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Performance Analysis: Economic Foundations and Trends

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Contents

Performance Analysis: Economic Foundations and Trends

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ABSTRACT

The goal of this monograph is to very concisely outline the economic theory foundations and trends of the field of Efficiency and Productivity Analysis, also sometimes referred to as Performance Analysis. I start with the profit maximization paradigm of mainstream economics, use it to derive a general profit efficiency measure and then present its special cases: revenue maximization and revenue efficiency, cost minimization and cost efficiency. I then consider various types of technical and allocative efficiencies (directional and Shephard's distance functions and related Debreu–Farrell measures as well as non-directional measures of technical efficiency), showing how they fit into or decompose the profit maximization paradigm. I then cast the efficiency and productivity concepts in a dynamic perspective that is frequently used to analyze the productivity changes of economic systems (firms, hospitals, banks, countries, etc.) over time. I conclude this monograph with an overview of major results on aggregation in productivity and efficiency analysis, where the aggregate productivity and efficiency measures are theoretically connected to their individual analogues.

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1

Introduction

The goal of this monograph is to give a relatively concise overview of Efficiency and Productivity Analysis—a very important field of research and practice, spanning over and engaging with many disciplines, most prominently Economics (theoretical and applied), Statistics (and therefore Econometrics), Operations Research (OR) and Management Science (MS), as well as Business Analytics and Business Information Systems, Computer Science and Engineering, etc. Methods developed in this field became very popular in practice for analyzing the efficiency of various economic systems: firms or its distinct departments, branches or plants, entire industries or sub-industries, countries, regions or provinces, as well as various groups or unions of countries, such as APEC, EU, OECD, etc.

While many very good articles and books have been written for this fascinating field, often they were either focusing on certain aspects (in case of articles), which made them narrow (to be sharp and deep) or, the opposite, they were too broad and too long (in case of books), or they were written a while ago and may need an update. All these reasons motivated this work. Thus, this monograph aims to complement and update the existing literature, aiming to be fairly broad, yet with

some rigor, and yet also be relatively concise, with numerous references where more details can be found.

Along the way I realized the goal was too ambitious for one monograph and so I had to split it into two parts. In this first part that you are reading now I focus on the economic theory foundation for the Efficiency and Productivity Analysis. In the second part I plan to focus on the Data Envelopment Analysis (DEA). Again, the goal is to be concise and so, rather than trying to "cover everything" (which is impossible), I try to give a foundation that an interested reader may find useful before learning more, whether it be in terms of theory or in terms of empirical work involving any estimator, and whether it is the DEA or its alternatives, such as Stochastic Frontier Analysis (SFA) or a symbiosis of them. This (hopefully) makes this part useful by itself, even if the reader decides not to pursue with DEA, SFA or any other particular approach.

This monograph can also be viewed as a much shorter (yet still fairly rigorous) discussion as those in Chapters 1–5 and 7 of Sickles and Zelenyuk (2019) and even shorter than the classic book of Färe and Primont (1995). It is also cast at a different angle than these sources which, *inter alia*, I try to follow myself and often refer to here for further details.

Specifically, in Section 2, I start with the profit maximization paradigm of mainstream economics, using it to derive a general profit efficiency measure and to present some of its interesting variants, related decompositions and various intuitive interpretations. Then, in Section 3, I present key special cases of the profit maximization and related efficiency measures: (i) revenue maximization and revenue efficiency, (ii) cost minimization and cost efficiency, and (iii) various types of technical efficiency and allocative efficiency. In particular, I consider directional distance functions, Shephard's distance functions (related Debreu–Farrell measures), and non-directional measures of technical efficiency. In that chapter I also show how the technical efficiency measures fit into (or help decompose) the profit maximization paradigm. Furthermore, in Section 4, I outline the efficiency and productivity concepts in a dynamic perspective that can be deployed for analyzing the productivity changes of economic systems (firms, hospitals, banks,

countries, etc.) over time. Finally, Section 5 provides an overview of major recent results on aggregation in efficiency and productivity analysis, where the aggregate efficiency and productivity measures are theoretically linked to their individual analogues. Section 6 closes with brief concluding remarks.

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