
**Inference in the Presence
of Weak Instruments:
A Selected Survey**

Inference in the Presence of Weak Instruments: A Selected Survey

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Inference in the Presence of Weak Instruments: A Selected Survey

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Abstract

Here we present a selected survey in which we attempt to break down the ever burgeoning literature on inference in the presence of weak instruments into issues of estimation, hypothesis testing and confidence interval construction. Within this literature a variety of different approaches have been adopted and one of the contributions of this survey is to examine some of the links between them. The vehicle that we will use to establish these links will be the small concentration results of Poskitt and Skeels (2007), which can be used to characterize various special cases when instruments are weak. We make no attempt to provide an exhaustive survey of all of the literature related to weak instruments. Contributions along these lines can be found in, *inter alia*, Stock et al. (2002), Dufour (2003), Hahn and Hausman (2003), and Andrews and Stock (2007), and we view this survey as complementary to those earlier works.

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1

Introduction

This survey is concerned with inference in the linear simultaneous equations model. The ideas developed for this model, which rose to prominence through the early activities of the Cowles Commission (see, *inter alia*, Malinvaud, 1988) have remained central to econometric practice, with the use of instrumental variables estimation having served as a unifying paradigm in econometrics for decades. Indeed, the model is of such fundamental importance as a well-understood benchmark that the study of this model and its extensions has continued largely unabated since the late 1930s.

For most of this period the literature could be viewed as belonging to one of two strands, either large-sample asymptotics or finite sample analysis.¹ Of these two strands the former matured more quickly and has had far greater impact on empirical practice than the latter. In contrast, the finite sample literature took some 20 years longer to develop, by which time empirical practice was largely entrenched. In any event, the consensus view was that the asymptotic results were considerably simpler to interpret than the exact results obtained and are

¹Not all developments could be so classified; see, for example, the work of Kadane (1971) and Morimune (1983), Basmann (1965), Anderson (1977), and Bekker (1994).

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notionally more general as they are predicated on weaker distributional assumptions.

The ease of interpretation of asymptotic results is perhaps overstated. One lesson from the finite sample literature is that sample size, by and of itself, is not a natural parameter of the sampling distributions of any of the commonly used estimators in this model; see, for example, Rothenberg (1984). Indeed, the work of Owen (1976) and Buse (1992), both asymptotic in nature, illustrates the complicated ways in which the parameters of the model interact in these distributions making *ceteris paribus* arguments problematic.

Towards the end of the 1980s both strands of the literature focussed attention on models that were either unidentified or close to unidentified. There had been earlier discussions in the literature of the role of identification, but these had tended to focus on characterizations of identification (Fisher, 1966; Hsiao, 1983) rather than the implications of under-identification on inference, although notable exceptions include Liu (1960), Liu and Breen (1969), Liu and Breen (1972), Fisher and Kadane (1972), and Sims (1980). Such interest was motivated by three distinct considerations. First, there was a growing understanding of the empirical consequences of using weak instruments; see, for example, Rotemberg (1984), Hall (1988), Campbell and Mankiw (1989), Moazzami and Buse (1990), Buse and Moazzami (1991), McClellan et al. (1994), and the analyses by Bound et al. (1995) and Staiger and Stock (1997) of the results of Angrist and Krueger (1991). Indeed, such analyses continue to appear across the literature, including, *inter alia*, Fuhrer et al. (1995), Pagan and Robertson (1998), Hotchkiss and Moore (1999) and Mavroeidis (2004, 2005).

Second, the finite sample results developed throughout the 1980s invariably involved multiple infinite series of invariant polynomials of matrix argument which, to say the least, were typically not very revealing. Consequently, simplifying special cases were typically explored to illustrate the results contained within the more general expressions. It was observed that the leading terms of these series expansions corresponded to totally unidentified models and so the analyses of these models became a commonly used expository device in this literature; see Phillips (1983, 1984a,b), Hillier et al. (1984), Hillier (1985),

Hillier and Skeels (1993) and Skeels (1995b). These totally unidentified models can be thought of as limiting cases of weak instruments.

Finally, it was becoming clear that the existing large-sample asymptotic results were providing very poor approximations to the true sampling behaviour of various statistical procedures.² Explanations for some of this behaviour were presented in Phillips (1989) and were quickly followed by a series of related observations in papers by Nelson and Startz (1990b,a) and Maddala and Jeong (1992); Buse (1992), Choi and Phillips (1992), Bekker (1994), Bound et al. (1995), Staiger and Stock (1997), and Dufour (1997). More recently, the literature has been devoted to analysing potential remedies to the problem of weak instruments, including the papers by Hall et al. (1996), Shea (1997), Wang and Zivot (1998), Zivot et al. (1998), Angrist et al. (1999), Blomquist and Dahlberg (1999), Godfrey (1999), Bekker and van der Ploeg (2000), Hahn and Hausman (2002) and Zivot et al. (2006). Similarly, the literature is extending to closely related models; see Stock and Wright (2000) for an exploration of weak instruments in GMM models.³

It should be noted that, although led to similar places, the motivations of the finite sample literature differed from that of the large-sample literature. In the former case, the difficulties in obtaining results at all have narrowed the focus of the literature to obtaining distributional results for estimators and exact expressions for moments of these estimators. In particular, there have been very few results on testing.⁴ One notable exception is the remarkable result of Anderson and Rubin

²In some sense this outcome had been anticipated by Basman et al. (1974, Footnote 2) some years earlier as weak instruments constitute a simple example where a large sample size need not correspond to a large concentration parameter (defined in Equation (2.23)), for it is the latter quantity that is the key parameter for the sampling distributions of various statistics of interest (*cf.* Rothenberg, 1984).

³Fundamental characteristics of the ‘weak instruments’ problem arise in other contexts too. For example, Forchini and Hillier (2003) explore the relationship between weak instruments and the Fieller–Creasy problem; see Fieller (1954); Creasy (1954) and, for a modern treatment, Koschat (1987). Similar structures arise wherever one is estimating ratios of parameters, as illustrated by Hirschberg and Lye (2005) when estimating extrema of quadratic models, Staiger et al. (1997) in the context the NAIRU, and Eika et al. (1996) and Ericsson et al. (1998) in respect of a monetary condition index. A nice discussion of these latter examples is provided in Schweder and Hjort (2003, Section 13.1.3).

⁴Phillips (1983, Section 3.8) provides a useful discussion of early literature. See also Phillips (1986) and Hillier (1987) for some more recent contributions.

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(1949) who provided a likelihood ratio test on the rank of part of the reduced form coefficient matrix, which subsequently formed the basis of the Koopmans and Hood (1953) likelihood ratio test of over-identifying restrictions where identification of a single equation is obtained solely through exclusion restrictions. For further discussion and critique of these classical identification tests see Hausman (1983, Section 8) and Dhrymes (1994, Sections 2.8 and 4.8). In contrast, the emphasis of the large-sample asymptotic literature, in respect of weak instruments, has been the construction of confidence intervals and hypothesis testing. Consequently many of the results obtained can be thought of as complementary and so, like a modern-day Goldilocks, practitioners will dip into one intellectual tradition or another until they find an approach that is “just right” for them.

The structure of this survey is as follows. The issues of estimation, hypothesis testing and confidence interval construction will be considered in Sections 4–6, respectively. Section 7 will examine the question of assessing the weakness of instruments. Prior to this the model and notation will be outlined in Section 2, while Section 3 will provide a simple classification of weak instrument scenarios.

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