Channel Coding Methods for Non-Volatile Memories

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Contents

Abstract

Non-volatile memories (NVMs) have emerged as the primary replacement of hard-disk drives for a variety of storage applications, including personal electronics, mobile computing, intelligent vehicles, enterprise storage, data warehousing, and data-intensive computing systems. Channel coding schemes are a necessary tool for ensuring target reliability and performance of NVMs. However, due to operational asymmetries in NVMs, conventional coding approaches - commonly based on designing for the Hamming metric - no longer apply. Given the immediate need for practical solutions and the shortfalls of existing methods, the fast-growing discipline of coding for NVMs has resulted in several key innovations that not only answer the needs of modern storage systems but also directly contribute to the analytical toolbox of coding theory at large.

This monograph discusses recent advances in coding for NVMs, covering topics such as error correction coding based on novel algebraic and graph-based methods, write-once memory (WOM) codes, rank modulation, and constrained coding. Our goal in this monograph is multifold: to illuminate the advantages - as well as challenges - associated with modern NVMs, to present a succinct overview of several exciting recent developments in coding for memories, and, by presenting numerous potential research directions, to inspire other researchers to contribute to this timely and thriving discipline.

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1

Data-Driven Need For More Reliable Memories

Welcome to the age of data! The amount of data generated in 2014 was estimated to be an unprecedented 4 Zettabytes (which is four billion Terabytes!), and is expected to increase to a whopping 40 Zettabytes by 2020 [2]. This data deluge has provided opportunities for scientific discoveries and technological innovations. To make full use of new data, the ability to store large-scale data reliably and efficiently has become the key challenge. Yet, available data storage systems have been grossly outpaced by data generation – the capacity of current storage systems is estimated at only 30% of generated data, and this percentage will decrease even further [2]. It is thus paramount to develop new approaches that make data storage systems fast, ultra-reliable, dense, and affordable.

Non-volatile memories (NVMs) are a class of memories that maintain stored data even after being disconnected from a power supply. NVMs offer faster data access, reduced power consumption, and improved physical resiliency relative to conventional hard disks. As a result, NVMs have emerged as the primary replacement of hard-disk drives for a variety of storage applications, including personal electronics, mobile computing, intelligent vehicles, enterprise storage, data warehousing, and data-intensive computing systems.

Flash memories are the most popular NVM technology today. Other NVM types, such as phase-change memory (PCM), magnetoresistive random access memory (MRAM), and spin-transfer-torque random access memory (STT-RAM), are also being actively pursued and may eventually offer viable alternatives to flash. Given the maturity and the ubiquity of flash devices, we will primarily focus on coding methods for flash memories; as appropriate, we will comment on the usage of related techniques for other technologies as well.

A flash device consists of floating gate transistors, organized in a two- or three-dimensional array. Each memory cell stores information: the amount of charge stored in the cell corresponds to a certain digital value. Improvements in fabrication technology over the past decade have resulted in decreased per-unit cost and increased areal densities of NVMs, and have thus made flash memories the primary replacement of hard disk drives in a range of modern applications. However, these positive trends have come at the expense of reduced reliability and limited device lifetime.

Unlike hard disks, NVMs can only be used for a certain number of program and erase (P/E) cycles, after which a device is deemed unusable. Currently, there is an unfavorable trade-off between storage density and endurance: as the cell density doubles, the lifetime drops by 10x - 20x [67]. Additionally, with the increase in storage capacity, due to a variety of physical impairments, memory reliability also rapidly decreases.

A particularly promising approach to overcome these physical limitations in NVMs is to apply error correction schemes. The field of channel coding deals with the development of practical error correction methods that introduce controlled, carefully designed redundancy into a data stream to make transmission or storage over a noisy medium as reliable as possible. Channel coding methods have been used with great success in data storage technologies and have helped make computer storage ubiquitous. However, NVMs present unique operational constraints that make existing channel coding methods inadequate. In particular, due to operational asymmetries in NVMs, conventional coding approaches – commonly based on designing for the Hamming metric – no longer apply. Given the immediate need for practical solutions and the shortfalls of the existing methods, the fast-growing discipline of coding for NVMs has resulted in several key innovations that not only answer the needs of modern storage systems but also directly contribute to the analytical toolbox of coding theory at large.

This manuscript discusses recent advances in coding for NVMs, covering topics such as error correction coding based on novel algebraic and graph-based methods, write-once memory (WOM) codes, rank modulation, and constrained coding. Our goal for this work is multifold: to illuminate the advantages – as well as challenges – associated with modern NVMs, to present a succinct overview of several exciting recent developments in coding for memories, and, by presenting several possible research directions, to inspire other researchers to contribute to this timely and thriving discipline.

In Chapter 2 we will discuss the key operational features associated with flash memories. Motivated by these idiosyncrasies, subsequent chapters will overview recent developments in channel coding methods (Chapter 3), WOM codes (Chapter 4), rank modulation schemes (Chapter 5), and constrained codes (Chapter 6). The concluding Chapter 7 will summarize key results and will also list several research topics of interest to broad coding/information theory and signal processing community.

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