to be sum-rate sub-optimal using the limit expressions [1], even though Gaussian input distribution indeed achieves the sum-rate capacity for strong interference. As such, one can not expect that the limit expression for the GIC capacity region collapses into a single-letter expression that is easy to evaluate, and provides insight to capacity achieving coding schemes.

Many efforts have thus been devoted to the design of various coding schemes that lead to achievable regions with single-letter expressions. The best inner bound was obtained by Han and Kobayashi [24] using superposition encoding and joint decoding. This region was later simplified by Chong, Motani, Garg and El Gamal in [14] and by Kramer in [29]. Early outer bounds on the capacity region can be found in [37],[38],[12]. Cooperation and genie-aided outer bounds can be found in [28], [22], [51], [34], [3]. Etkin, Tse, and Wang showed that the Han and Kobayashi achievable region is within a half bit per user of the capacity region [22].

The first capacity region of the GIC was obtained by Carleial in [10] for the very strong interference case, in which the capacity is achieved by decoding and subtracting interference before decoding the useful signals. This result was extended to the strong interference case in [24] and [39], in which receivers jointly decode the interference and the useful signal to achieve capacity. The sum-rate capacity of the degraded GIC was obtained in [38]. It was shown in [15] that the capacity region of a Gaussian Z interference channel (GZIC) is equivalent to that of a degraded GIC. Therefore, the sum-rate capacity of a GZIC is obtained directly from [38] (see [36, Theorem 2]). Recently, it was shown in [51], [34], and [3] that the sum-rate capacity is achieved by treating interference as noise if the GIC satisfies a simple condition. This kind of GIC is said to have noisy interference. The sum-rate capacity for GICs with mixed interference was determined in [34] and [61].

On the study of the capacity for the multiple-input multiple-output (MIMO) GIC and its two special cases, namely, the multiple-input single-output (MISO) GIC and single-input multiple-output (SIMO) GIC, Telatar and Tse [58] showed that the Han-Kobayashi region is within a half bit per user and per receive antenna of the capacity region. Vishwanath and Jafar [60] determined the capacity region for the SIMO GIC with strong interference. Recent work in [45] and [47] derived the capacity region of a MIMO GIC for the strong interference case, and the sum-rate capacity for strong Z interference, weak Z interference, noisy interference and mixed interference. Other noisy interference conditions were obtained in [4] for MIMO GIC with full-rank optimal input covariance matrices and symmetric MISO and SIMO

GICs, and in [48] for parallel GICs (diagonal channel matrices). The noisy interference sum-rate capacity in [45],[47], [4], and [48] was later generalized in [54]. Realizing the fact that very strong interference is not necessarily a special case of strong interference for a MIMO GIC (a sharp contrast to the single-antenna GIC) [47], [46], a new category of interference type, referred to as generally strong interference, which includes both strong and very strong interferences as special cases, was introduced in [53]. The capacity region of the MIMO GIC with generally strong interference is achieved by jointly decoding the signal and interference. This new generally strong interference condition can determine the capacity region of MIMO GICs that cannot be determined by the traditional strong interference condition (e.g., MISO GICs).

1.3 Outline of Monograph

In this chapter, we have introduced the system model and the standard form for the single-antenna GIC. A brief summary of existing capacity results for GIC has also been given.

In Chapter 2, we present the Han-Kobayashi (H-K) achievable rate region and some of its subregions. Several important results, including the role of time sharing, the equivalence in capacity region between the GZIC and the degraded GIC, the so-called 'noiseberg' approach for the GZICs [31], are discussed in details.

In Chapter 3, we introduce several outer bounds on the capacity region. These outer bounds lead to the bounded gap between the H-K region and the capacity region, as well as various capacity results, including the capacity region for strong and very strong interference, the sum-rate capacity for noisy interference, Z interference, and mixed interference.

In Chapter 4, we introduce the standard form channel model for MIMO GICs and an effective way to compute the H-K region for MISO GICs. Subsequently, we extend all the capacity results for single-antenna GICs to multiple-antenna GICs. These include the capacity region for strong interference and very strong interference, and the sum-rate capacity for noisy interference. In addition, we introduce a new concept called generally strong interference, which generalizes the notion of strong interference, and obtain the capacity region under such conditions. The generally strong interference also allows us to obtain the sum-rate capacity for mixed interference. We also determine part of the capacity boundary for the MIMO GIC with weak Z interference and mixed interference.

References

[1] E. Abbe and L. Zheng. A coordinate system for Gaussian networks. *IEEE Trans. Inf. Theory*, 58:721 – 733, Feb. 2012.

[2] R. Alshwede. Multi-way communication channels. In *Proc. IEEE International Symposium on Information Theory*, Budapest, Hungaria, 1971.

[3] V. S. Annapureddy and V. V. Veeravalli. Gaussian interference networks: Sum capacity in the low interference regime and new outer bounds on the capacity region. *IEEE Trans. Inf. Theory*, 55(7):3032–3050, Jul. 2009.

[4] V. S. Annapureddy and V. V. Veeravalli. Sum capacity of MIMO interference channels in the low interference regime. *IEEE Trans. Inf. Theory*, 57(5):2565–2581, May 2011.

[5] A. S. Avestimehr, S. N. Diggavi, and D. N. C. Tse. Wireless network information flow: a deterministic approach. *IEEE Trans. Inf. Theory*, 57:1872 – 1905, Apr. 2011.

[6] R. H. Bartels and G. W. Stewart. Solution of the matrix equation AX + XB = C. *Commun. ACM*, 15:820–826, Sep. 1972.

[7] D. P. Bertsekas. *Nonlinear Programming*. Athena Scientific, Belmont, MA, 2003.

[8] G. Bresler, A. Parekh, and D. N. C. Tse. The approximate capacity of the many-to-one and one-to-many Gaussian interference channels. *IEEE Trans. Inf. Theory*, 56:4566 – 4592, Sep. 2010.

[9] V. Cadambe and S. Jafar. Interference alignment and the degrees of freedom of the k-user interference channel. *IEEE Trans. Inf. Theory*, 49:1073–1096, Aug. 2008.

[10] A. B. Carleial. A case where interference does not reduce capacity. *IEEE Trans. Inf. Theory*, 21:569–570, Sep. 1975.

[11] A. B. Carleial. Interference channels. *IEEE Trans. Inf. Theory*, 24:60–70, Jan. 1978.

[12] A. B. Carleial. Outer bounds on the capacity of interference channels. *IEEE Trans. Inf. Theory*, 29:602–606, July 1983.

[13] R. S. Cheng and S. Verdú. On limiting characterizations of memoryless multiuser capacity regions. *Trans. Inf. Theory*, 39:609–612, Mar, 1993.

[14] H. F. Chong, M. Motani, H. K. Garg, and H. El Gamal. On the Han-Kobayashi region for the interference channel. *IEEE Trans. Inf. Theory*, 54:3188–3195, Jul. 2008.

[15] M. H. M. Costa. On the Gaussian interference channel. *IEEE Trans. Inf. Theory*, 31:607–615, Sept. 1985.

[16] M. H. M. Costa. Noisebergs in z Gaussian interference channels. In *Proc. Information Theory and Applications Workshop*, pages 1–6, La Jolla, CA, Feb. 2011.

[17] M. H. M. Costa and A. A. El Gamal. The capacity region of the discrete memoryless interference channel with strong interference. *IEEE Trans. Inf. Theory*, 33:710–711, Sep. 1987.

[18] T. M. Cover and J. A. Thomas. *Elements of Information Theory.* New York: Wiley, second edition, 2006.

[19] I. Csiszár and J. Körner. *Coding Theorems for Discrete Memoryless Systems.* New York: Cambridge University Press, second edition, 2011.

[20] J. C. Engwerda, A. C. M. Ran, and A. L. Rijkeboer. Necessary and sufficient conditions for the existence of a positive definite solution of the matrix equation $\mathbf{X}+\mathbf{A}^*\mathbf{X}^{-1}\mathbf{A} = \mathbf{Q}$. *Linear Algebra and Its Applications*, 186:255–275, 1993.

[21] R. H. Etkin. New sum-rate upper bound for the two-user Gaussian interference channel. In *Proc. IEEE International Symposium on Information Theory*, pages 2582 – 2586, Seoul, Korea, Jun. 2009.

[22] R. H. Etkin, D. Tse, and H. Wang. Gaussian interference channel capacity to within one bit. *IEEE Trans. Inf. Theory*, 54(12):5534–5562, Dec. 2008.

[23] B. Hajek. *An Exploration of Random Processes for Engineers.* University of Illinois at Urbana Champaign, 2008. available at `http://www.ifp.uiuc.edu/~hajek/Papers/randomprocJan08.pdf`.

[24] T. S. Han and K. Kobayashi. A new achievable rate region for the interference channel. *IEEE Trans. Inf. Theory*, 27:49–60, Jan. 1981.

[25] R. A. Horn and C. R. Johnson. *Matrix Analysis.* Cambridge University Press, New York, 1985.

[26] S. A. Jafar. Interference alignment: A new look at signal dimensions in a communication network. *Foundations and Trends in Communications and Information Theory*, 7:1–136, 2011.

[27] S. A. Jafar and S. Vishwanath. Generalized degrees of freedom of the symmetric Gaussian k-user interference channel. *IEEE Trans. Inf. Theory*, 56:3297 – 3303, Jul. 2010.

[28] G. Kramer. Outer bounds on the capacity of Gaussian interference channels. *IEEE Trans. Inf. Theory*, 50:581–586, Mar. 2004.

[29] G. Kramer. Review of rate regions for interference channels. In *Proc. International Zurich Seminar*, pages 162–165, Zurich, Switzerland, Feb. 2006.

[30] T. Liu and P. Viswanath. An extremal inequality motivated by multiterminal information-theoretic problems. *IEEE Trans. Inf. Theory*, 53(5):1839–1851, May 2006.

[31] Y. Liu and E. Erkip. On the sum capacity of k-user cascade Gaussian z-interference channel. In *Proc. IEEE International Symposium on Information Theory*, pages 1382 – 1386, St. Petersburg, Russia, Jul. 2011.

[32] M. Maddah-Ali, A. Motahari, and A. Khandani. Communication over MIMO x channels: interference alignment, decomposition, and performance analysis. *IEEE Trans. Inf. Theory*, 54:3457–3470, Aug. 2008.

[33] R. Mochaourab and E. Jorswieck. Optimal beamforming in interference networks with perfect local channel information. *IEEE Trans. Signal Processing*, 59:1128–1141, Mar. 2011.

[34] A. S. Motahari and A. K. Khandani. Capacity bounds for the Gaussian interference channel. *IEEE Trans. Inf. Theory*, 55(2):620–643, Feb. 2009.

[35] R. T. Rockafellar. *Convex Analysis.* Princeton, NJ: Princeton University Press, 1997.

[36] I. Sason. On achievable rate regions for the Gaussian interference channels. *IEEE Trans. Inf. Theory*, 50:1345–1356, June 2004.

[37] H. Sato. Two-user communication channels. *IEEE Trans. Inf. Theory*, 23:295–304, May 1977.

[38] H. Sato. On degraded Gaussian two-user channels. *IEEE Trans. Inf. Theory*, 24:634–640, Sep. 1978.

[39] H. Sato. The capacity of the Gaussian interference channel under strong interference. *IEEE Trans. Inf. Theory*, 27:786–788, Nov. 1981.

[40] X. Shang. *On the Capacity of Gaussian Interference Channels*. PhD thesis, Syracuse University, Syracuse, NY, Aug. 2008.

[41] X. Shang and B. Chen. A new computable achievable rate region for the Gaussian interference channel. In *Proc. IEEE International Symposium on Information Theory 2007*, pages 2191–2195, Nice, France, Jun. 2007.

[42] X. Shang and B. Chen. Achievable rate region for downlink beamforming in the presence of interference. In *Proc. 41st Asilomar Conference on Signals, Systems, and Computers*, pages 1684–1688, Pacific Grove, CA, Nov. 2007.

[43] X. Shang, B. Chen, and M. J. Gans. On achievable sum rate for MIMO interference channels. *IEEE Trans. Inf. Theory*, 52(9):4313–4320, Sep. 2006.

[44] X. Shang, B. Chen, and M.J. Gans. On achievable sum rate for vector Gaussian interference channels. In *Proc. IEEE International Symposium on Information Theory*, pages 2215–2219, Seattle, WA, Jul. 2006.

[45] X. Shang, B. Chen, G. Kramer, and H. V. Poor. On the capacity of MIMO interference channels. In *Proc.* 46*th Annual Allerton Conference on Communication, Control, and Computing*, pages 700–707, Monticello, IL, Sep. 2008.

[46] X. Shang, B. Chen, G. Kramer, and H. V. Poor. MIMO Z-interference channels: Capacity under strong and noisy interference. In *Proc. 43rd Asilomar Conference on Signals, Systems and Computers*, pages 667–671, Pacific Grove, CA, Nov. 2009.

[47] X. Shang, B. Chen, G. Kramer, and H. V. Poor. Capacity regions and sum-rate capacities of vector Gaussian interference channels. *IEEE Trans. Inf. Theory*, 56(10):5030–5044, Oct. 2010.

[48] X. Shang, B. Chen, G. Kramer, and H. V. Poor. Noisy-interference sum-rate capacity of parallel Gaussian interference channels. *IEEE Trans. Inf. Theory*, 57(1):210–226, Jan. 2011.

[49] X. Shang, B. Chen, and H. V. Poor. Multi-user MISO interference channels with single-user detection: Optimality of beamforming and the achievable rate region. *IEEE Trans. Inf. Theory*, 57(7):4255–4273, Jul. 2011.

[50] X. Shang, G. Kramer, and B. Chen. Throughput optimization for multi-user interference channels. In *Proc. IEEE Military Communications Conference*, pages 1–7, San Diego, CA, Nov. 2008.

[51] X. Shang, G. Kramer, and B. Chen. A new outer bound and the noisy-interference sum-rate capacity for Gaussian interference channels. *IEEE Trans. Inf. Theory*, 55(2):689–699, Feb. 2009.

[52] X. Shang and H. V. Poor. Capacity for MIMO Gaussian interference channels with generally strong and noisy interference. In *Proc. IEEE International Symposium on Information Theory*, pages 2786–2790, Cambridge, MA, Jun. 2012.

[53] X. Shang and H. V. Poor. Capacity region of vector Gaussian interference channels with generally strong interference. *IEEE Trans. Inf. Theory*, 58(6):3472–3496, Jun. 2012.

[54] X. Shang and H. V. Poor. Noisy-interference sum-rate capacity for vector Gaussian interference channels. *IEEE Trans. Inf. Theory*, 59(1):132–153, Jan. 2013.

[55] C. E. Shannon. Two-way communication channels. In *Proc. 4th Berkeley Symp. Math. Stat. and Prob.*, volume 1, pages 611–644, Berkeley, CA, 1961. Also available in *Claude E. Shannon: Collected Papers*, IEEE Press, New York, 1993.

[56] S. Sridharan, A. Jafarian, S. Vishwanath, and S. A. Jafar. Capacity of symmetric k-user Gaussian very strong interference channels. In *Proc. IEEE Global Telecommunications Conference*, pages 1–5, New Orleans, LO, Nov. 2008.

[57] H. Stark and J. Woods. *Probability and Random Processes with Applications to Signal Processing.* New Jersey: Prentic Hall, third edition, 2002.

[58] E. Telatar and D. Tse. Bounds on the capacity region of a class of interference channels. In *Proc. IEEE International Symposium on Information Theory*, pages 2871–2874, Nice, France, Jun. 2007.

[59] J. A. Thomas. Feedback can at most double Gaussian multiple access channel capacity. *IEEE Trans. Inf. Theory*, 33(5):711–716, Sep. 1987.

[60] S. Vishwanath and S. A. Jafar. On the capacity of vector Gaussian interference channels. In *Proc. IEEE Information Theory Workshop*, pages 365–369, San Antonio, TX, Oct. 2004.

[61] Y. Weng and D. Tuninetti. On Gaussian interference channels with mixed interference. In *Proc. Information Theory and Applications Workshop*, San Diego, CA, Jan. 2008. [Online]. Available: `http://ita.ucsd.edu/workshop/08/files/paper/paper_264.pdf`.

[62] F. Willems and E. van der Meulen. The discrete memoryless multiple access channel with cribbing encoders. *IEEE Trans. Inf. Theory*, 31:313–327, May 1985.

[63] J. Xu, H. Chen, and B. Chen. New observations on interference channels under strong and very strong interference. In *Proc. IEEE Global Telecommunications Conference*, pages 1–5, Miami, FL, Dec. 2010.

[64] R. Zhang and S. Cui. Cooperative interference management with MISO beamforming. *IEEE Trans. Signal Processing*, 58:5450–5458, Oct. 2010.

[65] Y. Zhao, F. Zhu, and B. Chen. The Han-Kobayashi region for a class of Gaussian interference channels with mixed interference. In *Proc. IEEE International Symposium on Information Theory*, pages 2791 – 2795, Boston, MA, Jul. 2012.

[66] L. Zhou and W. Yu. On the capacity of the k-user cyclic Gaussian interference channel. *IEEE Trans. Inf. Theory*, 59:154–165, Jan. 2013.