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# Unsourced Random Access

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ABSTRACT

Current wireless networks are designed to optimize spectral efficiency for human users, who typically require sustained connections for high-data-rate applications like file transfers and video streaming. However, these networks are increasingly inadequate for the emerging era of machine-type communications (MTC). With a vast number of devices exhibiting sporadic traffic patterns consisting of short packets, the grant-based multiple access procedures utilized by existing networks lead to significant delays and inefficiencies. To address this issue the unsourced random access (URA) paradigm has been proposed. This paradigm assumes the devices to share a common encoder thus simplifying the reception process by eliminating the identification procedure. The URA paradigm not only addresses the computational challenges but it also considers the random access (RA) as a coding problem, i.e., takes into account both medium access protocols and physical layer effects. In this monograph we provide a comprehensive overview of the URA problem in noisy channels, with the main task being to explain the major ideas rather than to list all existing solutions.

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# Acronyms

| | |
|---|---|
| **3GPP** | Third-Generation Partnership Project |
| **5G NR** | Fifth-generation New Radio |
| **ACK** | Acknowledgment |
| **AMP** | Approximate message passing |
| **AoA** | Angle-of-arrival |
| **ARQ** | Automatic Repeat reQuest |
| **ASR** | Approximate support recovery |
| **AWGN** | Additive white Gaussian noise |
| **BAC** | Binary adder channel |
| **BCH** | Bose-Chaudhuri-Hocquenghem |
| **BP** | Belief propagation |
| **BPSK** | Binary phase-shift keying |
| **BS** | Base station |
| **CCS** | Coded compressed sensing |
| **CDMA** | Code division multiple-access |
| **CNOP** | Check-node operation |
| **CRC** | Cyclic redundancy check |
| **CRDSA** | Contention resolution diversity slotted ALOHA |
| **CS** | Compressed sensing |
| **CSI** | Channel state information |
| **CSMA** | Carrier-sense multiple-access |

| | |
|---|---|
| **CSMA/CA** | Carrier-sense multiple-access with collision avoidance |
| **CSS** | Chirp spread spectrum |
| **DBPSK** | Differential binary phase-shift keying |
| **DE** | Density evolution |
| **DFT** | Discrete Fourier transform |
| **DSSS** | Direct-sequence spread spectrum |
| **FAR** | False alarm rate |
| **FBL** | Finite blocklength |
| **FCFS** | First-come, first-serve |
| **FDMA** | Frequency division multiple-access |
| **FFT** | Fast Fourier transform |
| **FT** | Fourier transform |
| **G-URA** | URA over Gaussian MAC |
| **GMAC** | Gaussian MAC |
| **IDMA** | Interleave division multiple-access |
| **IoT** | Internet of Things |
| **IRSA** | Irregular repetition slotted ALOHA |
| **IRSA-B** | Basic IRSA protocol |
| **IRSA-F** | IRSA with per-frame preambles |
| **IRSA-S** | IRSA with per-slot preambles |
| **JSC** | Joint successive cancellation |
| **LDPC** | Low-density parity check |
| **LDS** | Low-density signature |
| **LLR** | Log-likelihood ratio |
| **LMMSE** | Linear MMSE |
| **LPWAN** | Low-power wide area network |
| **MAC** | Multiple-access channel |
| **MF** | Matched filter |
| **MIMO** | Multiple input, multiple output |
| **ML** | Maximum likelihood |
| **MMSE** | Minimum mean squared error |
| **MMV** | Multiple measurement vector |
| **MPA** | Message passing algorithm |
| **MSE** | Mean squared error |
| **MTC** | Machine-type communications |

| | |
|---|---|
| **MUD** | Multi-user detector |
| **NNLS** | Non-negative least squares |
| **ODMA** | On-off division multiple access |
| **OFDM** | Orthogonal frequency-division multiplexing |
| **OMP** | Orthogonal matching pursuit |
| **PAN** | Personal-area network |
| **PEXIT** | Protograph extrinsic information transfer |
| **PO** | Physical uplink shared channel occasion |
| **PUPE** | Per-user probability of error |
| **PUSCH** | Physical uplink shared channel |
| **QoS** | Quality of service |
| **QPSK** | Quadrature phase-shift keying |
| **RA** | Random access |
| **RACH** | Random access channel |
| **RCB** | Random coding bound |
| **RFID** | Radio-frequency identification |
| **RIP** | Restricted isometry property |
| **RM** | Reed-Muller |
| **RS** | Reed-Solomon |
| **RSMA** | Rate-splitting multiple-access |
| **SA** | Slotted ALOHA |
| **SC** | Successive cancellation |
| **SCL** | Successive cancellation list |
| **SCMA** | Sparse-coded multiple-access |
| **SF** | Spreading factor |
| **SIC** | Successive interference cancellation |
| **SINR** | Signal-to-interference-plus-noise ratio |
| **SoIC** | Soft SIC |
| **SPARCs** | Sparse regression codes |
| **SPC** | Single-parity-check |
| **TDMA** | Time-division multiple-access |
| **TIN** | Treat interference as noise |
| **TIN-SIC** | TIN followed by SIC |
| **UE** | User equipment |
| **URA** | Unsourced random access |
| **VNOP** | Variable-node operation |

# Notation

Throughout the monograph we use the following notations and abbreviations:

| | |
|---|---|
| $x$, $X$ | deterministic scalar value |
| $\boldsymbol{x}$ | deterministic column-vector |
| $\boldsymbol{X}$ | deterministic matrix |
| $\boldsymbol{I}_n$ | $n \times n$ identity matrix |
| diag | diagonal matrix |
| $\lVert\boldsymbol{x}\rVert_2$ | Euclidean norm of vector $\boldsymbol{x}$ |
| $\mathrm{supp}\,(\boldsymbol{x})$ | support of vector $\boldsymbol{x}$ |
| $\mathrm{wt}\,(\boldsymbol{x})$ | the number of nonzero components in vector $\boldsymbol{x}$ |
| $\mathbb{N}$ | set of natural numbers |
| $\mathbb{C}$ | set of complex numbers |
| $\mathbb{F}_q$ | finite field with $q$ elements |
| $\mathcal{X}$ | set |
| $\boldsymbol{\mathcal{X}}$ | sequence of sets |
| $\bigsqcup$ | disjoint union |
| $[n]$ | $[n] = \{1, \ldots, n\}$, where $n \in \mathbb{N}$ |
| $\boldsymbol{a}_{\mathcal{I}}$ | $\boldsymbol{a}_{\mathcal{I}} = (a_{i_1}, \ldots, a_{i_s})$, where $\boldsymbol{a} = (a_1, \ldots, a_n)$ and $\mathcal{I} = \{i_1, \ldots, i_s\} \subseteq [n]$ with $i_1 < \ldots < i_s$ |
| $\boldsymbol{A}_{\mathcal{I}}$ | $\boldsymbol{A}_{\mathcal{I}} = (\boldsymbol{a}_{i_1}, \ldots, \boldsymbol{a}_{i_s})$, where $\boldsymbol{A} = (\boldsymbol{a}_1, \ldots, \boldsymbol{a}_n)$ and $\mathcal{I} = \{i_1, \ldots, i_s\} \subseteq [n]$ with $i_1 < \ldots < i_s$ |

| | |
|---|---|
| $\mathcal{CN}(\mathbf{0}, \boldsymbol{I}_n)$ | circularly symmetric complex standard normal distribution |
| $\mathrm{Bern}(p)$ | Bernoulli distribution with parameter $p$ |
| $\mathrm{Unif}([Q])$ | uniform distribution on $[Q]$ |
| $\mathsf{x}$, $\mathsf{X}$ | random scalar value |
| $\mathbf{x}$ | random column-vector |
| $\mathbf{X}$ | random matrix |
| $E$ | event |
| $E^c$ | complementary event to $E$ |
| $\mathbb{1}_E$ | indicator of the event $E$ |
| $\Pr[E]$ | probability of the event $E$ |
| $\mathbb{E}$ | expectation operator |
| $H(\mathsf{x})$ | entropy of discrete random variable $\mathsf{x}$ |
| $I(\mathsf{x}, \mathsf{y})$ | mutual information of random variables $\mathsf{x}$ and $\mathsf{y}$ |
| $h(p)$ | $h(p) = -p\log_2(p) - (1-p)\log_2(1-p)$, $0 \leq p \leq 1$, where $h(1) = h(0) = 0$ |
| $\tau(x)$ | binary phase-shift keying (BPSK) modulation $\tau(x) = (1-2x)\sqrt{P}$ |
| w.l.o.g. | "without loss of generality" |
| r.v. | "random variable" |
| i.i.d. | "independent identically distributed" |
| p.m.f. | "probability mass function" |

# 1

# Massive Machine-type Communications

Machine-type communications (MTC) dramatically change traffic patterns. Instead of focusing on peak data rates and low latencies, massive connectivity becomes a key requirement. The MTC concept involves a massive number of autonomous devices and sensors being connected to a gateway: a node (or a set of nodes) responsible for data collection. MTC is a crucial component of the Internet of Things (IoT) paradigm, which defines the infrastructure and scenarios for interconnecting devices rather than humans. IoT encompasses various tasks, including monitoring, remote and automated control, data collection, and data-related services. MTC is a communication technology specifically designed to support this paradigm, enabling connectivity for a vast number of devices. In this monograph, we focus on the communication aspects and use the terms IoT and MTC interchangeably.

IoT applications encompass a wide range of use cases, including:

- Environmental and health monitoring.
- Smart homes, cities, and industries.
- Road traffic monitoring and tracking to improve efficiency and safety.

Typical MTC transmissions involve short measurement reports generated either regularly or sporadically, resulting in additional requirements.

1. Improved *battery life* is essential. Wiring a large number of devices to the electricity grid would require expensive cabling, making it preferable for these devices to be autonomous. Moreover, monitoring the battery status of thousands (or even millions) of devices may also be prohibitively costly. Therefore, the battery lifetime must match the device lifetime (approximately 10 years). As a result, energy-efficient solutions must utilize simple radio-frequency devices with low-complexity signal processing algorithms.

2. There is a need for *improved coverage*. Many sensors may be located in so-called deep indoor environments (e.g., building basements), leading to significant signal loss between the transmitter and receiver.

3. *Low cost* is a critical factor. The challenge of low cost is twofold:

   - since IoT devices generate only small amounts of data, subscription fees should be much lower compared to those for ordinary smartphones, minimizing operational expenses;
   - the massive deployment of IoT devices necessitates a low cost per device, reducing capital expenses.

According to Ericsson's forecast [1], the number of cellular IoT devices (see Figure 1.1) is expected to reach 6.1 billion by 2029, while the total number of connected devices across all IoT technologies is projected to reach approximately 39 billion. Currently, the IoT industry exhibits a compound annual growth rate of approximately 16%. Given this rapid growth, standardization plays a crucial role in ensuring sustainable IoT development.

## 1.1 Internet of Things Standardization

There are various standards for IoT, each fulfilling different requirements specified above. We distinguish three main branches of IoT technologies: short-range, wide-area, and cellular.

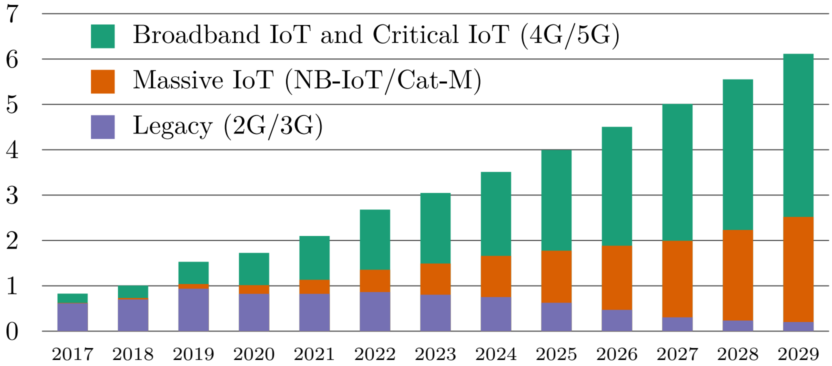


**Figure 1.1:** Predicted number of worldwide IoT cellular subscriptions (billions) in accordance with Ericsson mobility report [1] (Nov. 2023).

Short-range IoT encompasses a variety of technologies, including radio-frequency identification (RFID) and personal-area networks (PANs) such as Bluetooth and Zigbee. These technologies typically operate in unlicensed spectrum and within a very short range. Massive connectivity in this case is limited by the small coverage area (on the order of several meters) of the corresponding radio devices [2].

In contrast, wide-area technologies have the potential to connect millions of autonomous devices to a single base station (BS). In this short overview, we focus on Sigfox and LoRa technologies, which are described in Section 1.1.2.

### 1.1.1 Short-range IoT

Short-range, or PAN, provides a communication environment for various IoT applications, such as communication between wearable devices, short-range location tags, home controllers, and more. A typical network consists of at most a few dozen devices, making communication between them relatively simple in terms of channel access. The low communication range is beneficial for energy efficiency and security.

An extreme example of energy efficiency (particularly regarding remote device battery life) is RFID technology, where the energy required to transmit data is induced by an interrogation pulse from a nearby reader device. The short communication range also prevents signals from being detected over large distances between the transmitter and receiver, which significantly simplifies security protocols.

Due to the absence of massive connectivity issues and the wide variety of PAN technologies, we will not consider them in the remainder of this monograph.

### 1.1.2 Wide-area IoT

To fulfill the requirements described above, several solutions are commonly employed in wide-area IoT systems – low-power wide area networks (LPWANs). To improve system range, narrowband signals are typically utilized. Since thermal noise power is proportional to the processed bandwidth, narrowband signals can tolerate a greater link loss for the same transmitter power and hence enable coverage of a wider area. Additionally, due to the extended communication distances, narrowband signals benefit from the reduced frequency selectivity of the wireless channel. Consequently, there is no need for complex multicarrier modulations or high-complexity signal processing algorithms at the transmitter.

To simplify remote devices, wide-area IoT communication systems often have limited or even completely absent downlink functionality. This limitation reduces the ability to coordinate transmitting devices and typically eliminates the possibility of employing Automatic Repeat reQuest (ARQ) mechanisms. Additionally, the use of ARQ in scenarios involving sporadic data transmission by a massive number of devices can overwhelm the control channel.

To address this issue, transmissions can be repeated multiple times, potentially at different frequencies, to exploit both frequency and time diversity, assuming the retransmission delay exceeds the channel coherence time. Further transmitter simplifications often include the use of constant-envelope modulation formats, which do not require expensive or power-inefficient signal amplifiers.

At the receiver, additional solutions are applied to enhance performance. The simplest way to increase diversity is through *multiple receptions*. When multiple nodes collect the transmitted data, many can detect, demodulate, and decode the message, thereby enhancing *spatial diversity*.

These typical solutions are implemented in Sigfox and LoRa, the most popular wide-area IoT technologies, which are described below. Both approaches rely on distributed resource coordination mechanisms[1] based on ALOHA and carrier-sense multiple-access with collision avoidance (CSMA/CA), rather than the centralized coordination employed in cellular systems.

### Sigfox – An Ultra-narrowband IoT Technology

The main feature of Sigfox is its extremely narrow transmission bandwidth of just 100 Hz. This narrow bandwidth significantly reduces in-band thermal noise to very low levels (approximately −154 dBm), which is a key enabler of its long-range communication. Downlink functionality is extremely limited, with a complete absence of synchronization, coordination, and ARQ mechanisms.

To send a packet, a device selects a transmission frequency within a 192 kHz band and transmits the packet, followed by two replicas at different randomly selected frequencies to enhance diversity. Multiple reception is achieved by deploying multiple BSs, which continuously scan the entire 192 kHz bandwidth to detect uplink messages.

The transmitted packets are exceptionally short. Each packet consists of:

- a 4-byte preamble,
- a 2-byte frame-synchronization sequence,
- a 4-byte device identifier,
- a payload of up to 12 bytes,
- a variable-length hash code for packet authentication within the Sigfox network, and
- a 2-byte cyclic redundancy check (CRC).

[1]These mechanisms are often referred to as medium access control (MAC). In this monograph, however, we interpret the abbreviation MAC as multiple-access channel.

Uplink packets are typically modulated using differential binary phase-shift keying (DBPSK). Differential modulation is chosen to allow for non-coherent detection. The combination of simple modulation and coding, along with a very low sampling rate, results in a highly cost-effective solution. Additionally, the high link budget enables a reduction in transmit power, thereby fulfilling battery efficiency requirements with ease.

Downlink messages (if configured) are triggered by a transmitting device in the form of a callback. The BS's response has a fixed delay and is transmitted at the reception frequency of the request plus a predefined frequency shift. The payload size for a downlink message is fixed at 8 bytes.

### LoRa

The LoRa (Long Range) protocol is another IoT alternative. Similar to the previously described ultra-narrowband solutions, this communication standard assumes a BS and end devices connected to it, managed by a simple medium access protocol. LoRa supports wider communication bandwidths (125 or 500 kHz) based on the chirp spread spectrum (CSS) technique, which utilizes linear frequency modulation.

Let $B$ denote the total transmission bandwidth at a carrier frequency $f_c$, and let $T$ represent the symbol duration. The instantaneous frequency $f(t), \quad t \in [0, T]$, of the CSS signal changes linearly with a rate of $^B/_T$, wrapping around when it reaches the edges of the transmitted bandwidth:

$$f(t) = f_c - \frac{B}{2} + \left\{ \frac{B}{T} t + \frac{B}{M} i \right\} \bmod B,$$

where $i \in [M]$ is the transmitted message index, and the transmission rate is $\log_2 M$ bits per symbol. Different values of $M$ correspond to different spreading factor (SF) values [3].

Different SFs are supported by LoRa, and signals corresponding to different SFs are almost orthogonal [3]. This property reduces interference between transmissions with different SFs, thereby improving the communication range. The medium access mechanism also allows for downlink transmissions. The LoRa standard supports three different classes of devices.

Devices of the first class (A) behave similarly to Sigfox: they send data as it becomes available, and downlink messages can be sent during *receive windows*, whose timings are configurable. After an uplink transmission, the device waits for the start of a downlink message within two receive windows. If no downlink message is detected, the device enters sleep mode. For this type of device, the sender spends most of its time in sleep mode, enabling long battery life. Class-A functionality is basic and must be supported by all devices.

Devices of the second class (B) extend the downlink functionality by opening periodic receive windows (or ping slots). To manage this functionality, periodic beacons are sent by the network to maintain synchronization.

Finally, devices of the third class (C) enhance the capabilities of Class-A devices by keeping the receive windows open unless transmitting an uplink message. As a result, Class-C devices can receive downlink messages almost any time, offering very low latency for downlink transmissions.

Typical use cases for Class-A devices include sensors that periodically report measurements or send data triggered by an alarm event. Class-B devices are useful for applications requiring measurements on request, while Class-C devices are ideal for remote control mechanisms powered by a continuous power source.

### 1.1.3 Cellular IoT

Cellular systems are highly attractive for massive deployments due to their wide coverage, straightforward subscription procedures, operation over licensed spectrum with effective interference management, and robust security protocols.

However, cellular systems employ centralized coordination algorithms (as opposed to distributed CSMA/CA), which are better suited for high data rates among a fixed and relatively small number of active users within the coverage area of a single BS.

Cellular networks did not support machine-type devices prior to Third-Generation Partnership Project (3GPP) Release 12. Earlier releases assumed that any device connecting to the BS would support the full bandwidth of 20 MHz. This large bandwidth requirement was not suitable for achieving the long battery life needed by IoT devices.

### Different Device Types

The 3GPP standards define various categories of devices based on their capabilities. These categories are represented by numbers, where higher numbers indicate devices that support higher peak uplink and downlink rates, a greater number of supported antennas, and so on. However, these categories primarily pertain to devices operated by humans, while the requirements for IoT devices can differ significantly.

3GPP Release 13 introduced the Cat-M category (with “M” denoting MTC). This user equipment category was the first narrowband device type, supporting a bandwidth of 6 resource blocks (1.08 MHz). This new device type required novel approach to control channel design.

Further optimization for MTC devices was achieved in Release 13 with the introduction of the NB (narrowband) device category. Devices in this category reduced the total supported bandwidth to 200 kHz. Additionally, optimizations enabled narrowband transmissions with bandwidths reduced to a single subcarrier, referred to as single-tone transmission. The subcarrier bandwidth for these transmissions is either 15 kHz or 3.75 kHz.

The Fifth-generation New Radio (5G NR) standard introduced broadband IoT, allowing sensing devices to transmit larger amounts of data. In Release 17, the RedCap (Reduced Capabilities) network type was introduced. The RedCap standard aims to support all industrial applications by enabling broadband communication services for machine-type devices. This standard assumes the utilization of up to 20 MHz bandwidth in the frequency range below 6 GHz.

### Random access Procedures in Cellular Systems

To initiate a connection with a cellular network, each device must proceed with random access (RA) procedure. The introduction of IoT devices and their massive deployments could overload the control channel. To address this issue, a new RA procedure should be developed, aiming to reduce the overall communication overhead.

As specified in 3GPP TS 138.321, the original RA procedure follows a four-way handshake, as illustrated in Figure 1.2. This handshake consists of the following phases:

1. RA through preamble transmission to identify users.
2. Resource allocation provided by the BS.
3. Data transmission using orthogonal resources assigned to the identified users.
4. Final acknowledgment (ACK).

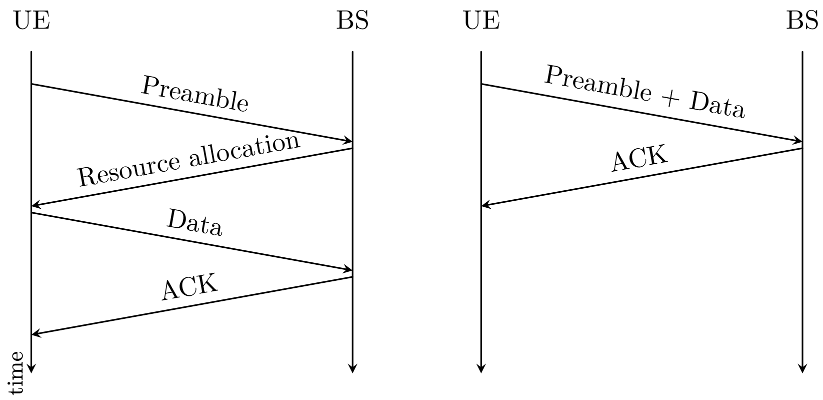


**Figure 1.2:** Four-step RA (left) and two-step RA procedures specified in 3GPP TS 138.321. A time diagram corresponds to message exchange between user equipment (UE) and base station (BS).

The described above procedure separates the RA phase from data transmission. Starting with 3GPP Release 16, this procedure was simplified, requiring only a two-way handshake. In this updated procedure, the preamble transmission also announces the resources to be used for data transmission, which follows immediately. Users select preambles from a predefined orthogonal set. Data is then transmitted during specified positions of the physical uplink shared channel occasion (PO)s. If the BS successfully receives the data, it sends an acknowledgment. Otherwise, the traditional four-way handshake is performed. This transmission scheme is depicted in Figure 1.3 (see the detailed description in [4]).

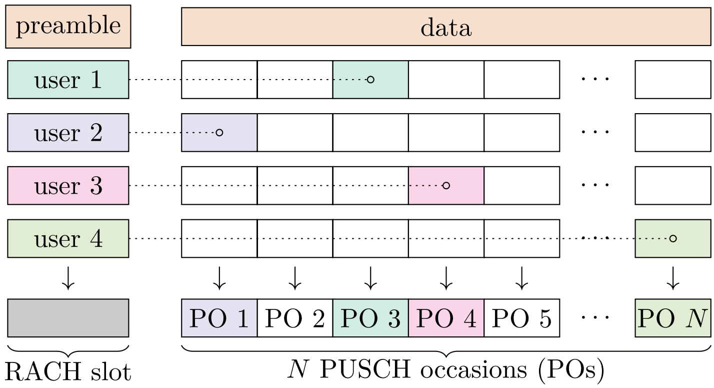


**Figure 1.3:** Two-step RA procedure with data transmission.

## 1.2 Challenges for the Next-generation Cellular Systems

The key challenge for the next generation of radio-access networks is managing the massive number of infrequently communicating sensors. Current solutions are inadequate due to their reliance on centralized resource allocation, which orthogonalizes access for different users. For MTC, this approach is unacceptable as it results in significant control-layer overhead and latency. Therefore, a new communication solution is required.

As evidence, we will first demonstrate the performance of the two-step RA procedure presented above. Following the exposition in [4], we reproduce several numerical results from the referenced manuscript. The objective of the numerical setup is to highlight the significant gap between the energy efficiency of the two-step RA procedure and the achievable energy efficiency in a massive RA scenario (see Theorem 4.1). Energy efficiency is defined as the minimum energy required to transmit a single information bit (or energy per bit) under certain quality of service (QoS) constraints. The exact definition of energy efficiency will be provided in Section 3.

The technical details of this numerical experiment are as follows. The preamble dictionary consists of 64 Zadoff-Chu sequences with varying lengths. For short preambles, the length is 139, while for long preambles, it is 839. Each preamble corresponds to a PO. Within a PO, data is transmitted using low-density parity check (LDPC) codes as specified in 5G NR standards. The additive white Gaussian noise (AWGN) channel model is considered, and reference signals are not required in this setup. The system parameters outlined in Table 1.1, and the energy efficiency as a function of the number of simultaneously active users is presented in Figure 1.4.

**Table 1.1:** Simulation parameters for numerical comparison presented in Figure 1.4

| Parameter | Value |
|---|---|
| Preamble length | $2 \times 139$ (A1 configuration) |
| Error-correcting code | $(500, 100)$ LDPC (5G NR base graph 2) |
| Modulation | Quadrature phase-shift keying (QPSK) |
| Decoding algorithm | TIN / TIN-SIC |
| Pilot configuration | Pilot-free |
| Number of POs | 64 |
| Overall frame length | 16278 channel uses |

Current schemes that are part of existing standards exhibit significantly lower energy efficiency compared to theoretical bounds. Moreover, the two-step RA procedure proposed by 3GPP remains optional. The lack of energy-efficient schemes has motivated many researchers to extensively study this new massive MTC scenario.

In this monograph, we outline the core ideas behind these theoretical bounds and provide a brief overview of the challenges in designing low-complexity schemes. We demonstrate that some of these schemes closely approach the achievability bounds.

In our introductory example in Figure 1.4, we considered the simple case of a Gaussian channel. However, real wireless channels are affected by multipath propagation, which introduces additional random effects during signal transmission. In the subsequent sections, we address various challenges posed by real propagation environments.

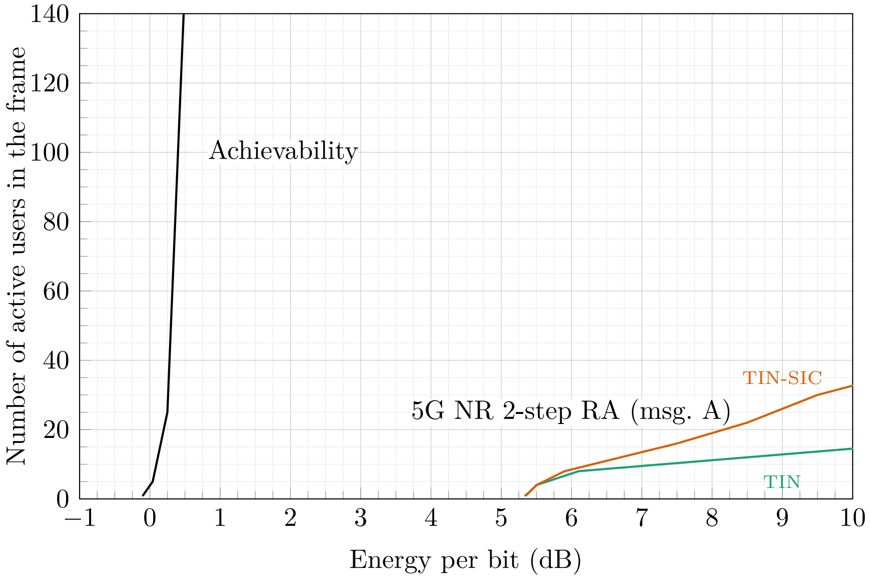


**Figure 1.4:** Energy efficiency of the proposed 2-step RA procedure in 5G NR versus achievability bound on energy efficiency [5]. AWGN channel model, reference signals are not considered [4].

## 1.3 Monograph Organization

The monograph is organized as follows:

- Section 2 explores MAC problems and their formulations, emphasizing the differences between classical MAC scenarios and the unsourced random access (URA) setup.
- Section 3 defines the URA problem and introduces per-user probability of error (PUPE), the primary measure of the system's operational quality. It also revisits the definition of energy efficiency (see (3.3)) previously discussed in Figure 1.4. Additionally, this section establishes a connection between the URA problem and the well-known compressed sensing (CS) problem.
- Section 4 investigates the fundamental limits of energy efficiency under PUPE constraints in the Gaussian channel.

- Section 5 focuses on low-complexity schemes for the Gaussian channel.
- Section 6 examines more realistic channels with fading effects, specifically the quasi-static Rayleigh fading channel. It covers scenarios where the BS is equipped with either a single antenna or multiple antennas.
- Section 7 concludes the monograph by highlighting the remaining challenges and open problems.

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