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Estimation of Spatial Panels*

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Abstract

Spatial panel models have panel data structures to capture spatial interactions across spatial units and over time. There are static as well as dynamic models. This text provides some recent developments on the specification and estimation of such models. The first part will consider estimation for static models. The second part is devoted to the estimation for spatial dynamic panels, where both stable and unstable dynamic models with fixed effects will be considered.

For the estimation of a spatial panel model with individual fixed effects, in order to avoid the incidental parameter problem due to the presence of many individual fixed effects, a conditional likelihood or partial likelihood approach is desirable. For the model with both fixed individual and time effects with a large and long panel, a conditional likelihood might not exist, but a partial likelihood can be constructed. The partial likelihood approach can be generalized to spatial panel

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models with fixed effects and a space–time filter. If individual effects are independent of exogenous regressors, one may consider the random effects specification and its estimation. The likelihood function of a random effects model can be decomposed into the product of a partial likelihood function and that of a between equation. The underlying equation for the partial likelihood function can be regarded as a within equation. As a result, the random effects estimate is a pooling of the within and between estimates. A Hausman type specification test can be used for testing the random components specification vs. the fixed effects one. The between equation highlights distinctive specifications on random components in the literature.

For spatial dynamic panels, we focus on the estimation for models with fixed effects, when both the number of spatial units n and the number of time periods T are large. We consider both quasi-maximum likelihood (QML) and generalized method of moments (GMM) estimations. Asymptotic behavior of the estimators depends on the ratio of T relative to n. For the stable case, when n is asymptotically proportional to T, the QML estimator is $\sqrt{nT}$-consistent and asymptotically normal, but its limiting distribution is not properly centered. When n is large relative to T, the QML estimator is T-consistent and has a degenerate limiting distribution. Bias correction for the estimator is possible. When T grows faster than $n^{1/3}$, the bias corrected estimator yields a centered confidence interval. The n and T ratio requirement can be relaxed if individual effects are first eliminated by differencing and the resulting equation is then estimated by the GMM, where exogenous and predetermined variables can be used as instruments. We consider the use of linear and quadratic moment conditions, where the latter is specific for spatial dependence. A finite number of moment conditions with some optimum properties can be constructed. An alternative approach is to use separate moment conditions for each period, which gives rise to many moments estimation.

The remaining text considers estimation of spatial dynamic models with the presence of unit roots. The QML estimate of the dynamic coefficient is $\sqrt{nT^3}$-consistent and estimates of all other parameters are $\sqrt{nT}$-consistent, and all of them are asymptotically normal. There are cases that unit roots are generated by combined temporal and spatial correlations, and outcomes of spatial units are cointegrated. The asymptotics of the QML estimator under this spatial cointegration case can be analyzed by reparameterization. In the last part, we propose a data transformation resulting in a unified estimation approach, which can be applied to models regardless of whether the model is stable or not. A bias correction procedure is also available.

The estimation methods are illustrated with two relevant empirical studies, one on regional growth and the other on market integration.

Contents

1

Introduction

The last decade has seen a growing literature on panel data models with cross sectional dependence. The current text presents some recent developments in the specification and estimation of panel data models with spatial interactions. Spatial econometrics consists of econometric techniques dealing with interactions of economic units in space, which can be of physical or economic characteristics. The spatial autoregressive (SAR) model by Cliff and Ord (1973) has received the most attention in economics. Early development in estimation and testing for cross sectional data in econometrics can be found in Anselin (1988, 1992), Cressie (1993), Kelejian and Robinson (1993), Anselin and Florax (1995), Anselin and Rey (1997) and Anselin and Bera (1998), among others. Under the panel data setting, spatial panel data models are of great interest, because they enable researchers to take into account dynamics and control for unobservable heterogeneity.

For static models, the following spatial panel model with both spatial lag and spatial disturbances is a typical one:

$$\begin{aligned} Y_{nt} &= \lambda_0 W_n Y_{nt} + X_{nt}\beta_0 + \mathbf{c}_{n0} + U_{nt}, \\ U_{nt} &= \rho_0 M_n U_{nt} + V_{nt}, \quad t = 1, 2, \ldots, T, \end{aligned} \tag{1.1}$$

where $Y_{nt} = (y_{1t}, y_{2t}, \ldots, y_{nt})'$ and $V_{nt} = (v_{1t}, v_{2t}, \ldots, v_{nt})'$ are $n \times 1$ vectors, and v_{it} is *i.i.d.* across i and t with zero mean and variance σ_0^2. W_n is an $n \times n$ nonstochastic spatial weights matrix that generates the spatial dependence on y_{it} among cross sectional units, which may or may not be row-normalized. X_{nt} is an $n \times k$ matrix of nonstochastic time varying regressors, $\mathbf{c}_{n0}$ is an $n \times 1$ vector of individual effects, M_n is an $n \times n$ spatial weights matrix for the disturbance process. In practice, M_n may or may not be W_n.

For static panel data models with spatial interactions, we can have random effects or fixed effects specifications. For the random effects specification, Anselin (1988) provides a panel regression model with error components and SAR disturbances, and Baltagi et al. (2003) consider specification tests for spatial correlation in that spatial panel regression model. The Anselin and Baltagi et al. model is $Y_{nt} = X_{nt}\beta_0 + \mathbf{c}_{n0} + U_{nt}$, $U_{nt} = \lambda_0 W_n U_{nt} + V_{nt}$, where $\mathbf{c}_{n0}$ is an $n \times 1$ vector of individual error components, and the spatial correlation is in U_{nt}. Kapoor et al. (2007) propose a different specification with error components and a SAR process in the overall disturbance, and suggest a method of moments (MOM) estimation. The specification in Kapoor et al. (2007) is $Y_{nt} = X_{nt}\beta_0 + U_{nt}^+$ and $U_{nt}^+ = \lambda_0 W_n U_{nt}^+ + \mathbf{d}_{n0} + V_{nt}$, where $\mathbf{d}_{n0}$ is a vector of individual error components. Fingleton (2008) adopts a similar approach to estimate a spatial panel model with SAR dependent variables, random components and a spatial moving average (SMA) structure in the overall disturbance. By the transformation $(I_n - \lambda_0 W_n)$, the data generating process (DGP) of Kapoor et al. (2007) becomes $Y_{nt} = X_{nt}\beta_0 + \mathbf{c}_{n0} + U_{nt}$ where $\mathbf{c}_{n0} = (I_n - \lambda_0 W_n)^{-1}\mathbf{d}_{n0}$ and $U_{nt} = U_{nt}^+ - (I_n - \lambda_0 W_n)^{-1}\mathbf{d}_{n0}$. The $U_{nt} = \lambda_0 W_n U_{nt} + V_{nt}$ forms a SAR process. This model implies spatial correlations in both the individual and disturbance components, $\mathbf{c}_{n0}$ and U_{nt}, having the same spatial effect parameter. Nesting the Anselin (1988) and Kapoor et al. (2007) models, Baltagi et al. (2007) suggest an extended model without restrictions on implied SAR structures in the error component and the remaining disturbance.

As an alternative to the random effects specification, Lee and Yu (2010b) investigate the quasi-maximum likelihood (QML) estimation of spatial panel models under the fixed effects specification. The fixed effects model has the advantage of robustness in that fixed effects are allowed to depend on included regressors in the model. It also provides a unified model framework because different random effects models in Anselin (1988), Kapoor et al. (2007) and Baltagi et al. (2007) reduce to the same fixed effects model.

We have two approaches to estimate the spatial panel data models with individual fixed effects. The first is called the "direct approach", where common parameters and the individual effects are jointly estimated. The second is called the "transformation approach", where the individual effects are eliminated first before estimation. For the direct ML approach, it will yield consistent estimates for the spatial and regression coefficients, except for the variance parameter when T is finite. Thus, the results are similar to Neyman and Scott (1948).[1] The transformation approach is the method of conditional likelihood, which is applicable when sufficient statistics can be found for the fixed effects. For the linear regression and logit panel models, the time average of the dependent variables for each cross sectional unit provides a sufficient statistic (see Hsiao, 1986). For the normal panel regression model, the conditional likelihood can be constructed from some transformed data. We investigate the use of similar transformations to the spatial panel model. By using the deviation from the time mean transformation, individual effects can be eliminated. The transformed equation can then be estimated by the QML approach. This transformation approach can be justified as a conditional likelihood approach. For the model with both individual and time fixed effects, one may combine the transformations by deviations from time means and also deviations from cross section means to eliminate those effects. The transformed equation can be regarded as well-defined equation system when the spatial weights matrix is row-normalized. The resulting likelihood function can be interpreted as a partial likelihood (Cox, 1975; Wong, 1986).

The spatial panel data models have a wide range of applications such as agricultural economics (Druska and Horrace, 2004), transportation research (Frazier and Kockelman, 2005), public economics (Egger et al.,

[1] As is illustrated in Neyman and Scott (1948), for the linear panel regression model with fixed effects, the ML estimates of the regression coefficients are consistent, while the MLE of the variance parameter is inconsistent when T is finite.

2005), and good demand (Baltagi and Li, 2006), to name a few. The above panel models are static ones which do not incorporate time-lagged dependent variables in the regression equation.

Spatial panel data can include both spatial and dynamic effects to investigate the state dependence and spatial correlations. To include time dynamic features in spatial panel data, an immediate approach is to use the time lag term as an explanatory variable. In a conventional dynamic panel data model with individual fixed effects, the MLE of the autoregressive coefficient is biased and inconsistent when n tends to infinity but T is fixed (Nickell, 1981; Hsiao, 1986). By taking time differences to eliminate the fixed effects in the dynamic equation and by the construction of instrumental variables (IVs), Anderson and Hsiao (1981) show that IV methods can provide consistent estimates. When T is finite, additional IVs can improve the efficiency of the estimation. However, if the number of IVs is too large, the problem of many IVs arises as the asymptotic bias would increase with the number of IVs.

For spatial dynamic models, Korniotis (2010) investigates a spatial time lag model with fixed effects, and considers a bias adjusted within estimator, which generalizes Hahn and Kuersteiner (2002). Elhorst (2005) estimated a dynamic model with spatial disturbances by unconditional maximum likelihood method, and Mutl (2006) investigates the model using a three-step GMM. Su and Yang (2007) derive the QMLEs of the above model under both fixed and random effects specifications. Yang et al. (2006) propose a generalized dynamic error component model that accounts for the effects of functional form and spatial dependence. For a general model with both time and space dynamics, we term it the spatial dynamic panel data (SDPD) model to better link the terminology to the dynamic panel data literature (see, e.g., Hsiao, 1986; Alvarez and Arellano, 2003). Yu et al. (2008) study the stable SDPD models where the individual time lag, spatial time lag and contemporaneous spatial lag are all included. For the estimation of SDPD models, we can use the QMLE when the number of periods T is large. When T is relatively small, we can rely on GMM where lagged values can be used as IVs. Elhorst (2010) uses Monte Carlo to investigate small sample performances of various ML and GMM estimators when T is finite.

When both n and T are large, the incidental parameter problem in the MLE becomes less severe as each individual fixed effect can be consistently estimated. However, the presence of asymptotic bias may still cause the distribution of estimates not centered properly. Similar issue on asymptotic bias occurs for estimates of SDPD models. As the presence of asymptotic bias is an undesirable feature of these estimates, a bias correction procedure is needed. Kiviet (1995), Hahn and Kuersteiner (2002), and Bun and Carree (2005) have constructed bias corrected estimators for the conventional dynamic panel data model by analytically modifying the within estimator. For the QMLE of the SDPD model, analytic bias correction is also possible (Yu et al., 2008).

A general SDPD model can be specified as, for $t = 1, 2, \ldots, T$,

$$\begin{aligned} Y_{nt} &= \lambda_0 W_n Y_{nt} + \gamma_0 Y_{n,t-1} + \rho_0 W_n Y_{n,t-1} + X_{nt}\beta_0 \\ &\quad + \mathbf{c}_{n0} + \alpha_{t0} l_n + V_{nt}, \end{aligned} \tag{1.2}$$

where $\mathbf{c}_{n0}$ is an $n \times 1$ column vector of fixed effects and α_{t0}'s are time effects. Comparing it to the static model, we have included the dynamic terms $Y_{n,t-1}$ and $W_n Y_{n,t-1}$ in (1.2). Here, γ_0 captures the pure dynamic effect and ρ_0 captures the spatial-time effect.

These SDPD models can be applied to various fields such as growth convergence of countries and regions (Ertur and Koch, 2007), regional markets (Keller and Shiue, 2007), labor economics (Foote, 2007), public economics (Revelli, 2001; Tao, 2005; Franzese, 2007).

To investigate the dynamics of the SDPD model, a reduced form of (1.2) is of interest. Define $S_n(\lambda) = I_n - \lambda W_n$ and $S_n \equiv S_n(\lambda_0)$. Then, presuming that S_n is invertible[2] and denoting $A_n = S_n^{-1}(\gamma_0 I_n + \rho_0 W_n)$, (1.2) can be rewritten as

$$Y_{nt} = A_n Y_{n,t-1} + S_n^{-1} X_{nt}\beta_0 + S_n^{-1}\mathbf{c}_{n0} + \alpha_{t0} S_n^{-1} l_n + S_n^{-1} V_{nt}. \tag{1.3}$$

To study the dynamics of this model, one may investigate eigenvalues of A_n under the assumption that W_n is diagonalizable. Let $\varpi_n = \mathrm{diag}\{\varpi_{n1}, \varpi_{n2}, \ldots, \varpi_{nn}\}$ be the $n \times n$ diagonal eigenvalues matrix of W_n such that $W_n = \Gamma_n \varpi_n \Gamma_n^{-1}$ where Γ_n is the corresponding eigenvector matrix. As $A_n = S_n^{-1}(\gamma_0 I_n + \rho_0 W_n)$, the eigenvalues matrix of A_n

[2] As an example in practice with a row-normalized W_n, $|\lambda_0| < 1$ will guarantee that S_n is invertible.

is $D_n = (I_n - \lambda_0 \varpi_n)^{-1}(\gamma_0 I_n + \rho_0 \varpi_n)$ such that $A_n = \Gamma_n D_n \Gamma_n^{-1}$. When W_n is row-normalized, all the eigenvalues are less than or equal to 1 in absolute value, where it definitely has some eigenvalues being 1 (Ord, 1975). Let m_n be the number of unit eigenvalues of W_n and let the first m_n eigenvalues of W_n be the unity. Then, D_n can be decomposed into two parts, one corresponding to the unit eigenvalues of W_n, and the other corresponding to the remaining eigenvalues of W_n smaller than 1. Define $\mathbb{J}_n = \mathrm{diag}\{l'_{m_n}, 0, \ldots, 0\}$ with l_{m_n} being an $m_n \times 1$ vector of ones and $\tilde{D}_n = \mathrm{diag}\{0, \ldots, 0, d_{n,m_n+1}, \ldots, d_{nn}\}$, where $|d_{ni}| < 1$, for $i = m_n + 1, \ldots, n$, are assumed.[3] As $\mathbb{J}_n \cdot \tilde{D}_n = \mathbf{0}$, we have $A_n^h = (\frac{\gamma_0+\rho_0}{1-\lambda_0})^h \Gamma_n \mathbb{J}_n \Gamma_n^{-1} + B_n^h$ where $B_n^h = \Gamma_n \tilde{D}_n^h \Gamma_n^{-1}$ for any $h = 1, 2, \ldots$.

Denote $W_n^u = \Gamma_n \mathbb{J}_n \Gamma_n^{-1}$. Then, for $t \geq 0$, Y_{nt} can be decomposed into a sum of a possible stable part, a possible unstable or explosive part, and a time effect part:

$$Y_{nt} = Y_{nt}^s + Y_{nt}^u + Y_{nt}^\alpha, \tag{1.4}$$

where

$$\begin{aligned}
Y_{nt}^s &= \sum_{h=0}^{\infty} B_n^h S_n^{-1}(\mathbf{c}_{n0} + X_{n,t-h}\beta_0 + V_{n,t-h}), \\
Y_{nt}^u &= W_n^u \left\{ \left(\frac{\gamma_0 + \rho_0}{1 - \lambda_0} \right)^{t+1} Y_{n,-1} \right. \\
&\quad \left. + \frac{1}{(1-\lambda_0)} \left[\sum_{h=0}^{t} \left(\frac{\gamma_0 + \rho_0}{1 - \lambda_0} \right)^h (\mathbf{c}_{n0} + X_{n,t-h}\beta_0 + V_{n,t-h}) \right] \right\}, \\
Y_{nt}^\alpha &= \frac{1}{(1-\lambda_0)} l_n \sum_{h=0}^{t} \alpha_{t-h,0} \left(\frac{\gamma_0 + \rho_0}{1 - \lambda_0} \right)^h.
\end{aligned}$$

The Y_{nt}^u can be an unstable component when $\frac{\gamma_0+\rho_0}{1-\lambda_0} \geq 1$. With $\lambda_0 < 1$, $\gamma_0 + \rho_0 + \lambda_0 > 1$ is equivalent to $\frac{\gamma_0+\rho_0}{1-\lambda_0} > 1$ and, in that case, Y_{nt}^u can be explosive. The component Y_{nt}^α captures the time effect due to the time dummies. The Y_{nt}^α can be rather complicated as it depends on what

[3] We note that $d_{ni} = (\gamma_0 + \rho_0 \varpi_{ni})/(1 - \lambda_0 \varpi_{ni})$. Hence, if $|\gamma_0| + |\lambda_0| + |\rho_0| < 1$, we have $d_{ni} < 1$ as $|\varpi_{ni}| \leq 1$.

the time dummies would represent. The Y_{nt} can be explosive when α_{t0} represents some explosive functions of t, even when $\frac{\gamma_0+\rho_0}{1-\lambda_0}$ were smaller than 1. Without a specific time structure for α_{t0}, it is desirable to eliminate this component for the estimation. If the absolute values of the elements in $\tilde{D}_n$ are less than 1, Y_{nt}^s will be a stable component. The Y_{nt}^s can be a stable component unless $\gamma_0 + \rho_0 + \lambda_0$ is much larger than 1. If the sum $\gamma_0 + \rho_0 + \lambda_0$ were too big, some of the eigenvalues d_{ni} in Y_{nt}^s might become larger than 1.

The spatial cointegration case is the situation where $\gamma_0 + \rho_0 + \lambda_0 = 1$ but $\gamma_0 \neq 1$. The unit eigenvalues of A_n correspond exactly to those unit eigenvalues of W_n via the relation $D_n = (I_n - \lambda_0 \varpi_n)^{-1}(\gamma_0 I_n + \rho_0 \varpi_n)$. W_n has some unit eigenvalues, but not all of them are equal to 1 because $\mathrm{tr}(W_n) = 0$, and hence the sum of eigenvalues of W_n is zero. Hence, some eigenvalues of A_n, but not all, are equal to 1. If $\mathbf{c}_{n0}$ and/or the time mean of $X_{nt}\beta_0$ are nonzero, the $\sum_{h=0}^{t}(\mathbf{c}_{n0} + X_{n,t-h}\beta_0)$ will generate a time trend. The $\sum_{h=0}^{t} V_{n,t-h}$ will generate a stochastic trend. These imply the unstability of Y_{nt}^u.

The unit roots case has all eigenvalues of A_n being 1. It occurs when $\gamma_0 + \rho_0 + \lambda_0 = 1$ and $\gamma_0 = 1$, because $A_n = (I_n - \lambda_0 W_n)^{-1}(\gamma_0 I_n + \rho_0 W_n) = (I_n - \lambda_0 W_n)^{-1}(I_n - \lambda_0 W_n) = I_n$. For this unit roots case, the unit eigenvalues of A_n are not linked to the eigenvalues of W_n. Because W_n^u is defined completely from W_n, the decomposition in (1.4) is not revealing for the unit roots case; instead, one has

$$Y_{nt} = Y_{n,t-1} + S_n^{-1}(X_{nt}\beta_0 + \mathbf{c}_{n0} + \alpha_{t0} l_n + U_{nt}). \tag{1.5}$$

Some implications of spatial and dynamic effects in terms of the coefficients λ_0, γ_0 and ρ_0 can be revealed via marginal impacts of regressors. Suppose that we are interested in (an average) total (expected) impact resulting from changing a regressor by the same amount across all spatial units in some time periods, say, from the time period t_1 to t, where $t_1 \leq t$. For simplicity, we assume that x_{nt} is a single regressor, and consider the situation that W_n is row-normalized and does not depend on x. Thus, we have from the reduced form equation that $\frac{\partial E(Y_{nt})}{\partial x} = \beta_0 \sum_{h=0}^{t-t_1} A_n^h S_n^{-1} l_n$, where l_n is an n-dimensional vector of ones. As W_n is row-normalized such that $W_n l_n = l_n$, $\frac{\partial E(Y_{nt})}{\partial x} = \sum_{h=0}^{t-t_1} A_n^h S_n^{-1} l_n \beta_0 = l_n \sum_{h=0}^{t-t_1} (\frac{\gamma_0+\rho_0}{1-\lambda_0})^h \cdot \frac{\beta_0}{1-\lambda_0}$, where every unit will receive the same impact.

There are several cases of interest:

(1) $t_1 = t$, i.e., the marginal change of x occurs for all spatial units at the current period t. In that case, $\frac{\partial E(Y_{nt})}{\partial x} = l_n \frac{\beta_0}{1-\lambda_0}$, which is the marginal impact due to spatial interactions. The β_0 is the marginal effect of x and $\frac{1}{1-\lambda_0}$ represents the spatial multiplier effect.

(2) $t_1 < t$, i.e., the marginal change of x occurs for all spatial units from the past period t_1 to the current period t. In this case, $\frac{\partial E(Y_{nt})}{\partial x} = l_n[1 + (\frac{\gamma_0+\rho_0}{1-\lambda_0}) + \cdots + (\frac{\gamma_0+\rho_0}{1-\lambda_0})^{t-t_1}]\frac{\beta_0}{1-\lambda_0}$. If $t_1 = t-1$, the marginal impact for each spatial unit becomes $[1 + (\frac{\gamma_0+\rho_0}{1-\lambda_0})]\frac{\beta_0}{1-\lambda_0}$. This marginal impact is composed of the marginal impact $\frac{\beta_0}{1-\lambda_0}$ of changing x in the current period t to $E(Y_{nt})$ and also an impact due to changing x in the last period $t-1$. The change of x at $t-1$ has the marginal impact with spatial multiplier $\frac{\beta_0}{1-\lambda_0}$ on $Y_{n,t-1}$. This marginal change of $Y_{n,t-1}$ generates its marginal impact $\frac{\gamma_0+\rho_0}{1-\lambda_0}$ on $E(Y_{nt})$ through both the time filter $(\gamma_0 I_n + \rho_0 W_n)$ and the space filter S_n^{-1}. Thus, the marginal impact on changing x in the last period is the product $(\frac{\gamma_0+\rho_0}{1-\lambda_0})\frac{\beta_0}{1-\lambda_0}$. The marginal impact on changing x from a past period can be deducted recursively, and the total impact accumulates effects of those changes. For both the unit roots case ($\gamma_0 = 1$ and $\lambda_0 + \rho_0 = 0$) and the spatial cointegration case ($\lambda_0 + \gamma_0 + \rho_0 = 1$ with $\gamma_0 \neq 1$), they imply $\frac{\gamma_0+\rho_0}{1-\lambda_0} = 1$; hence, their total marginal impact is simply the product of the marginal impact $\frac{\beta_0}{1-\lambda_0}$ at each time period multiplied by the total number of time periods of changing x, i.e., $\frac{\partial E(Y_{nt})}{\partial x} = l_n \frac{\beta_0}{1-\lambda_0}(t - t_1 + 1)$.

(3) $t_1 = -\infty$, i.e., the marginal change of x occurs from infinite past to the current period. For the stable SDPD process, one has a convergent series. The total marginal impact would be $\frac{\partial E(Y_{nt})}{\partial x} = l_n \frac{\beta_0}{1-(\lambda_0+\gamma_0+\rho_0)}$. The spatial and dynamic effects are combined into the multiplier effect $\frac{1}{1-(\lambda_0+\gamma_0+\rho_0)}$.

For estimation of those models with QML approaches, the QMLEs may have different rates of convergence. For the stable case, the rates of convergence of QMLEs are $\sqrt{nT}$, as shown in Yu et al. (2008) and reported in a subsequent section. For the spatial cointegration case, Yu et al. (2007) show that the QMLEs for such a model are $\sqrt{nT}$ consistent and asymptotically normal, but, the presence of the unstable components will make the estimators' asymptotic variance matrix singular. Consequently, a linear combination of the spatial and dynamic effects estimates can converge at a higher rate. For the unit roots case, the QMLEs of γ_0 is $\sqrt{nT^3}$ consistent and other estimates are $\sqrt{nT}$ consistent; however, the estimate of sum of $\rho_0 + \lambda_0$ is $\sqrt{nT^3}$ consistent. For the explosive case, we will rely on a data transformation in order to estimate the model.

The rest of the text is organized as follows. *Static Spatial Panels — Fixed Effects Models* uses either the conditional likelihood or the partial likelihood approaches to estimate the spatial panel model with individual fixed effects. *Static Spatial Panels — Random Effects Models* investigates the spatial panel model with a general space–time filter under random effects specification. It is shown that the estimates under the random effects specification is a pooling of the within and between estimates, and a Hausman type specification test can be used for testing the random components specification vs. the fixed effects one. *Spatial Dynamic Panels — Stable Models with Fixed Effects* study both QML and GMM estimation of stable SDPD models with fixed effects. The QML approach is applicable when T is large, and a bias correction procedure can eliminate the dominant bias of the QMLE. The n and T ratio requirement can be relaxed if individual effects are first eliminated by differencing and the resulting equation is then estimated by the GMM, where exogenous and predetermined variables can be used as instruments. *Spatial Dynamic Panels — Unstable Models with Fixed Effects* cover QML estimation of SDPD models in the presence of unit roots. A data transformation is proposed to estimate various SDPD models, and can provide regular $\sqrt{nT}$-consistent and asymptotic normal estimates as long as the stable component is present. Finally, two empirical applications are presented to illustrate the proposed estimation methods. Some technical theorems and notations are provided in Appendices.

Even though we have different model specifications, there are some basic common features for all of them. The following common assumptions will be used throughout the text for both static and dynamic models.[4] In addition to these, specific assumptions for different models will be listed when needed.

Assumption 1. All the spatial weights matrices (i.e., W_n) are nonstochastic with zero diagonals and are uniformly bounded in both row and column sums in absolute value (for short, UB).[5]

Assumption 2. The relevant disturbances (i.e., v_{it} or e_{it}) are *i.i.d.* across i and t with zero mean and finite variance, and their higher than fourth moments exist.

Assumption 3. The true spatial effect coefficients (i.e., λ_0) are in the interior of their parameter spaces. The spatial transformation matrices (i.e., $I_n - \lambda W_n$) are invertible on the compact parameter spaces of spatial effects, and their inverses are UB uniformly in the parameter spaces.

Assumption 4. The elements of X_{nt} are nonstochastic and bounded, uniformly in n and t. When we have $n \times k_z$ time invariant regressor z_n, it is also nonstochastic and bounded uniformly in n.

For some cases, we focus on row-normalized spatial weights matrices that are popular in empirical applications. Under these situations, we make that explicit and extend Assumption 1 to

Assumption 1′. All the spatial weights matrices are row-normalized and satisfy Assumption 1.

Assumption 5. n goes to infinity, where T can be finite or an increasing function of n.

Assumption 5′. T goes to infinity, where n is an increasing function of T.

[4] Matlab codes for those estimation methods are available upon request for readers who are interested.

[5] We say a (sequence of $n \times n$) matrix P_n is uniformly bounded in row and column sums in absolute value if $\sup_{n \geq 1} \|P_n\|_\infty < \infty$ and $\sup_{n \geq 1} \|P_n\|_1 < \infty$, where $\|P_n\|_\infty = \sup_{1 \leq i \leq n} \sum_{j=1}^n |p_{ij,n}|$ is the row sum norm and $\|P_n\|_1 = \sup_{1 \leq j \leq n} \sum_{i=1}^n |p_{ij,n}|$ is the column sum norm.

Assumption 5$''$. T goes to infinity, where n can be finite or an increasing function of T.

The zero diagonal assumption for the W_n matrix helps the interpretation of the spatial effect, as self-influence shall be excluded in practice. As a result, the trace of a spatial weights matrix is zero and hence the sum of all its eigenvalues is zero. In many empirical applications with a non-negative spatial weights matrix, each of the rows of that spatial weights matrix sums to 1, which ensures that all the weights are between 0 and 1. Row-normalized spatial weights matrix provides simple interpretation of the spatial interaction effect as an average neighborhood effect. For such a spatial weights matrix, because its spectral radius (the largest eigenvalue in absolute value) is 1, dynamic features of the SDPD model is easier to understand. Assumption 2 provides *i.i.d.* regularity assumptions for the disturbances. If there is unknown heteroskedasticity, the MLE (QMLE) will not be consistent. Methods such as the GMM in Lin and Lee (2010) and the G2SLS in Kelejian and Prucha (2010) may be designed for that situation. The UB condition in Assumption 3 is originated by Kelejian and Prucha (1998, 2001) and also used in Lee (2004, 2007), which limits the spatial correlation to a manageable degree. Invertibility of spatial transformation matrices in Assumption 3 guarantees that the reduced form of the spatial process is valid and the true parameter lies in the interior of the parameter space, which rules out spatial (near) unit roots problem in a cross section setting. As usual, compactness is a condition for theoretical analysis on nonlinear functions. When W_n is row-normalized, a compact subset of $(-1, 1)$ has often been taken as the parameter space for λ in theory. When exogenous variables X_{nt} are included in the model, it is convenient to assume that they are uniformly bounded as in Assumption 4. If elements of X_{nt} are allowed to be stochastic and unbounded, appropriate moment conditions can be imposed instead. Assumption 5 specifies that we have a large number of spatial units, while the time period T could be either large or small. For some direct estimation approaches, we need both n and T large, as in Assumption 5$'$. When we have a dynamic feature in the panel data model, we need a large T condition as in Assumption 5$''$, unless we specify a separate process for the initial value observation.

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