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S. Sandeep Pradhan
Department of Electrical Engineering and Computer Science
University of Michigan
USA
pradhanv@umich.edu

Arun Padakandla
Department of Electrical Engineering and Computer Science
University of Tennessee
USA
arunpr@utk.edu

Farhad Shirani
Department of Electrical and Computer Engineering
North Dakota State University
USA
f.shiranichaharsoogh@ndsu.edu

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Contents

An Algebraic and Probabilistic Framework for Network Information Theory

S. Sandeep Pradhan[1], Arun Padakandla[2] and Farhad Shirani[3]

[1]*Department of Electrical Engineering and Computer Science, University of Michigan, USA; pradhanv@umich.edu*
[2]*Department of Electrical Engineering and Computer Science, University of Tennessee, USA; arunpr@utk.edu*
[3]*Department of Electrical and Computer Engineering, North Dakota State University, USA; f.shiranichaharsoogh@ndsu.edu*

ABSTRACT

In this monograph, we develop a mathematical framework based on asymptotically good random structured codes, i.e., codes possessing algebraic properties, for network information theory. We use these codes to propose new strategies for communication in multi-terminal settings. The proposed coding strategies are applicable to arbitrary instances of the multi-terminal communication problems under consideration. In particular, we consider four fundamental problems which can be considered as building blocks of networks: distributed source coding, interference channels, multiple-access channels with distributed states and multiple description source coding. We then develop a systematic framework for characterizing the performance limits of these strategies for these problems from an information-theoretic viewpoint. Lastly, we identify several examples of the multiterminal communication problems studied herein, for which structured codes

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attain optimality, and provide strictly better performance as compared to classical techniques based on unstructured codes. In summary, we develop an algebraic and probabilistic framework to demonstrate the fundamental role played by structured codes in multiterminal communication problems. This monograph deals exclusively with discrete source and channel coding problems.

1

Introduction

1.1 Overview

Many components of modern infrastructure such as transportation systems, power systems, climate and environment monitoring systems, education systems and even government are being increasingly interconnected through information networks. Information is playing an ever bigger role in our lives than just a decade ago. There is a need to gather, store, process and communicate information across several distributed devices. These devices continually and simultaneously perform one or more of these information processing tasks. In doing so, they share resources, co-ordinate to achieve their objectives, and even at times overcome individual constraints through the pooling of networked resources. Central to the functioning of current-day information networks are strategies that facilitate these network information processing objectives. In this monograph, we address the overarching challenge of designing efficient information processing strategies from a fundamental network information theory viewpoint. Specifically, we aim to identify the broad ideas that enable efficient processing of information in networks. Network information theory [1]–[3] is a comprehensive theory of information storage, transmission, and processing in networks. The foundation of network information was laid down by the pioneering works of Ahlswede, Berger, Bergmans, Cover, Csiszar, El Gamal, Gelfand, Han, Körner, Marton, Pinsker, Shannon, Slepian, Wolf and Wyner, and was developed by many follow-up works.

In this monograph, we examine several network communication problems which can be considered as building blocks of networks. We consider these problems from both the channel coding (data transmission) as well as the source coding (data storage) perspectives. We have twin objectives of (a) understanding the basic physics of information coding and processing strategies for these problems, and (b) developing the mathematics of characterizing fundamental and optimal performance limits (called achievable regions) of these strategies as applied to the corresponding problems. We look at multiterminal communication setups under different scenarios of collaboration among the terminals or lack of it. In particular we consider the following cases: (a) The multiterminal encoders are distributed: distributed source coding and multiple access channel with distributed channel state information. (b) The multiterminal decoders are distributed: multiple description coding. (c) Both the multiterminal encoders and decoders are distributed: communication over interference channels. In this monograph, we restrict our attention to the discrete memoryless setting. In particular, we focus on providing information-theoretic computable inner bounds to the performance limits by devising structured coding schemes for the finite alphabet cases of these problems. For each problem, we provide at least one example where we prove that the structured coding scheme is optimal, whereas the unstructured coding scheme is strictly suboptimal. We do not provide outer bounds to the performance limits of these problems.

Toward studying the information-theoretic performance limits in each of these communication scenarios, in the following, we consider two key concepts: common information and code structure, uncover a new fundamental connection between them, and then develop the key elements of a unified coding framework.

1.2 Structure of Codes

Characterizations of fundamental performance limits in communication over networks require devising optimal transmitters and receivers at distributed network terminals. Mathematically, the operations of transmitters and receivers are characterized using encoding functions and decoding functions, respectively, also referred to as codes. The image of an encoding function in channel coding and a decoding function in source coding is referred to as a codebook. Hence, one of the main objectives in network information theory is to design optimal codes. The conventional technique of deriving the performance limits for any communication problem in information theory is via random coding [2] involving so-called Independent Identically Distributed (IID) random codebooks. Since a *typical* code possesses only single-letter empirical properties, coding techniques are constrained to exploit only these using the law of large numbers for enabling efficient communication. Since random coding does not guarantee any structural properties on the codebooks other than these single-letter properties, we refer to them as unstructured code ensembles. A code ensemble is a collection of codes and a probability distribution defined on the collection. In most cases, only the average performance of such an ensemble is analytically tractable using law of large numbers.

Techniques based on IID unstructured code ensembles have been proven to achieve asymptotic performance limits in certain point-to-point and multiterminal communication problems in the discrete memoryless setting. Examples include the capacity of point-to-point (PTP) channels [4], multiple access channels [5]–[9] (MAC) and degraded broadcast channels [10]–[12], and the rate-distortion function of PTP source coding [13], successive refinement source coding [14], [15], and the rate region of the lossless distributed source coding [16]. A groundbreaking development in the realm of unstructured coding in network communications is the concept of random binning: random (unstructured) partition of codebooks. This, pioneered by Slepian and Wolf [16], led to the distributed source coding paradigm [17]–[26] which was solved completely in the lossless case. The next conceptual leap was taken by Cover and Bergmanns [10], [12], [27], [28], where they introduced a global structure in the codes. They constructed random superposition codes toward addressing broadcast channels. This also led to the concept of the auxiliary random variables that capture this structure [2], [29]. For the case of degraded broadcast channels, they were shown to be optimal [11], [12]. These codes have a global structure, and the structure is that of a cloud surrounded by satellites. The auxiliary random variables characterize the cloud centers of the superposition codes. The superposition codes were then employed for several other multi-terminal communication scenarios such as interference channels [30], [31], relay channels [32], multiple-description coding [33]–[36] and so on. Soon, random superposition codes were either shown to be optimal or were achieving the best performance for most scenarios [37]–[39]. Based on these initial successes, it was widely believed that one can achieve the performance limits of any network communication problem using IID superposition codes with random binning.

Stepping beyond these ideas that are based on unstructured code ensembles, Körner and Marton [40] proposed an ingenious technique based on statistically correlated codebooks (in particular, identical random linear codes) possessing algebraic closure properties, henceforth referred to as (random) structured code ensembles. They showed that their proposed scheme outperformed all techniques based on (random) unstructured code ensembles for the specific problem of distributed computation of modulo-2 sum of binary correlated sources. In other words, the average performance of the former ensemble is better than that of the latter. More recently, several examples [41]–[51] in the context of specific *symmetric and additive* communication scenarios have employed structured codes, and devised new coding techniques that outperform techniques that are based on IID unstructured codes. It appears that even if computation is a non-issue, algebraic structured codes may be necessary, in a deeply fundamental way, to achieve optimality in transmission and storage of information in networks. Structured codes appear to facilitate information cooperation among distributed terminals more efficiently. What is the fundamental reason behind this?

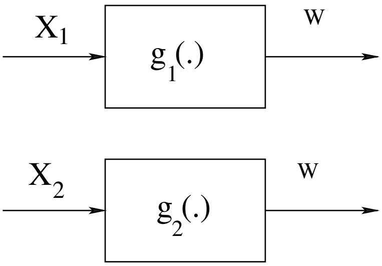


Figure 1.1: Common information between two correlated sources.

1.3 Common Information and Code Structure: 2 Terminal Case

Toward answering the above question, we appeal to the notion of common information which we explain through a simple example. Consider a pair of correlated information sources X_1 and X_2, distributed among two terminals. Terminal 1 observes only X_1 and terminal 2 observes only X_2. The common information [52], [53] (in the sense of Gacs, Körner and Witsenhausen) between X_1 and X_2 is the largest amount of common random bits per sample of the source that can be generated distributively by the two terminals by processing their respective information sources (see Figure 1.1). In other words, it is given by the maximum value of $H(W)$ such that $W = g_1(X_1) = g_2(X_2)$ with probability 1, where $H(\cdot)$ is the entropy function. We denote the common information between X_1 and X_2 as $C(X_1; X_2)$. Conventionally, common information has been seen as a measure of degeneracy of the joint probability matrix of X_1 and X_2. Rather than viewing common information as a form of degeneracy, we can look at it optimistically as a form of desirable and hence useful structure that certain random variables are endowed with.

If the sources can be expressed as $X_1 = (W, \tilde{X}_1)$ and $X_2 = (W, \tilde{X}_2)$ for a non-trivial random variable W, then the common information between them is non-zero. This concept plays a fundamental role in network information theory. To see its significance, for example, consider a peer-to-peer interference network where there are two pairs of encoder–decoder terminals. Each encoder wishes to transmit some information (that is independent of that of the other) reliably to its decoder. The encoders do not communicate with each other, nor do the decoders.

Let X_1 and X_2 denote the signals transmitted by the transmitters 1 and 2, respectively, taking values in a common alphabet X. The signal X_1 meant for decoder 1 interferes with the signal X_2 that is meant for decoder 2. What should be their communication strategy to maximize the system throughput? One approach that has been studied extensively involves partial interference decoding using random superposition codes. In this approach, each receiver decodes a part of the signal meant for the other, and then decodes the signal meant for it. A simpler version of this approach is illustrated as follows. Decoder 1 decodes a univariate function $U_2 = g_2(X_2)$ of the interfering signal X_2, before decoding X_1, where g_2: $\mathsf{X} \to \mathsf{U}$, and U is an auxiliary alphabet. Similarly decoder 2 decodes $U_1 = g_1(X_1)$ before decoding X_2, where g_1: $\mathsf{X} \to \mathsf{U}$. So after decoding, the information available at the receivers are given by (U_2, U_1, X_1), and (U_1, U_2, X_2). Hence they have non-trivial common information given by (U_1, U_2) (with high probability). Although, U_1 (respectively U_2) is not needed to be decoded at decoder 2 (respectively at decoder 1), the overall system throughput increases by doing so, thus inducing non-trivial common information between the decoder terminals. To facilitate such communication, the codes employed by the terminals should have a global structure that reflects the relations characterized by the univariate functions g_i.

In the conventional random superposition codes studied in information theory this is manifest as described below. For example, for the code associated with Encoder i, there are cloud center codewords $\{U_i^n(1), \ldots, U_i^n(M_{i1})\}$, generated IID from a distribution P_{U_i}. For each codeword, $U_i^n(j)$, there are satellite codewords $\{X_i^n(j,1), \ldots, X_i^n(j, M_{i2})\}$, generated IID from $P_{X_i \mid U_i}$ conditioned on $U_i^n(j)$, where M_{i1} and M_{i2} determine the rate of transmission, and n denotes the block-length. It can be argued that the superposition codes—with a cloud and satellite structure—have such a global structure, where the cloud center is associated with the random variable U_i and the satellite is associated with the random variable X_i. The structure of the superposition code associated with the ith encoder–decoder pair is that it is closed under the univariate function g_i, for $i = 1, 2$. In other words, if an n-length word X_i^n is a codeword, then the vector obtained by applying g_i on each component of X_i^n is also a codeword. The thesis is that the functional characterization of common information be reflected in the global structure of the code. It is this thesis that we plan to extend to the case of more than two terminals.

1.4 Common Information and Code Structure: 3 Terminal Case

Next we argue why algebraic structure emerges naturally from the perspective of common information for a system with more than two terminals. At the heart of an algebraic code such as a linear code or a group code [54], [55], there is an abstract group, and at the heart of it, there is a bivariate function [56]. This bivariate function is the addition operation associated with the group. For instance consider a linear code constructed over the ternary field $\mathbb{F}_3 = \{0, 1, 2\}$. The corresponding bivariate function in this example is addition modulo-3.

Recall that common information between two random variables is captured via a pair of univariate functions g_1 and g_2. What is the common information among three information sources, say X_1, X_2 and X_3? We answer this question, by noting that common information is a vector consisting of common information between every pair of sources, i.e., $C(X_1; X_2)$, $C(X_2; X_3)$, $C(X_1; X_3)$, and the information that is common to all three, $C(X_1; X_2; X_3)$, where $C(X_1; X_2; X_3)$ is given by the maximum value of $H(W)$ such that $W = g_1(X_1) = g_2(X_2) = g_3(X_3)$, with probability one. For example, consider $X_1 = \{U_1, U_2, U_3\}$, $X_2 = \{U_1, U_4\}$, and $X_3 = \{U_1, U_2, U_3 \oplus_2 U_4\}$, where the U_i's are IID binary symmetric random variables, and $\oplus_2$ denotes addition modulo-2. We have $C(X_1; X_2) = H(U_1)$, $C(X_2; X_3) = H(U_1)$, $C(X_1; X_3) = H(U_1, U_2)$, and $C(X_1; X_2; X_3) = H(U_1)$. All these four components are captured via univariate functions. Is that it? What about common information between the pair (X_1, X_2) and X_3? A moment's thought reveals that this structure in the joint probability matrix captured by $C(X_1, X_2; X_3)$ must be included in the common information among the three. For the above example, the random variable $U_3 \oplus_2 U_4$ is a bivariate function of (X_1, X_2) and a univariate function of X_3. Hence, the common information between (X_1, X_2) and X_3 is given by $H(U_1, U_2, U_3 \oplus_2 U_4)$.

Similarly, the common information between (X_1, X_3) and X_2 and that between (X_2, X_3) and X_1 must be included in the common information among X_1, X_2 and X_3 as well. So rather than a 4-dimensional vector, the common information is more like a 7-dimensional vector. The last three components are fundamentally different from the first four, because, in the latter, there is conferencing between a pair of sources. For example, to characterize $C(X_1, X_2; X_3)$, we need to evaluate the common randomness generated distributively by X_3 and by a "conference" between X_1 and X_2. The conference happens via a "bivariate" function, say, $g_{12}(X_1, X_2)$. So, the last three components of the common information are captured via bivariate functions. We call this conferencing common information. Common information has also been looked at from related but different perspectives in [57]–[59].

This structure cannot be manifested through univariate common information. The next question that comes up is can all these components be exploited in a multiterminal communication problem involving three or more transmitters, or receivers or both. The answer is yes. It turns out that to exploit these components of common information, we need codes which are closed under the corresponding bivariate function. This is the fundamental connection between common information and algebraic structured codes. This is the thesis which the present monograph is woven around. Let us revisit the interference network example, now with three pairs of encoder–decoder terminals. The signals X_2 and X_3 meant for Decoder 2 and 3 interfere with X_1. Suppose that the interference is characterized as $g_{12}(U_2, U_3)$, where $U_i = g_i(X_i)$ for $i = 2, 3$. Then Decoder 1 may wish to decode $g_{12}(U_2, U_3)$, instead of the pair (U_2, U_3), before decoding X_1. To facilitate such a decoding, the codes associated with Encoder 2 and 3 need to be closed with respect to $g_{12}(\cdot, \cdot)$. This induces a conferencing common information between the messages decoded at Decoder 1 that decoded at Decoder 2 and 3. This global structure of the codes need to reflect the function characterizing the conferencing common information. A code may be called structured if it is closed with respect to a bivariate function.

To see this more clearly, let us assume that the bivariate function is given by the addition operation corresponding to a finite abelian group (which is defined formally in Section 2). It is well known [60] that primary cyclic groups form the building blocks of all finite abelian groups. A primary cyclic group $\mathbb{Z}_{p^r}$ is given by the set $\{0, 1, 2, \ldots, p^r - 1\}$ with addition modulo-p^r, and p is a prime number. The primary cyclic group $\mathbb{Z}_{p^r}$ has a natural ring structure with multiplication modulo-p^r. When $r = 1$, we get a primary finite field, $\mathbb{F}_p$. Consider a code $\mathbb{C}$ of block-length n built on the alphabet $\mathbb{Z}_{p^r}$. Let us suppose that the code $\mathbb{C}$ is a coset of a subgroup of the n-product group $\mathbb{Z}_{p^r}^n$, i.e., $\mathbb{C} - B^n \leq \mathbb{Z}_{p^r}^n$, for some vector $B^n \in \mathbb{Z}_{p^r}^n$. Note that the size of sum of $\mathbb{C}$ and $\mathbb{C}$ is equal to that of $\mathbb{C}$, i.e., $|\mathbb{C} + \mathbb{C}| = |\mathbb{C}|$. In contrast, if $\mathbb{C}$ were a random subset of $\mathbb{Z}_{p^r}^n$, then, with high probability, we have $|\mathbb{C} + \mathbb{C}| \approx |\mathbb{C}|^2$. The algebraic structure helps in containing the size of $|\mathbb{C} + \mathbb{C}|$. Hence decoding in $\mathbb{C} + \mathbb{C}$ is much easier than decoding in $\mathbb{C} \times \mathbb{C}$.

This concept has also been studied in additive number theory and additive combinatorics [61], [62], where estimates of sums of subsets of integers such as $A + B = \{x + y \colon x \in A, y \in B\}$ are characterized. One should be able to exploit the powerful results available in this area to develop a theory of asymptotically good structured codes. For example, one of the most fundamental results in this area is the Davenport-Cauchy theorem which provides a lower bound on the sum of two subsets of a prime finite field: for any $A, B \subset \mathbb{F}_p$, we have $|A + B| \geq \min\{|A| + |B| - 1, p\}$. Another important result is Freiman theorem, a version [63] of which states that if A is a binary code of length n, i.e., $A \subset \mathbf{F}_2^n$, and $|A + A| \leq K|A|$, then A is contained in a coset of a subspace of size no larger than $K^2 2^{2K^2 - 2}|A|$. One could use these results toward deriving outer bounds to the achievable rate regions in multiterminal networks.

What is the information-theoretic cost of endowing a code $\mathbb{C}$ with the algebraic structure of a subgroup for the task of packing and covering? Put in a different way, what is the capacity of a point-to-point channel if one is restricted to use a code which is a subgroup of $\mathbb{Z}_{p^r}^n$? Toward answering this question, first consider a discrete memoryless symmetric channel with input alphabet $\mathbb{Z}_{p^r}$, where the capacity-achieving input distribution is uniform. From standard random coding arguments (see [64]), we know that any code ensemble where (i) the distribution of each codeword is uniform over $\mathbb{Z}_{p^r}^n$, and (ii) the codewords in the code ensemble are pairwise independent achieves the capacity. For the algebraic codes, consider first the case where $r = 1$. In this case, $\mathbb{Z}_p$ is a simple group i.e., the group does not contain non-trivial subgroups, and hence a finite field $\mathbb{F}_p$. Furthermore, any subgroup of $\mathbb{F}_p^n$ is endowed with a vector space structure with a generator matrix G of size $k \times n$ for some $k \leq n$ [54], [65]. Consider a code ensemble (called coset code ensemble) where a random structured code is constructed by choosing the generator matrix G and the shift vector B^n randomly and uniformly. For any four fixed vectors $u_1^k, u_2^k \in \mathbb{F}_p^k$, $x_1^n, x_2^n \in \mathbb{F}_p^n$, and $u_1^k \neq u_2^k$, we see that

$$
\begin{aligned}
P(u_1^k G + B^n &= x_1^n, u_2^k G + B^n = x_2^n) \\
&= P((u_1^k - u_2^k)G = x_1^n - x_2^n, u_1^k G + B^n = x_1^n) && (1.1) \\
&\overset{(a)}{=} P(u^k G = x^n)\frac{1}{p^n} && (1.2) \\
&\overset{(b)}{=} P\left(\sum_{i=1}^{k} u_i g_i^n = x^n\right)\frac{1}{p^n} && (1.3) \\
&\overset{(c)}{=} P\left(g_j^n = u_j^{-1}\left(x^n - \sum_{i\neq j} u_i g_i^n\right)\right)\frac{1}{p^n} && (1.4) \\
&= \frac{1}{p^n}\frac{1}{p^n}, && (1.5)
\end{aligned}
$$

where (a) follows by defining $u^k := u_1^k - u_2^k$, and $x^n := x_1^n - x_2^n$, (b) follows by denoting the ith row of G as g_i^n, and (c) follows because there exists an index j such that $u_j \neq 0$. Thus the two properties given above-uniformity and pairwise independence – are satisfied, and hence the coset code ensemble achieves the capacity. Similarly, one can show that coset code ensemble achieves the Shannon rate-distortion function of a symmetric source with additive distortion function. In this sense, the finite field structure comes for free.

Now consider the case when $r > 1$. In this case, $\mathbb{Z}_{p^r}$ is not a finite field because of the presence of non-trivial subgroups, and hence zero divisors, non-zero elements which do not have multiplicative inverses. For example $p^i\mathbb{Z}_{p^r}$ is a non-trivial subgroup for any $0 < i < r$. It can be noted that subgroups of $\mathbb{Z}_{p^r}^n$ have the structure of a module instead of a vector space. Although there are many details, to see the big picture, consider a simple code ensemble with a generator matrix G of dimension $k \times n$ and a shift vector B^n chosen independently and uniformly. Let us evaluate $P(u^kG = x^n)$, for $u^k \neq 0$, where the multiplication is modulo-p^r. First consider the case when at least one component of u^k is invertible, i.e., $u^k \notin p\mathbb{Z}^k_{p^r}$. Then as in the case of finite fields, the equation $u^kG = x^n$ has $p^{(k-1)rn}$ solutions, i.e., except one row, all the elements can be G chosen arbitrarily. Hence $P(u^kG = x^n) = \frac{1}{p^{rn}}$. There are $p^{rk} - p^{(r-1)k}$ such vectors u^k. Next consider the case when $u^k \in p\mathbb{Z}^k_{p^r}$, and $u^k \notin p^2\mathbb{Z}^k_{p^r}$. Then the equation $u^kG = x^n$ has p^n times as many solutions as before if $x^n \in p\mathbb{Z}^n_{p^r}$, and no solutions otherwise. Because, all but one row can be chosen arbitrarily, and the remaining row has p^n solutions. Hence $P(u^kG = x^n) = \frac{p^n}{p^{rn}}$. Continuing similarly, one can see that

$$P(u^kG = x^n) = \begin{cases} \dfrac{1}{p^{(r-i)n}} & \text{if } u^k \in p^i\mathbb{Z}^k_{p^r} \backslash p^{i+1}\mathbb{Z}^k_{p^r}, x^n \in p^i\mathbb{Z}^n_{p^r} \\ & \qquad 0 \leq i < r \\ 1 & \text{if } u^k \in p^r\mathbb{Z}^k_{p^r}, x^n \in p^r\mathbb{Z}^n_{p^r}. \end{cases} \tag{1.6}$$

Hence, pairwise independence is lost, and this results in an information-theoretic penalty. In some cases, it can be shown that the capacity of the channel cannot be achieved with such a code ensemble. Similarly, there is an asymptotic performance loss in source coding with a fidelity criterion. In other words, there is an information-theoretic cost for endowing the code with an algebraic structure of a module. Even then, in certain multiterminal communication problems, such a code ensemble performs better than the random code ensemble in terms of the overall system performance because of the property $|\mathbb{C} + \mathbb{C}| = |\mathbb{C}|$. Although in this monograph we restrict ourselves to the case $r = 1$ for simplicity, we give a brief overview of the performance limits for general case $r > 1$ in Section 7. We also look at the case when the group is not abelian.

For the more general nonsymmetric discrete memoryless channels where the capacity achieving input distribution is not symmetric, the coset code ensemble may not achieve the capacity. In such cases, we use either random binning of a coset code or unionizing of coset codes to achieve the capacity which is discussed in the next subsection.

For more than three terminals, we expect trivariate functions or more generally higher-order multivariate functions [66] to play significant roles. Examples include triple product $a \cdot (b \times c)$ used to define the volume of a parallelepiped [67], and the operation $(a \otimes b) + c$ used to define planar ternary rings with applications in projective geometries [68].

In spite of the several works that show that structured codes perform better than unstructured codes in several communication problems, there are still many questions remaining.

(i) Firstly, in contrast to the rich theory [1], [3] based on IID unstructured codebooks, structured codes have been studied only in the context of particular additive and symmetric channels, and we do not have a framework to treat general channels and sources. There is a lack of a general theory for arbitrary instances of the multi-terminal channels or multi-terminal sources. Linear codes and group codes [69] have been studied extensively toward achieving known performance limits in information theory [64], [70]–[81]. How does one reconcile an apparent contradiction between the negative result that linear codes cannot achieve the capacity of arbitrary point-to-point channels [82]–[85] and ingenious techniques of Körner and Marton based on linear codes.

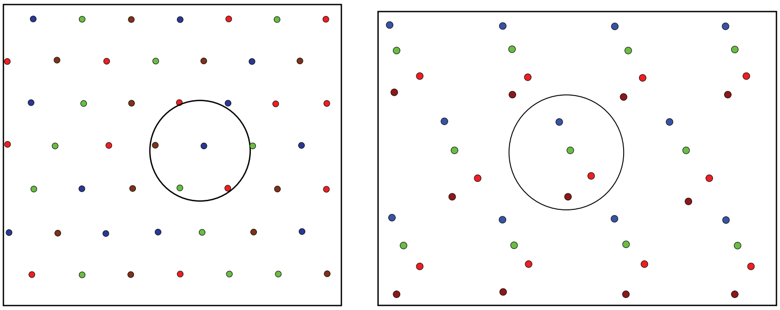
(ii) Secondly, the lack of wider applicability of structured codes to diverse communication scenarios has been a cause for concern. One of the appealing aspects of the theory based on IID codebooks is the ability to appropriately stitch together current known coding techniques – superposition [10], [27], binning [16], correlated quantization [33], block Markov superposition [86], [87] – to effect enhanced throughput and derive new achievable rate regions for any multi-terminal communication scenario. How does one stitch together current techniques based on structured codes to exploit algebraic closure properties over an arbitrary instance of a generic communication scenario?

(iii) Thirdly, there is a lack of a rich set of examples, beyond particular symmetric additive examples for which structured codes outperform current known techniques based on IID codes. Today, the question of whether structured codes are needed for non-additive problems is yet to be answered satisfactorily. To address these issues, we aim to develop a unified algebraic and probabilistic framework for network information theory.

1.5 Key Elements of the Framework

Firstly, we propose two new ensembles of coset codes (shifted linear codes): (i) partitioned coset codes (PCC), and (ii) unionized coset codes (UCC). These codes possess both the empirical properties present in unstructured codes [88] as well as algebraic closure properties. A PCC is a coset code that is randomly partitioned into bins. This means that each codeword in the shifted linear code is assigned a bin number chosen randomly and uniformly among a predetermined number of bins. Absent such partitioning, linear codes can only achieve a very limited set of empirical distributions (e.g., uniform distributions). As will be described in the subsequent sections, the partitioning ensures each bin possesses codewords with a specified empirical distribution and thereby achieves rates corresponding to non-uniform distributions. The overall code being a coset code possess (algebraic) closure properties (see Figure 1.2(a)). Alternatively, linear codes followed by nonlinear mapping have been used to achieve performance limits in point-to-point communication [64], [77]. We will not pursue this idea in this monograph.

On the other hand, a UCC is a collection of arbitrary cosets comprising a code (see Figure 1.2(b)). Similar to the binning operation in PCCs, the unionizing operation in UCCs allows them to achieve non-uniform empirical distributions. In many multiterminal communication systems, we see that both covering codes and packing codes are necessary in order to achieve the optimal performance. Moreover, they are used such that either a packing code is partitioned into covering codes or the other way around. In other words, we have nested codes with a denser code (packing/covering) containing a sparser code (covering/packing). As we have seen one can endow asymptotically optimal codes with the algebraic structure associated with a finite field with no cost. In other words, finite field structure comes for free. However, as we loosen the algebraic structure from that of a finite field to that of an arbitrary group, we have to pay a price for endowing a code with a group structure [89], [90] (see Section 7 for an example). So in PCC, we endow the



(a) (b)

Figure 1.2: (a) A depiction of PCC: The shaded circles denote the coset code. The circles belonging to a given color form a bin. Within the typical set (big circle), there is only one codeword with a given bin (color). (b) A depiction of UCC: The circles belonging to a given color form a bin. Each bin is a coset code. Within the typical set, there is only one codeword with a given bin (color).

denser code with an algebraic structure, whereas in UCC we endow the sparser code with an algebraic structure.

Secondly, in this monograph, we build upon the technique of typicality set encoding and decoding [2] to generalize coding techniques to arbitrary sources and channels. This approach has also been looked at in [91]. It can be noted that the theory developed in this framework can be further generalized to continuous-valued sources and channels [92], and in particular Gaussian multi-terminal channels and sources. Lattices have been studied extensively for source coding and channel coding in the linear quadratic Gaussian setting both in the point-to-point and multiterminal settings [93]–[106]. In particular, a framework involving lattice codes for communicating over real-valued channels can be obtained by mapping coset codes to lattice codes. These findings indicate that coset codes built over finite fields described here are only the first step in exploiting algebraic closure properties. Furthermore, it turns out that [73], [89], [90], [107]–[109], a framework based on codes built over groups can be employed to derive even larger achievable rate regions.

Thirdly, we develop new information-theoretic techniques to analyze performance of jointly related coset codes. The proposed algebraic framework develops all necessary tools to exploit algebraic closure properties in diverse communication scenarios. We believe that a good understanding of this framework will lead to a spurt of research activity in multi-terminal information theory and enable researchers develop new coding techniques based on coset codes for diverse communication scenarios.

Fourthly, the use of algebraic closure properties for enhanced *throughput* unfolds a new paradigm in communication. In practice, most communication systems employ structured codes and exploit structure in efficient encoding and decoding operations. Findings in information theory that involve structured codes indicate that structure can be exploited for enhanced throughput. This is a welcome sign for researchers aiming to build efficient communication systems.

The monograph is organized as follows. We apply the coding techniques to three network topologies: (a) many-to-one communication, (b) one-to-many communication and (c) many-to-many communication, from perspectives of source coding and channel coding. We choose the following problems for exposition as example scenarios for four different cases of the use of PCC and UCC for source coding and channel coding (see Figure 1.3). Section 2 will introduce the reader to these ensembles of codes and prove two fundamental results – UCC and PCC achieve both the capacity of arbitrary point-to-point channels and the rate-distortion function of an arbitrary source. These two results essentially establish the packing and covering properties of UCC and PCC. We address the following four specific multiterminal problems with distributed encoders and decoders. In Section 3, we consider the distributed source coding problem (many-to-one) where we use UCC with the covering code being partitioned into packing codes. The algebraic structure of the sparser packing code is exploited for efficient binning in this problem. Then we focus on the interference channel (many-to-many) in Section 4, where we use PCC with packing code being partitioned into covering codes. The algebraic structure of the denser packing code is exploited for interference alignment in this problem. Then we move on to the problem of multiple-access channels with distributed states (many-to-one)

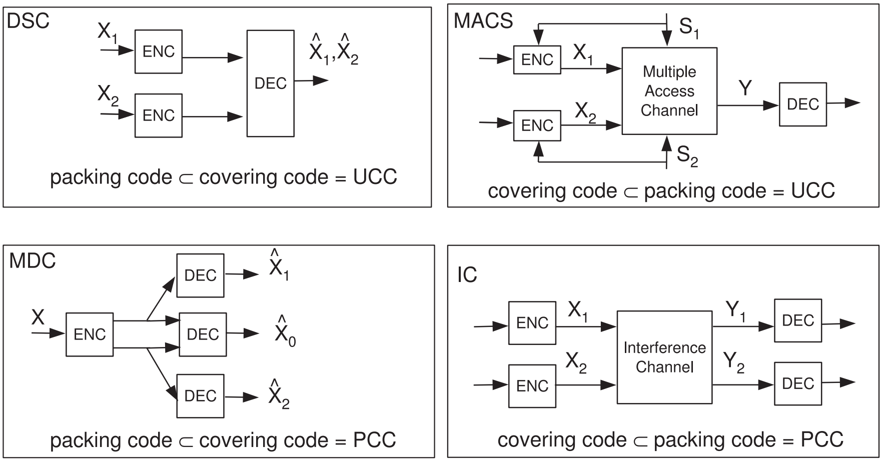


Figure 1.3: Four basic building blocks of networks: Distributed encoders: DSC (distributed source coding), MACS (multiple-access channel with states). Distributed Decoders: MDC (multiple description coding), IC (interference channel). Systems involving distributed encoders use UCC (unionized coset codes). Systems involving distributed decoders use PCC (partitioned coset codes). In source coding problems, covering codes are partitioned into packing codes. In channel coding problems, packing codes are partitioned into covering codes.

available at the encoders[1] in Section 5. In this problem, we use UCC with packing codes being partitioned into covering codes, where the algebraic structure of the sparser covering code is exploited for efficient side information covering. Finally, it is the multiple description problem (one-to-many) in Section 6, where we will be using PCC with covering codes being partitioned into packing codes. For this scenario, we exploit the algebraic properties of the denser covering codes for efficient multiple covering of the source. These observations also bring out a duality connections between source coding and channel coding problems. Our objective is to study both source coding and channel coding problems to get deep insights into the inner workings of network information theory. Duality between source coding and channel coding problems have been studied extensively in the literature [1], [13], [110]–[116].

[1]The performance limits of multiple-access channel with arbitrary number of transmitters and without states have been characterized using IID unstructured code ensembles.

One can obtain very interesting and deep insights into the structure of algebraic codes which give improved performance (over unstructured codes) for the problems which are building blocks of networks. (i) It turns out that systems with distributed encoders benefit from the use of UCC, and (ii) systems with distributed decoders benefit from the use of PCC. (iii) In source coding problems, covering code is partitioned into packing codes, and (iv) in channel coding problems, packing code is partitioned into covering codes. A brief summary of these finding is illustrated in Figure 1.3 for four basic multi-terminal networks.

1.6 Notation

We employ notation that has now been widely accepted in the information theory literature [1]–[3] supplemented with that given in Table 1.1. Upper case letters X, Y, Z denote random variables, and smaller case letters x, y, z denote the values taken by the random variables. All the random variables considered in this monograph are finite valued. The probability distribution of a triple of random variables (X, Y, Z) (XYZ for short) is denoted as P_{XYZ}.

In this monograph, we need to define multiple objects, mostly triples, of the same type. In order to reduce clutter, we use an underline to denote aggregates of objects of similar type. For example, (i) if $\mathsf{Y}_1, \mathsf{Y}_2, \mathsf{Y}_3$ denote (finite) sets, we let $\underline{\mathsf{Y}}$ either denote the Cartesian product $\mathsf{Y}_1 \times \mathsf{Y}_2 \times \mathsf{Y}_3$ or abbreviate the collection $(\mathsf{Y}_1, \mathsf{Y}_2, \mathsf{Y}_3)$ of sets, the particular reference being clear from context, (ii) if $y_k \in \mathsf{Y}_k$: $k = 1, 2, 3$, we let $\underline{y} \in \underline{\mathsf{Y}}$ abbreviate $(y_1, y_2, y_3) \in \mathsf{Y}$ (iii) if d_k: $\mathsf{Y}_k^n \to \mathsf{M}_k$: $k = 1, 2, 3$ denote (decoding) maps, then we let $\underline{d}(\underline{y}^n)$ denote $(d_1(y_1^n), d_2(y_2^n), d_3(y_3^n))$.

We write α_{M} to express the vector $(\alpha_1, \alpha_2, \ldots, \alpha_m)$ where $\mathsf{M} = \{1, 2, \ldots, m\}$. A collection whose elements are sets is called a family of sets and is denoted by the calligraphic typeface $\mathscr{M}$. For a given family of sets $\mathscr{M}$ we define a set $\widetilde{\mathscr{M}} = \bigcup_{\mathsf{M} \in \mathscr{M}} \mathsf{M}$ as the set formed by the elements of the sets in $\mathscr{M}$. The family of sets containing all subsets of M is denoted by 2^{M}. A collection whose elements are families of sets is denoted by the bold typeface $\mathbf{M}$. The collection of families of sets $\{\mathscr{A}_1, \mathscr{A}_2, \ldots \mathscr{A}_m\}$ is also represented by $\mathscr{A}_{\mathsf{M}}$. In some case, random variables are indexed by families of sets as in $U_{\mathscr{M}}$. For the purposes of brevity we will write

Table 1.1: Description of symbols used in the monograph

Symbol	Meaning
$\mathrm{cl}(A)$	Closure of $A \subseteq \mathbb{R}^k$
$\mathrm{cocl}(A)$	Closure of convex hull of $A \subseteq \mathbb{R}^k$
$h_b(x)$	Binary entropy function $-x \log x - (1-x)\log(1-x)$
$[K]$	The set $\{1, 2, \ldots, K\}$
$\mathbb{P}$	The set of all prime numbers
$\mathbb{F}_p$	Finite field of size p with addition $\oplus_p$ (also denoted as $+$)
$\mathbb{F}_p^+$	$\mathbb{F}_p \backslash \{0\}$
$a \ominus_q b$	$a \oplus_q (-b)$ for $a, b \in \mathbb{F}_q$
$a * b$	Binary convolution $a(1-b) + b(1-a)$
$\ll$	Absolutely continuous
X, Y	Finite alphabets of sources and channels
$\mathscr{C}$	The capacity region of a channel
$\mathscr{R}$	Rate-distortion region of a source
$\mathscr{R}_i$	Inner bound to capacity/rate-distortion region
M	A Message set or an index set
$\mathscr{P}$	A set of probability distributions
d	A distortion function d: $\mathsf{X} \times \hat{\mathsf{X}} \to \mathbb{R}^+$
κ	A cost function κ: $\mathsf{X} \to \mathbb{R}^+$
τ	Expected cost
ξ	Probability of decoding error
$A_\epsilon^{(n)}(Q)$	Frequency typical set of a random variable Q with parameter ϵ
$\mathbb{E}(\cdot)$	Expectation operation

$U_{\mathsf{M}_1, \mathsf{M}_2, \ldots, \mathsf{M}_n}$ instead of $U_{\mathscr{M}}$ where $\mathscr{M} = \{\mathsf{M}_1, \mathsf{M}_2, \ldots, \mathsf{M}_n\}$ wherever the notation doesn't cause ambiguity. $U_{\mathscr{M}}^n$ denotes a vector of length n of random variables, each distributed according to the distribution $P_{U_{\mathscr{M}}}$. Definitions of basic information measures and key results regarding typicality are collected in the Appendix.

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