

# Algorithmic Aspects of Parallel Data Processing

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## Abstract

In the last decade or so we have witnessed a growing interest in processing large data sets on large distributed clusters. The idea was pioneered by the MapReduce framework, and has been widely adopted by several other systems, including PigLatin, Hive, Scope, U-SQL, Dremmel, Spark and Myria. A large part of the complex data analysis performed by these systems consists of a sequence of relatively simple query operations, such as joining two or more tables. This survey discusses recent algorithmic developments for distributed data processing. It uses a theoretical model of parallel processing called the Massively Parallel Computation (MPC) model, which is a simplification of the BSP model where the only cost is given by the amount of communication and the number of communication rounds. The survey studies several algorithms for multi-join queries, for sorting, and for matrix multiplication, and discusses their relationships and common techniques applied across the different data processing tasks.



# 1

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## Introduction

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In the last decade we have witnessed a huge and growing interest in processing large data sets on large distributed clusters. This trend began with the MapReduce framework [31], and has been widely adopted by several other systems, including PigLatin [69], Hive [83], Scope [24], Dremmel [65], Spark [91] and Myria [88] to name a few. While the applications of such systems are diverse (e.g., machine learning, data analytics), most involve relatively standard data processing tasks, such as identifying relevant data, cleaning, filtering, joining, grouping, transforming, extracting features, and evaluating results [25, 35].

This has generated great interest in the study of algorithms for data processing on large distributed clusters. This survey reviews some of the recent theoretical results on efficient data processing on large distributed architectures, as well as some of the relevant classical results on parallel sorting and parallel matrix multiplication.

The survey begins in Chapter 2 with a review of parallel models used to analyze algorithms on large distributed clusters. Modern data analytics run on large, shared-nothing clusters, where the cost of communication during data reshuffling can dominate the running time. For example, individual jobs in Cosmos, Microsoft's distributed file sys-

tem, often execute on over 10k nodes [72]. We introduce a very simple model of parallel computation, called the Massively Parallel Computation model (MPC) where the cost of a distributed algorithm is measured in the amount of communication per processor and the number of communication rounds. This model is a simplification of Valiant's *Bulk Synchronous Parallel (BSP)* model [84], and allows us to separate the computation cost from the communication cost, and to focus solely on the latter. In this chapter we introduce the MPC model, then review several important classical models of parallel computation, and discuss their connection to the MPC model.

In Chapter 3 we present and analyze two different approaches for computing in parallel the join of two large relations. Join operations are the bread and butter of most database processing tasks, and the support of efficient join algorithms is a top priority for all major big data systems. We discuss Parallel Hash join, and Parallel Sort Join. The preferred algorithm in practice is the Parallel Hash join, because on most datasets this algorithm is very effective and scales up linearly with the number of processors. However, the Parallel Hash join performs poorly on skewed data, when a large number of records have the same value of the join attribute and, thus, are hashed to the same processor. We discuss in detail how to handle skewed data. In contrast, Parallel Sort join is simpler and less sensitive to skew, but requires extra communication rounds to do the actual sorting.

Next, we consider multi-join queries, and discuss a variety of hash-based algorithms in Chapter 4. In the standard architecture of a database system, a multi-join query is first converted into a query plan, which is then optimized, and finally the plan is executed. The plan consists of simple operators like join, selection, duplicate elimination, and each operator creates an intermediate result that, in distributed query processing, needs to be materialized and re-shuffled for the next operator. Afrati and Ullman [4] pioneered an alternative approach for computing a multi-join query on a distributed system, which computes the query using a single reshuffle operation. Their algorithm, initially described for the MapReduce system, organizes the processors (which correspond to *reducers* in a MapReduce job) in a multidimensional

cube, then partitions each input relation in a sub-cube. The theoretical aspects of the algorithm have been studied in [17], where the algorithm was called *HyperCube*, while extensions to skewed data and to multiple rounds of communication were further discussed in [18, 57]; these will be reviewed in this chapter. While these algorithms are appealing because of their strong theoretical guarantees, modern database systems compute multi-join queries in traditional ways, by converting the query into a join plan. We continue the chapter by discussing the theoretical aspects of join plans, which have a long history in database theory. We review Yannakakis' algorithm for computing acyclic queries [90], the concept of hypertree decomposition [42], and various notions of tree-width [43, 55], and describe how these have been put together in the GYM algorithm [3].

In Chapter 5 we discuss a few traditional aspects of parallel sorting algorithms. Similar to hashing, sorting is a core technique in database query processing, both in the sequential and in the parallel setting. Sort-based techniques suffer less than hash-based techniques from skew in the data. For example, recently Hu, Tao, and Yi [45] have shown how to use sorting to design a simple join algorithm that is provably optimal for any input data (reviewed in Chapter 3). In this chapter we review some fundamental lower bounds for sorting on a distributed system, and also review three classic parallel sorting algorithms: Batcher's odd-even sort [16], Cole's algorithm [27], and Goodrich's algorithm [40].

Finally, in Chapter 6 we discuss classic parallel algorithms for matrix multiplication. We focus on multiplication of dense square matrices and adopt the relational view of matrix multiplication as a join of two tables followed by a group-by-and aggregate computation. Using techniques similar to those used in proving lower bounds in sorting and multi-join queries, we review the communication and round lower bounds for matrix multiplication of square and dense matrices. Then, we review existing algorithms that match these lower bounds. The chapter ends with a very brief overview of other known results in linear algebra, such as multiplication of non-square and sparse matrices, or *LU* and Cholesky matrix factorization.

Table 1.1 summarizes the notations used in the survey.

**Table 1.1:** Notations Used Throughout the Survey.

Relation	$R_j$
Number of relations	$\ell$
Variable	$x_i$
Number of variables	$k$
Query	$q$
Input size	IN or $N$
Output size	OUT
Number of processors	$p$
Number of communication rounds	$r$
Load (incoming communication per processor)	$L$
Memory per processor	$M$
Total communication	$C$
Fractional edge cover or edge packing	$u_j$
Fractional vertex cover or vertex packing	$v_i$
Fractional edge packing number	$\tau^*$
Fractional edge covering number	$\rho^*$
Quasi-packing number	$\psi^*$

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