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Data Envelopment Analysis: From Foundations to Modern Advancements

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Data Envelopment Analysis: From Foundations to Modern Advancements

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ABSTRACT

Data envelopment analysis (DEA) is a mainstream method for efficiency and productivity analysis, widely applied in numerous fields, including the healthcare sector, banking, energy generation and distribution, and cross-country economic growth analysis. In this monograph, we aim to provide a compendious overview of DEA. We start with the DEA estimators in various scenarios, such as for estimating technology, cost, revenue, profit functions and related efficiency measures, and its popular variants based on different assumptions about the shape of technology. The statistical properties and extensions on DEA, such as analysis on covariates of efficiency, are also discussed and the practical tips for computations are provided.

Keywords: Efficiency; data envelopment analysis; productivity index; statistical properties; covariates of efficiency.

JEL Codes: C14; C24; C43; C61; D24; I11; I18.

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Introduction

Efficiency analysis methods are widely developed and applied in numerous fields such as healthcare, banking, energy, and agriculture, among others. The data envelopment analysis (DEA) (Charnes *et al.*, 1978) and stochastic frontier analysis (SFA) (Aigner *et al.*, 1977; Meeusen and Broeck, 1977) are the two mainstream approaches with many variants that can be at least traced back to the seminal works of Debreu (1951), Shephard (1953), and Farrell (1957). A variety of studies have been conducted with DEA and its variants in the past decades, such as in the systematic reviews by Emrouznejad *et al.* (2008) and Emrouznejad and Yang (2018), as well as the recent chapter by Ray (2020) and those regarding the DEA applications in banking (Miller, 2020), transportation (Wheat *et al.*, 2020), telecommunications (Bhattacharyya, 2020), etc. Here, we complement these studies in many respects, as explained below.

DEA has been found useful for analyzing many sectors of the economy in many countries. In the area of healthcare efficiency analysis, DEA was found as the most widely applied approach (e.g., in the review by Hollingsworth, 2008; O'Neill *et al.*, 2008; and more recently Kohl *et al.*, 2019; See *et al.*, 2024; and Wang *et al.*, 2024), which can be traced back to at least Nunamaker (1983), Banker *et al.* (1986), and Grosskopf and Valdmanis (1987). DEA is also widely used for analyzing efficiency

and productivity of banking, e.g., see Sherman and Gold (1985), English *et al.* (1993), Seiford and Zhu (1999), Alam (2001), Sathye (2003), Casu *et al.* (2013), and Du *et al.* (2018), to mention a few.¹ DEA is also among the key analytical tools in energy and environmental studies, for example, Färe *et al.* (1985a), Goto and Tsutsui (1998), Athanassopoulos *et al.* (1999), Reinhard *et al.* (2000), Agrell and Bogetoft (2005), Sueyoshi and Goto (2013), and Wu *et al.* (2019).² DEA also received a wide use and appreciation in agricultural research, e.g., O'Donnell (2012) and Mugerá *et al.* (2016). This list can go on for many pages, yet we need to get to business, introducing the DEA for a wide audience.

The main goal of this monograph is to provide a compendious overview of DEA and its major variations mainly from a practical perspective.³ The economic theory for the models behind the DEA estimators has been covered extensively in various sources, most recently in Zelenyuk (2021), whose notation we try to follow here (with some refinements) for the sake of consistency.

In the following sections, we start by introducing the canonical envelopment-type estimators in the production function in Section 2, their advancements in Section 3, and the estimation of cost, revenue, and profit efficiency in Section 4.1. The reader is then introduced to several more advanced streams of DEA literature: the productivity indexes with DEA (Section 5), the statistical properties (Section 6), including the recent development in aggregation, bias-correction, explanation of efficiency, etc., and the “two-stage DEA” for explaining the inefficiency (Section 7). Finally, we also provide demonstrations of some prevalent DEA estimators in R (R Core Team, 2023) and other software with the hospital data in Appendix C.

¹See Henriques *et al.* (2020) and Zelenyuk and Zelenyuk (2021) for related discussions.

²Also see more studies as surveyed by Zhou *et al.* (2008), Zhang and Choi (2014), and Sueyoshi *et al.* (2017).

³For another major alternative approach, see Sickles *et al.* (2020) and Nguyen *et al.* (2022a) for overviews of the SFA paradigm and applications in statistical tools, Sickles *et al.* (2024) for utilizing SFA in healthcare, and Sickles and Zelenyuk (2019, Chapter 11–16) for more comprehensive treatment.

Appendices

A

Steps of the SZ07 Bootstrapping Method for Efficiency Aggregation

- (1) **Estimate the aggregate efficiency:** Deploy the selected estimator on the original data set to estimate individual efficiency of all DMUs. Then obtain the aggregate estimate of each subgroup and of the population, denoted as $\widehat{\mathcal{T}\mathcal{E}}^l, l = 1, \dots, L$ and $\widehat{\mathcal{T}\mathcal{E}}$ with (6.5) and (6.7), respectively.
- (2) **Generate the b^{th} bootstrap sample:** For each subgroup l of size n_l ($l = 1, \dots, L$), determine a sample size $m_l = \lfloor n_l^\delta \rfloor, \delta \in (0, 1)$ and resample m_l out of the n_l allocations independently, uniformly, and with replacement. The generated bootstrap sample for each group can be denoted as $\chi_{m_l, b}^{*,l} = \{(x_b^{*,l,j}, y_b^{*,l,j}) | j = 1, \dots, m_l; l = 1, \dots, L\}$, meanwhile the pool of bootstrap samples is $\chi_{m, b}^* = \{\chi_{m_l, b}^{*,l}, l = 1, \dots, L; b = 1, \dots, B\}, m = \sum_{l=1}^L m_l$.
- (3) **Estimate the b^{th} bootstrap aggregate efficiency:** For the b^{th} bootstrap subsample, estimate the aggregate efficiency (i.e., $\widehat{\mathcal{T}\mathcal{E}}_b^*$ and $\widehat{\mathcal{T}\mathcal{E}}_b^{*,l}, l = 1, \dots, L$) as in the first step, but using the frontier constructed by the pooled bootstrap sample $\chi_{m, b}^*$.

68 *Steps of the SZ07 Bootstrapping Method for Efficiency Aggregation*

- (4) **Repeat:** Step 2 and 3 for B times and record B bootstrap estimates of the interested aggregate efficiency, e.g., $\{\widehat{\mathcal{T}\mathcal{E}}_b^*\}_{b=1}^B$ and $\{\widehat{\mathcal{T}\mathcal{E}}_b^{*,l}\}_{b=1}^B, l = 1, \dots, L$.
- (5) **Obtain the bootstrap goals:** Use the collected results to infer the true efficiency, e.g., construct the confidence intervals of $\overline{\mathcal{T}\mathcal{E}}$ and perform statistical tests with the distribution of $\{\widehat{\mathcal{T}\mathcal{E}}_b^*\}_{b=1}^B$, as well as correct the bias of $\widehat{\mathcal{T}\mathcal{E}} - \overline{\mathcal{T}\mathcal{E}}$ through $\{\widehat{\mathcal{T}\mathcal{E}}_b^* - \widehat{\mathcal{T}\mathcal{E}}\}_{b=1}^B$.

B

Steps of the Two-Stage Double Bootstrap Method in SW07

- (1) **Estimate the efficiency:** Apply the selected DEA/FDH estimator on the original data set χ^n to obtain the individual efficiency estimates $\widehat{\mathcal{OTE}}(x^i, y^i | \chi^n)$ of all DMUs.
- (2) **Estimate the parameters of the truncated regression:** Use the MLE to estimate the parameters specified in the function of efficiency and obtain $\widehat{\beta}$ and $\widehat{\sigma}_\varepsilon$.
- (3) **Obtain the bootstrap estimates of efficiency:** Run the bootstrap process of B_1 times as follows.
 - (3.1) Draw $\widehat{\varepsilon}_b^i$ for $i = 1, \dots, n$, from the distribution of $N(0, \widehat{\sigma}_\varepsilon^2)$ with the left truncation as $\widehat{\varepsilon}_b^i \geq 1 - g(z^i | \widehat{\beta})$.
 - (3.2) Estimate the bootstrap efficiency with the results above as $\mathcal{OTE}_b^*(x^i, y^i | \chi^n) = g(z^i | \widehat{\beta}) + \widehat{\varepsilon}_b^i$.
 - (3.3) For $i = 1, \dots, n$, define $x_b^{i*} = x^i$, $y_b^{i*} = y^i [\widehat{\mathcal{OTE}}(x^i, y^i | \chi^n) / \mathcal{OTE}_b^*(x^i, y^i | \chi^n)]$, $z_b^{i*} = z^i$.
 - (3.4) Estimate $\widehat{\mathcal{OTE}}_b^*(x^i, y^i | \chi^n)$ using the same estimator as in the first step using the bootstrap data set $\chi_b^{n*} = \{(x_b^{i*}, y_b^{i*}) | i = 1, \dots, n\}$.

- (4) **Correct the bias of the efficiency estimates:** For $i = 1, \dots, n$, obtain $\widehat{\mathcal{OTE}}_{bc}^i(x^i, y^i | \chi^n) = \widehat{\mathcal{OTE}}(x^i, y^i | \chi^n) - \widehat{\text{Bias}}(\widehat{\mathcal{OTE}}(x^i, y^i | \chi^n))$, where the bias is bootstrap-estimated in a procedure akin to the section 6.5.2.
- (5) **Estimate the parameters based on the corrected efficiency:** Apply MLE on the truncated regression of $\widehat{\mathcal{OTE}}_{bc}^i(x^i, y^i | \chi^n)$ on z^i to obtain the estimates $(\widehat{\beta}, \widehat{\sigma}_\varepsilon)$.
- (6) **Obtain the bootstrap estimates of the parameters:** Run the bootstrap process as follows for B_2 times.
- (6.1) Draw $\widehat{\varepsilon}_b^i$ for $i = 1, \dots, n$, from the distribution of $N(0, \widehat{\sigma}_\varepsilon^2)$ with the left truncation as $\widehat{\varepsilon}_b^i \geq 1 - g(z^i | \widehat{\beta})$.
- (6.2) Estimate the double-bootstrap efficiency as $\mathcal{OTE}_b^{**}(x^i, y^i | \chi^n) = g(z^i | \widehat{\beta}) + \widehat{\varepsilon}_b^i$.
- (6.3) Apply MLE as in the fifth step on the truncated regression of $\mathcal{OTE}_b^{**}(x^i, y^i | \chi^n)$ on z^i to obtain the estimates $(\widehat{\beta}_b^*, \widehat{\sigma}_{\varepsilon,b}^*)$.
- (7) **Construct the confidence intervals of the parameters:** The refined estimates $(\widehat{\beta}, \widehat{\sigma}_\varepsilon)$ and the bootstrap estimates $(\widehat{\beta}_b^*, \widehat{\sigma}_{\varepsilon,b}^*)$ can be used to construct a confidence interval and hence the other interested statistics accordingly. For a significance level of α , there are values (a_α, b_α) for the j th element of $\widehat{\beta}$ that $\Pr[a_\alpha \leq (\beta^j - \widehat{\beta}^j) \leq b_\alpha] = 1 - \alpha$. As the distribution of $(\beta^j - \widehat{\beta}^j)$ is unavailable, the bootstrap estimates can approximate the values that $\Pr[a_\alpha^* \leq (\widehat{\beta}^j - \widehat{\beta}_b^{*j}) \leq b_\alpha^*] \approx 1 - \alpha$. With $B_2 \rightarrow \infty$, the confidence interval of β^j can be estimated as $[\widehat{\beta}^j + a_\alpha^*, \widehat{\beta}^j + b_\alpha^*]$. The confidence interval of the elements in σ_ε can be constructed analogously.

C

Programming and Computations

C.1 DEA Estimators in Statistical Tools

As one of the primary approach in efficiency analysis, a considerable amount of effort has been devoted into the programming and application of the DEA estimators. Specialized software has been developed for deploying DEA, e.g., DEAP (Coelli, 1996) and PIM-DEA (Emrouznejad and Thanassoulis, 2014). One can also apply DEA with Excel Solver amalgamated with add-ins, e.g., DEA Solver Pro (Cooper *et al.*, 2007) and DEAFrontier (Zhu, 2009).¹

Among others, the DEA estimators can be conducted in a variety of prevalent statistical tools, e.g., R, MATLAB, Stata, etc. One option is utilizing the functional user-written commands to estimate the specified estimators directly. Prior to the illustrations of the widespread packages and commands, we first implement the LP problems of DEA step-by-step in a statistical programming environment. In this way, the code is also more flexible than the pre-programmed commands adapting to other variants of DEA.

For an illustration, we generate a sample of seven DMUs with one input and two outputs. As shown in Box C.1 and C.2, the output-oriented

¹See Appendix C.3 for an illustration of utilizing the Excel Solver with a numerical example.

```

clear all
close all
%%%% Illustration of Output-Oriented DEA in Matlab

%% Generate X and Y
y1 = [1, 2, 1, 0.5, 1.5, 2, 0]';
y2 = [2, 1, 1, 1.5, 0.5, 0, 2]';
x1 = [1, 1, 1, 1, 1, 1, 1]';
Y = [y1, y2];
X = [x1];

%% Define parameters
rts = 'CRS';
% rts = 'VRS';

M = size(Y,2); N = size(X,2); n = size(X,1);
lb = [1; zeros(n,1)]; % Lower bounds for decision variables
ub = []; % Upper bounds

theta_all = [];

% Choose between VRS and CRS
if strcmp('VRS', rts)
    Aeq = [0, ones(1,n)]; % Matrix for linear equality constraints;
    beq = [1]; % Vector for linear equality constraint;
elseif strcmp('CRS', rts)
    Aeq = []; % CRS: no extra equality constraint
    beq = [];
end

%% Estimate theta_j
for j=1:n
    x_j = X(j,:); % Select x and y for a DMU of interest, j
    y_j = Y(j,:);

    f = -[1; zeros(n,1)]; % Parameters of the objective function
    Aineq = [zeros(N,1), X'; y_j', -Y']; % Matrix for linear inequality
        constraints
    bineq = [x_j'; zeros(M,1)]; % Vector for linear inequality
        constraints

    [x,fval,exitflag,output,lambda] = linprog(f,Aineq,bineq,Aeq,beq,lb,ub);
    theta_DEA = -fval; % Minus, because "min" was used instead of "max"
    theta_all = [theta_all; theta_DEA]; % Collect estimates
end

%% Print the results
disp('Firm # Estimated efficiency')
disp([(1:n)', theta_all])

```

Box C.1: MATLAB code snippet for Output-Oriented DEA

```

### Illustration of Output-Oriented DEA in R ###

# Generate sample data
y1 = c(1, 2, 1, 0.5, 1.5, 2, 0)
y2 = c(2, 1, 1, 1.5, 0.5, 0, 2)
x1 = c(1, 1, 1, 1, 1, 1, 1)
Y = as.data.frame(cbind(y1, y2))
X = as.data.frame(x1)

# Define parameters
rts='CRS'
# rts='VRS'

M = length(Y)
N = length(X)
n = nrow(X)

# Choose between VRS and CRS
if (rts=='VRS'){
  Aeq = cbind(0, t(rep(1,n))) # Matrix for linear equality constraints;
  beq = 1 # Vector for linear equality constraint;
}
if (rts=='CRS'){
  Aeq <- vector(mode = "numeric", length = 0)
  beq <- vector(mode = "numeric", length = 0)
}

library(lpSolve)
solution <- vector(mode = "numeric", length = 0)
# Estimate theta-i for each individual
for (i in 1:n){
  xi = X[i,]
  yi = Y[i,]
  objx <- vector(mode = "numeric", length = 0)
  f = t(-cbind(1, t(rep(0,n)))) # Objective function
  Aineq = rbind(cbind(rep(0,N), t(X)),
               cbind(t(yi), -t(Y))) # Matrix for linear inequality
               constraints
  bineq = t(cbind(xi, t(rep(0,M)))) # Vector for linear inequality
               constraints
  direction = c(rep("<=",N+M),"=") # Set the directions including the
               equivalent constraint
  A = rbind(Aineq,Aeq)
  b = rbind(bineq,beq)

  lp = lp (direction = "min", objective.in = f, const.mat = A,
           const.dir = direction, const.rhs = b) # Note that the bounds of every
           variable has been assumed in lp
  solution = rbind(solution, lp[["solution"]][1]) # Collect estimates
}
solution

```

Box C.2: R code snippet for Output-Oriented DEA

DEA estimator can be programmed in a form of LP problems by transforming the constraints. The optimization problem can be then solved with the LP solver in MATLAB (command **lingprog**) and R (function **lp** in package **lpSolve** (Berkelaar *et al.*, 2023)), respectively. For example, when assuming CRS, the efficiency scores estimated using the two snippets of code are identical as {1.00, 1.00, 1.50, 1.33, 1.33, 1.00, 1.00}.

C.2 Applications with User-Written Packages

C.2.1 Sample Data

A demonstration with a real data set may illustrate the DEA techniques, covered in this work, more explicitly in practice. We use the data from Queensland Health (QH), regarding 95 public hospitals in Queensland, Australia, during a four-year period (FY 2012/13 to FY 2015/16). The input-output allocation follows the seminal practice on hospital efficiency analysis (e.g., Grosskopf and Valdmanis, 1987; Rosko, 2001) and the recent studies on Queensland public hospital (e.g., Nguyen and Zelenyuk (2021a,b; Wang and Zelenyuk, 2024b)).² It contains three inputs, representing the aggregated labor, consumable goods, and capital, and one output, the aggregated inpatient and outpatient services.³ Moreover, three environmental variables, i.e., the location, size, and teaching functions, are considered in the regression of efficiency estimates. The descriptive statistics of these variables are as summarized in Table C.1.

C.2.2 Practical Applications of Technical Efficiency

The user-written commands are developed in a range of popular statistical tools, which are practical for the commonly applied DEA estimators. We focus on the R environment (R Core Team, 2023) in this monograph,

²See also Wang and Zelenyuk (2024a) for overview and illustrations of the efficiency analysis techniques in healthcare.

³The aggregation of the labor input and output variables follows the process in Nguyen and Zelenyuk (2021a) and Wang and Zelenyuk (2024b), based on a Principal Component Analysis (PCA) type approach introduced by Daraio and Simar (2007c).

Table C.1: Descriptive statistics of Queensland public hospitals, FY 2012/13 to FY 2015/16

Variable	Description	Mean	Std Dev	Min	Max
<i>Input</i>					
LABOR	Aggregated labor input	0.76	1.57	0.01	8.71
BED	Number of beds	74.92	133.78	3.00	680.00
SUPP*	Consumable expenditure	7.83	19.20	0.03	164.00
<i>Output</i>					
OUT	Inpatient and outpatient service	0.54	1.03	0.01	5.05
Variable	Description	Frequency	Percentage		
<i>Environmental</i>					
REMOTE	Located in remote areas	108	28.42%		
SMALL	Small hospitals	300	78.95%		
TEACH	Teaching hospitals	70	18.42%		

Note: *AUD 1,000,000 in constant price of FY2012/2013.

where the conventional DEA models can be constructed with some powerful packages, e.g., **Benchmarking** by Bogetoft and Otto (2022).^{4, 5} The technical efficiency can be estimated with the DEA estimators discussed above as illustrated in the snippet of R code in Box C.3.^{6, 7}

Both the output-oriented and input-oriented technical efficiency are estimated with the DEA estimators in CRS, VRS, or NIRS, or the FDH estimators through the functions in **Benchmarking**, respectively. The function **dea** in **Benchmarking** is feasible of all these scenarios, where for a more comparable efficiency score between $[0, 1]$, the output-oriented results are reciprocally transformed.

⁴Some other packages in R, e.g., **FEAR** by Wilson (2020), **rDEA** by Simm and Besstremyannaya (2020), and **deaR** by Coll-Serrano *et al.* (2022), also provide extensive functions, including the advanced applications in DEA, such as the bias-corrected efficiency, Malmquist productivity index, bootstrapping DEA, etc., more detail of which will be discussed in later sections.

⁵DEA is also well developed in MATLAB, as introduced by Sickles and Zelenyuk (2019). Interested readers can also follow the code they provided at: <https://sites.google.com/site/productivityefficiency/home>. One can also use the commands by Ji and Lee (2010) to apply DEA estimators in Stata.

⁶See Appendix D for the complete R code of the illustrations on the sample data, including the data process procedures and plotting.

⁷See also Sickles *et al.* (2020) for a guidance of the DEA applications in the R environment.

```
##### Technical Efficiency #####

attach(data)

# Use the functions by Bogetoft and Otto (2022)
library(Benchmarking)

# Output-oriented
dea.crs.out.bench = 1/dea(X, Y, RTS="crs", ORIENTATION="out")$eff # CRS
dea.vrs.out.bench = 1/dea(X, Y, RTS="vrs", ORIENTATION="out")$eff # VRS
dea.nirs.out.bench = 1/dea(X, Y, RTS="drs", ORIENTATION="out")$eff # NIRS
dea.fdh.out.bench = 1/dea(X, Y, RTS="fdh", ORIENTATION="out")$eff # FDH

# Input-oriented
dea.crs.in.bench = dea(X, Y, RTS="crs", ORIENTATION="in")$eff
dea.vrs.in.bench = dea(X, Y, RTS="vrs", ORIENTATION="in")$eff
dea.nirs.in.bench = dea(X, Y, RTS="drs", ORIENTATION="in")$eff
dea.fdh.in.bench = dea(X, Y, RTS="fdh", ORIENTATION="in")$eff

# Summarize the estimations
Effi <- as.data.frame(cbind(dea.crs.out.bench, dea.vrs.out.bench,
                           dea.nirs.out.bench, dea.fdh.out.bench,
                           dea.crs.in.bench, dea.vrs.in.bench,
                           dea.nirs.in.bench, dea.fdh.in.bench))

summary(Effi)
```

Box C.3: R code snippet for technical efficiency

The summarized statistics and the distribution of the efficiency estimates are as reported in Figures C.1 and C.2. The estimates with different returns to scale specifications lead to significantly different

```
> summary(Effi)
dea.crs.out.bench dea.vrs.out.bench dea.nirs.out.bench dea.fdh.out.bench
Min. :0.1182      Min. :0.1454      Min. :0.1322      Min. :0.1733
1st Qu.:0.2703    1st Qu.:0.3698    1st Qu.:0.3529    1st Qu.:0.5444
Median :0.3565    Median :0.6192    Median :0.5842    Median :0.8330
Mean :0.3666      Mean :0.5924      Mean :0.5728      Mean :0.7477
3rd Qu.:0.4281    3rd Qu.:0.8102    3rd Qu.:0.7977    3rd Qu.:1.0000
Max. :1.0000      Max. :1.0000      Max. :1.0000      Max. :1.0000
dea.crs.in.bench dea.vrs.in.bench dea.nirs.in.bench dea.fdh.in.bench
Min. :0.1182      Min. :0.1391      Min. :0.1182      Min. :0.1556
1st Qu.:0.2703    1st Qu.:0.4300    1st Qu.:0.2824    1st Qu.:0.6376
Median :0.3565    Median :0.5872    Median :0.4204    Median :0.8571
Mean :0.3666      Mean :0.6068      Mean :0.4950      Mean :0.7966
3rd Qu.:0.4281    3rd Qu.:0.7656    3rd Qu.:0.7217    3rd Qu.:1.0000
Max. :1.0000      Max. :1.0000      Max. :1.0000      Max. :1.0000
```

Figure C.1: Screenshot of the summarized statistics of the technical efficiency estimates of different estimators in R

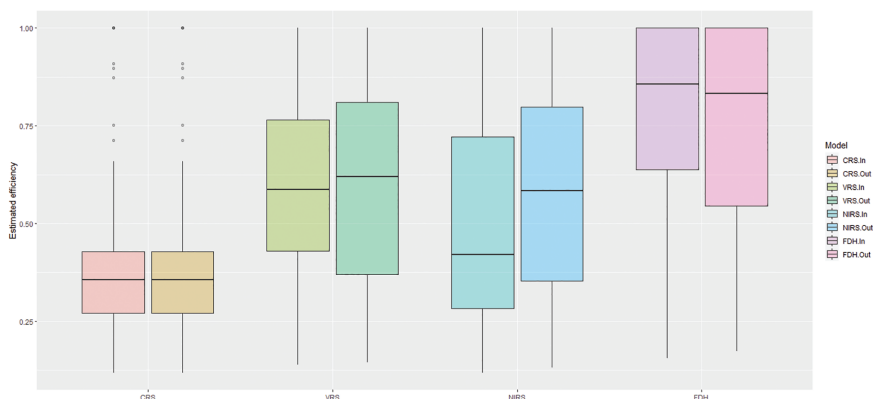


Figure C.2: Boxplot of technical efficiency estimates

results. The mean efficiency estimated by DEA-CRS is 36.66%, which may be due to the outliers in the sample, as indicated in the boxplot. Meanwhile in comparison, when NIRS assumption is applied, i.e., the outputs would not change equiproportionally as the inputs, the mean efficiency is estimated at 57.28%. The estimates of the FDH estimators are higher than the DEA estimators in general, and indicate a significant proportion of efficient units. Besides, as mentioned above, the reciprocal relationship between OTE and ITE is also exhibited. As the results of one orientation is in reciprocal, the output-oriented and input-oriented DEA-CRS estimates are identical to each other.

C.2.3 Practical Applications of Other Efficiency Measures

The estimators of cost, revenue, and profit efficiency are also feasible in the **Benchmarking** package (Bogetoft and Otto, 2022), which can be deployed with the **cost.opt**, **revenue.opt**, and **profit.opt** function, respectively. The applications are as illustrated in Box C.4.^{8,9} Besides the input-output allocations, vectors of prices are required for both inputs and outputs. Since the price variables are not available in our

⁸Inputs and outputs are predefined. See Appendix D for the complete R code including the data process procedures.

⁹CRS assumption is applied for an illustration, while the other commonly deployed assumptions on the returns to scale are also available.

```
##### Cost/Revenue/Profit Efficiency #####
attach(data)

# Generate artificial matrix of prices
w <- t(as.matrix(c(1,2,3)))
p <- as.matrix(4)

# Use the functions by Bogetoft and Otto (2022)
library(Benchmarking)

# Cost efficiency
xopt = cost.opt(X, Y, w, RTS='crs') #CRS
cobs <- X %*% t(w) # Observed Cost
copt <- xopt$x %*% t(w) # Optimal Cost
dea.crs.cost = copt/cobs # cost efficiency

# Revenue efficiency
yopt = revenue.opt(X, Y, p, RTS='crs') #CRS
yobs <- Y %*% p # Observed Revenue
yopt <- yopt$y %*% p # Optimal Revenue
dea.crs.revenue = yobs/yopt # Revenue efficiency

# Profit efficiency
popt = profit.opt(X, Y, w, p, RTS='crs') #CRS
pobs <- Y %*% p - X %*% t(w) # Observed Profit
popt <- popt$y %*% p - popt$x %*% t(w) # Optimal Profit
dea.crs.revenue = pobs/popt # Profit efficiency

# Other RTS assumptions can be analogously applied
```

Box C.4: R code snippet for cost, revenue, and profit efficiency

sample data, we generate simulated data for vectors w and p to represent the prices of the three inputs and one output, respectively.¹⁰ The functions return the optimal level of input or/and output regarding the efficiency type based on the price vectors. Consequently, the efficiency is estimated as a ratio between the optimal and real allocations as discussed in Section 4.1.

C.2.4 Practical Applications of MPI

The MPI can be directly estimated using the `malmq` function in **Benchmarking** (Bogetoft and Otto, 2022). As illustrated in Box C.5,

¹⁰Based on these artificial prices, the efficiency estimates are for examples only.


```
##### MPI #####

attach(data)

# Use the functions by Bogetoft and Otto (2022)
library(Benchmarking)

# Use period 1 and 2 as an example
X0 <- as.matrix(X[Year==1,])
X1 <- as.matrix(X[Year==2,])

Y0 <- as.matrix(Y[Year==1,])
Y1 <- as.matrix(Y[Year==2,])

IDO <- as.matrix(id[Year==1])
ID1 <- as.matrix(id[Year==2])

mpi.crs.in = malmq(X0, Y0, ID0, X1, Y1, ID1, RTS = "crs", ORIENTATION = "in")$m
mpi.vrs.in = malmq(X0, Y0, ID0, X1, Y1, ID1, RTS = "vrs", ORIENTATION = "in")$m
mpi.nirs.in = malmq(X0, Y0, ID0, X1, Y1, ID1, RTS = "drs", ORIENTATION = "in")$m
mpi.crs.out = malmq(X0, Y0, ID0, X1, Y1, ID1, RTS = "crs", ORIENTATION = "out")$m
mpi.vrs.out = malmq(X0, Y0, ID0, X1, Y1, ID1, RTS = "vrs", ORIENTATION = "out")$m
mpi.nirs.out = malmq(X0, Y0, ID0, X1, Y1, ID1, RTS = "drs", ORIENTATION = "out")$m

# Summarize the estimations
MPI <- as.data.frame(cbind(mpi.crs.in, mpi.vrs.in,
                           mpi.nirs.in, mpi.crs.out,
                           mpi.vrs.out, mpi.nirs.out))
summary(MPI)

# Estimate with DEA estimators (F re et al. 1992)
dea00<-dea(X0, Y0, RTS="crs", ORIENTATION="out", XREF=X0, YREF=Y0)$eff
dea11<-dea(X1, Y1, RTS="crs", ORIENTATION="out", XREF=X1, YREF=Y1)$eff
dea10<-dea(X0, Y0, RTS="crs", ORIENTATION="out", XREF=X1, YREF=Y1)$eff
dea01<-dea(X1, Y1, RTS="crs", ORIENTATION="out", XREF=X0, YREF=Y0)$eff

mpi.dea<-sqrt(dea01/dea00*dea11/dea10)
summary(mpi.dea) # Same results as above
summary(mpi.crs.out)
```

Box C.5: R code snippet for MPI

using the first two periods (FY 2012/13 and FY 2013/14) as an example, the MPIs of the 95 hospitals in Queensland are estimated under different returns to scale assumptions (CRS, VRS, and NIRS) and in input- and output-orientation, respectively.

The summarized statistics of the outcome is as shown in Figure C.3. The mean MPI of the hospitals is similar across different specifications, which indicate a moderate rise of productivity in general between the two periods. The highest MPI in different models is around 1.7, while the minimum value of input-oriented VRS and NIRS models are 0, which may be due to the specified model not conforming the data generating process in reality. Nevertheless, the Pearson correlation between each pair of the models is higher than 0.7.

As indicated above, DEA estimators can be applied in the estimation of the distance functions (Färe *et al.*, 1992). In the second part of the R code snippet, we deploy the DEA estimators by the aforementioned **dea** function in **Benchmarking**, and estimate the MPI accordingly. Using the output-oriented VRS specification as an example, the results are identical to the estimates of the **malmq** function. Note that as indicated above, the Farrell-type technical efficiency is reciprocal of the Shephard's distance function, and hence the results of the **dea** function is not reciprocally transformed as when they are used to estimate the technical efficiency in Box C.5.

For a more intuitive visualization of the productivity change, we can estimate the MPI of hospitals between each pair of adjacent periods.¹¹ As shown in the heatmap in Figure C.4, the MPIs are aggregated by the 14 Local Hospital Networks.¹² The hospitals in HHS 327 between FY 2013/14 and FY 2014/15 exhibit the highest productivity improvement, followed by HHS 315 and 317 in the earlier period. In contrast, hospitals in HHS 313, 317, and 326 in the last period of study are estimated with a decreasing productivity in general with a MPI estimate close to 0.5.

¹¹The input-oriented MPI under CRS assumption is displayed as an demonstration. See Appendix D for the complete R code of the illustrations, including the plotting of the heatmap.

¹²Local Hospital Networks in Queensland are often referred to as Hospital and Health Services (HHSs). There are 15 geographical HHSs, and 14 of them are recorded in our sample.

```

> summary(MPI)
  mpi.crs.in   mpi.vrs.in   mpi.nirs.in   mpi.crs.out   mpi.vrs.out   mpi.nirs.out
Min.   :0.6092   Min.   :0.0000   Min.   :0.0000   Min.   :0.6202   Min.   :0.3428   Min.   :0.6199
1st Qu.:0.9017   1st Qu.:0.9184   1st Qu.:0.8929   1st Qu.:0.9186   1st Qu.:0.9362   1st Qu.:0.9319
Median :0.9787   Median :1.0118   Median :0.9947   Median :1.0218   Median :1.0002   Median :1.0054
Mean   :1.0005   Mean   :1.0151   Mean   :0.9965   Mean   :1.0359   Mean   :1.0329   Mean   :1.0339
3rd Qu.:1.0887   3rd Qu.:1.0926   3rd Qu.:1.0890   3rd Qu.:1.1090   3rd Qu.:1.0959   3rd Qu.:1.1008
Max.   :1.6124   Max.   :1.6195   Max.   :1.7214   Max.   :1.6414   Max.   :1.7236   Max.   :1.6272
NA's   :4

> summary(mpi.dea) # Same results as above
  Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
0.6202 0.9186  1.0218  1.0359  1.1090  1.6414

> summary(mpi.crs.out)
  Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
0.6202 0.9186  1.0218  1.0359  1.1090  1.6414

```

Figure C.3: Screenshot of the summarized statistics of MPI estimated by different functions in R

```

##### Bias-correction #####

attach(data)

# Use the functions by Wilson (2020)
library(FEAR)

# Bias-corrected efficiency (CRS-Output-oriented)
Bootstrap.fear = boot.sw98(Xt, Yt, NREP = 2000, RTS = 3, ORIENTATION = 2,
  alpha = 0.05, CI.TYPE=2)
cdea = 1/Bootstrap.fear$dhat.bc

```

Box C.6: R code snippet for bias correction

Consequently, MPI is a useful method in evaluating the hospital performance, especially in a perspective of measuring the productivity change. A cross-sectional comparison of the MPI of hospitals in different groups (i.e., different regions, types, sizes, etc.) would also be one attractive application (i.e., Aragón *et al.* (2019)) as well as evaluating the growth rate of different groups in a study period, when a panel data is available.

C.2.5 Practical Applications of Bias Correction, Kernel Density Estimation, and Aggregation

Some aforementioned techniques in Section 6 can be implemented in R, by cooperating with some user-written packages. For example as shown in Box C.6, the seminal bootstrap method adapted in the DEA/FDH estimators for bias-correction by Simar and Wilson (1998) can be

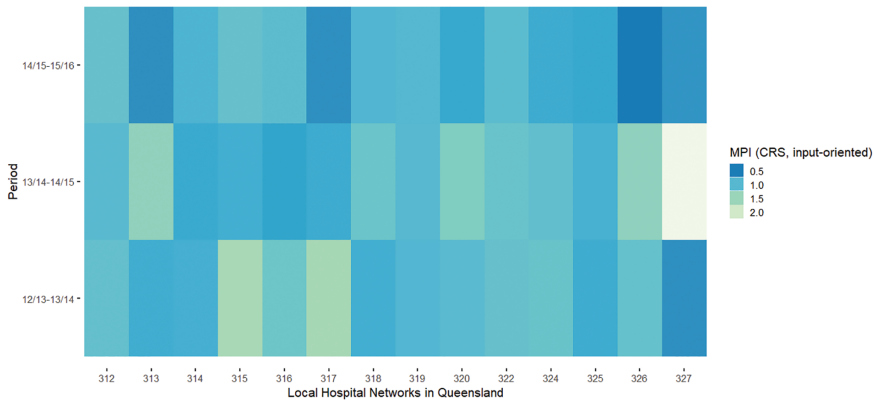


Figure C.4: Heatmap of MPI of HHS across periods

deployed by the `boot.sw98` function in **FEAR** (Wilson, 2020), or the `dea.robust` function in **rDEA** (Simm and Besstremyannaya, 2020), or more alternative functions.¹³ The results of different functions are close to each other, i.e., correlations higher than 90%, but not identical due to the random sampling process.

Taking the output-oriented DEA-CRS estimator as an example, as shown in the kernel density plot in Figure C.5,¹⁴ the efficiency bias-corrected by the `boot.sw98` function is relatively higher than the original estimates of the `dea` function. This phenomenon is as expected due to that the estimator of the output-oriented efficiency is downward biased ($1 \leq \widehat{OTE}(x, y|\hat{\Psi}) \leq OTE(x, y|\Psi) \leq \infty$).

The aggregate of efficiency is also feasible in R. Due to unavailable price variables in our sample, we generate a simple vector of price p . As shown in Box C.8, the aggregated efficiency of the population following (6.5) and (6.7) ($L = 1$) is applicable with few lines of code using the basic commands in R. Taking output-oriented DEA-CRS as an example, we obtain an efficiency level of 0.4038 in aggregate, which is slightly higher than the simple mean of the individual estimates (0.3666).

¹³An alternative is the `dea.boot` function in **Benchmarking**, which is however slower in computation than `boot.sw98` (Bogetoft and Otto, 2022).

¹⁴Using Gaussian kernel and bandwidth selected by cross-validation as shown in Box C.7.

```
##### Kernel density plot #####

# Compare the original and bias-corrected estimates
dea = dea(Xt, Yt, RTS=3, ORIENTATION=2)
cdea = cbind(cdea,rep(1,380))
dea = cbind(dea,rep(0,380))

correction = as.data.frame(rbind(cdea,dea))
colnames(correction) = c("DEA","method")

correction$Estimates[correction$method==1]='Bias-corrected'
correction$Estimates[correction$method==0]='Original'

attach(correction)
require("ggplot2")
.df <- na.omit(data.frame(x = correction$DEA))
.nbins <- pretty(range(.df$x), n = nclass.FD(.df$x), min.n = 1)
.dea <- ggplot(data = .df, aes(x = x, y = ..density..)) +
  # Epanechnikov kernel and CV bandwidth
  geom_density(
    kernel = "gaussian",
    bw = "ucv",
    alpha = 0.5,
    aes(color = Estimates, fill = Estimates)
  ) +
  scale_y_continuous(expand = c(0.01, 0)) +
  xlab("Estimated inefficiency") +
  ylab("Estimated Density") +
  labs(colour = "Estimates",
       shape = "Estimates",
       fill = "Estimates") +
  RcmdrPlugin.KMggplot2::theme_simple(base_size = 14, base_family = "sans")
print(.dea)
rm(.df, .nbins)

# Use the functions by Simm and Besstremyannaya (2020)
library(rDEA)
Bootstrap.rDEA = dea.robust(X, Y, W=NULL, model="output", RTS="constant", B
  =2000, alpha=0.05, bw="bw.ucv")
```

Box C.7: R code snippet for kernel density plot

C.2.6 Practical Applications of Two-Stage DEA

The SW07 methods are predominantly applied in the literature, which can be conveniently applied in various mainstream environments using the user-written packages. For example in Stata, the two algorithms in SW07 can be conducted through the **simarwilson** command by (Badunenko and Tauchmann, 2019) with the **teradial** command

```
##### Aggregate #####
# Generate artificial matrix of prices
p <- as.matrix(4)

# Calculate weight
weight = p%*Yt/sum(p%*rowSums(Yt))

# Aggregate
aggregate = sum(dea.crs.out.fear%*t(weight))
aggregate
mean(dea.crs.out.bench)
```

Box C.8: R code snippet for aggregate efficiency

(Badunenko and Mozharovskyi, 2016) in estimating the efficiency scores in prior.¹⁵

In the focused R environment in this monograph, the steps of SW07 can be achieved by incorporating multiple functions. For example, the bias-corrected efficiency score can be estimated with the functions illustrated in the last section. Meanwhile, the bootstrap truncated regression can be conducted with the **truncSP** package by Karlsson and Lindmark (2014). The **treg** function in **FEAR** (Wilson, 2020) is also applicable in the estimation of a truncated regression with MLE. Nevertheless, the second algorithm in SW07, as discussed above, can be directly deployed with the **dea.env.robust** function in the **rDEA** package (Simm and Bestremyannaya, 2020). As illustrated in Box C.9, in our sample, we define a vector of environmental variable as $Z = \{\text{TEACH}, \text{SMALL}, \text{REMOTE}\}$, which exhibits the teaching functions, the size, and the location of the hospitals, respectively. Taking the output-oriented technical efficiency under CRS as an example, by setting $B_1 = 100, B_2 = 2000, \alpha = 0.05$ (the bootstrap time of efficiency and parameter estimation and the significance level), we obtain a result as in Figure C.6.¹⁶ As the Farrell-type technical efficiency is employed, a higher score reflects a lower level of efficiency (further from the frontier). Hence, the negative coefficient of the teaching functions

¹⁵See also the MATLAB code for the two-stage DEA provided by Sickles and Zelenyuk (2019) at <https://sites.google.com/site/productivityefficiency/home>.

¹⁶See more detailed steps of SW07 in Appendix A.

```
##### SW07 #####
attach(data)

# Use the functions by Simm and Besstremyannaya (2020)
library(rDEA)

# Define environmental variables
Z = as.matrix(cbind(TEACH, Small, Remote))

# Output-oriented & CRS
sw07 = dea.env.robust(X, Y, W=NULL, Z, "output", RTS="constant", L1=100, L2
=2000, alpha=0.05)
sw07$beta_hat_hat
sw07$beta_ci
```

Box C.9: R code snippet for SW07

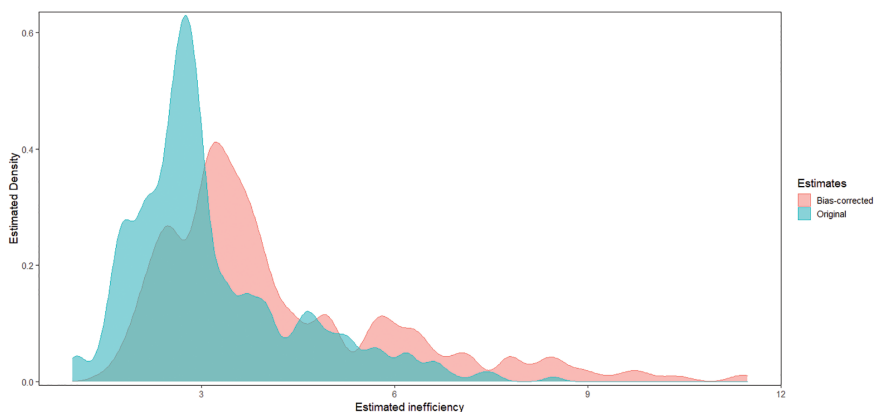


Figure C.5: Estimated kernel densities of the original and bias-corrected efficiency level

indicate that teaching hospitals tend to perform more efficiently, whereas the positive coefficients of the other two environmental variables indicate that hospitals in smaller sizes or in remote areas exhibit a tendency of being estimated as more inefficient. As shown in the confidence interval estimated below, the coefficients are at least significant at a 5% level of significance.

```

> sw07$beta_hat_hat
(Intercept)      TEACH      Small      Remote
  1.1840621 -1.3537605  2.1275255  0.5960517
> sw07$beta_ci
              2.5%      97.5%
(Intercept)  0.15897451  2.4377391
TEACH        -2.36600439 -0.2055785
Small        0.92978447  3.1491399
Remote       0.09072817  1.0893119

```

Figure C.6: Screenshot of the results of SW07 method in R

C.3 Illustrations with Excel Solver

The DEA estimators, e.g., in (2.12), (2.20), (2.23), and (2.29), are essentially LP problems, while the FDH estimator is a mixed $\{0,1\}$ -integer and LP problem. Therefore, the Excel Solver is naturally an alternative option for solving the LP problems due to its accessibility, adaptability, and functional stability. Utilizing the numerical example in Section 2.2.5, the DEA and FDH estimators in (2.30) and (2.31) can be modeled in Excel as illustrated in the left panel from Figure C.7 to Figure C.10.

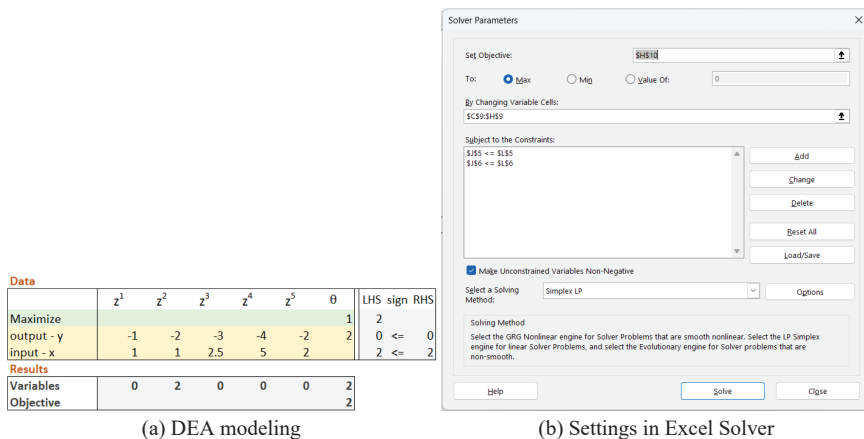


Figure C.7: Illustration of Excel Solver with the numerical sample in Table 2.1 under CRS

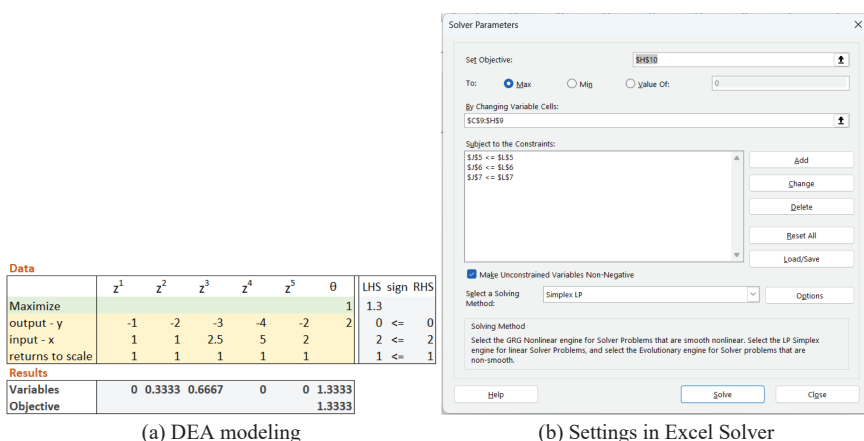


Figure C.8: Illustration of Excel Solver with the numerical sample in Table 2.1 under NIRS

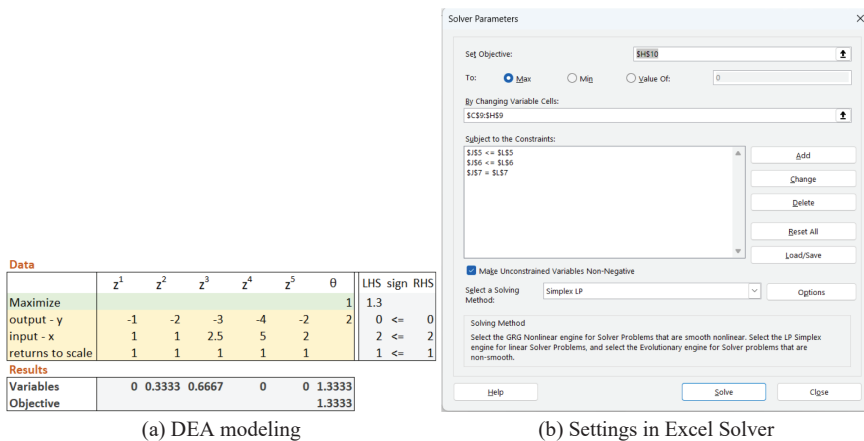


Figure C.9: Illustration of Excel Solver with the numerical sample in Table 2.1 under VRS

Each row presents a constraint of an input/output or the assumption of returns to scale. By changing the intensity variables and the efficiency score, the Excel Solver would optimize the efficiency following the corresponding constraints as illustrated in the right panel from Figure C.7 to Figure C.10.

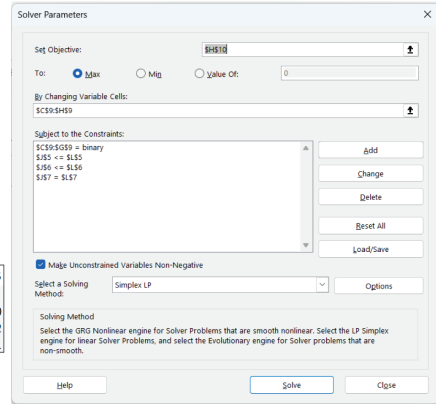
Data

	z^1	z^2	z^3	z^4	z^5	θ	LHS	sign	RHS
Maximize							1		
output - y	-1	-2	-3	-4	-2	2	0	<=	0
input - x	1	1	2.5	5	2		1	<=	2
returns to scale	1	1	1	1	1		1	=	1

Results

Variables	0	1	0	0	0	1
Objective						1

(a) FDH modeling



(b) Settings in Excel Solver

Figure C.10: Illustration of Excel Solver with the numerical sample in Table 2.1 under FDH

As can be seen, the Excel Solver provides the same estimates of the efficiency scores and intensity variables as we demonstrated in Section 2.2.5.

D

Full R Code for the Illustrations

```
#####
####      Illustration of DEA on Queensland hospitals      ####
#####

rm(list=ls())
graphics.off()

##### Data Process #####

# Read data
data <- read.csv("QLD.csv")
names(data)[names(data)=="HOSID"] <- "id"
names(data)[names(data)=="Yeardummy"] <- "Year"

# Convert to panel data
library(plm)
paneldata<- pdata.frame(data, c("id", "Year"))

attach(data)
# Input/Output for "Benchmarking"
X = as.matrix(cbind(BEDS, Agglabours, SUPP))
Y = as.matrix(Aggout)

# Input/Output for "FEAR"
Xt = t(X)
Yt = t(Y)

##### Technical Efficiency #####

attach(data)
```

```

# Use the functions by Bogetoft and Otto (2022)
library(Benchmarking)

# Output-oriented
dea.crs.out.bench = 1/dea(X, Y, RTS="crs", ORIENTATION="out")$eff # CRS
dea.vrs.out.bench = 1/dea(X, Y, RTS="vrs", ORIENTATION="out")$eff # VRS
dea.nirs.out.bench = 1/dea(X, Y, RTS="drs", ORIENTATION="out")$eff # NIRS
dea.fdh.out.bench = 1/dea(X, Y, RTS="fdh", ORIENTATION="out")$eff # FDH

# Input-oriented
dea.crs.in.bench = dea(X, Y, RTS="crs", ORIENTATION="in")$eff
dea.vrs.in.bench = dea(X, Y, RTS="vrs", ORIENTATION="in")$eff
dea.nirs.in.bench = dea(X, Y, RTS="drs", ORIENTATION="in")$eff
dea.fdh.in.bench = dea(X, Y, RTS="fdh", ORIENTATION="in")$eff

# Summarize the estimations
Effi <- as.data.frame(cbind(dea.crs.out.bench, dea.vrs.out.bench,
                           dea.nirs.out.bench, dea.fdh.out.bench,
                           dea.crs.in.bench, dea.vrs.in.bench,
                           dea.nirs.in.bench, dea.fdh.in.bench))
summary(Effi)

##### Cost/Revenue/Profit Efficiency #####

attach(data)

# Generate artificial matrix of prices
w <- t(as.matrix(c(1,2,3)))
p <- as.matrix(4)

# Use the functions by Bogetoft and Otto (2022)
library(Benchmarking)

# Cost efficiency
xopt = cost.opt(X, Y, w, RTS='crs') #CRS
cobs <- X %>% t(w) # Observed Cost
copt <- xopt$x %>% t(w) # Optimal Cost
dea.crs.cost = copt/cobs # cost efficiency

# Revenue efficiency
yopt = revenue.opt(X, Y, p, RTS='crs') #CRS
yobs <- Y %>% p # Observed Revenue
yoct <- yopt$y %>% p # Optimal Revenue
dea.crs.revenue = yobs/yoct # Revenue efficiency

# Profit efficiency
popt = profit.opt(X, Y, w, p, RTS='crs') #CRS
pobs <- Y %>% p -X %>% t(w) # Observed Profit
popt <- popot$y %>% p - popot$x %>% t(w) # Optimal Profit
dea.crs.revenue = pobs/popt # Profit efficiency

# Other RTS assumptions can be analogously applied.

##### MPI #####

```

```

attach(data)

# Use the functions by Bogetoft and Otto (2022)
library(Benchmarking)

# Use period 1 and 2 as an example
X0 <- as.matrix(X[Year==1,])
X1 <- as.matrix(X[Year==2,])

Y0 <- as.matrix(Y[Year==1,])
Y1 <- as.matrix(Y[Year==2,])

ID0 <- as.matrix(id[Year==1])
ID1 <- as.matrix(id[Year==2])

mpi.crs.in = malmq(X0, Y0, ID0, X1, Y1, ID1, RTS = "crs", ORIENTATION = "in")$m
mpi.vrs.in = malmq(X0, Y0, ID0, X1, Y1, ID1, RTS = "vrs", ORIENTATION = "in")$m
mpi.nirs.in = malmq(X0, Y0, ID0, X1, Y1, ID1, RTS = "drs", ORIENTATION = "in")$m
mpi.crs.out = malmq(X0, Y0, ID0, X1, Y1, ID1, RTS = "crs", ORIENTATION = "out")$m
mpi.vrs.out = malmq(X0, Y0, ID0, X1, Y1, ID1, RTS = "vrs", ORIENTATION = "out")$m
mpi.nirs.out = malmq(X0, Y0, ID0, X1, Y1, ID1, RTS = "drs", ORIENTATION = "out")$m

# Summarize the estimations
MPI <- as.data.frame(cbind(mpi.crs.in, mpi.vrs.in,
                           mpi.nirs.in, mpi.crs.out,
                           mpi.vrs.out, mpi.nirs.out))

summary(MPI)

# Estimate with DEA estimators (Fare et al. 1992)
# Same results by manual MPI (when implosion, using: (X2,Y2))
dea00<-dea(X0, Y0, RTS="crs", ORIENTATION="out", XREF=X0, YREF=Y0)$eff
dea11<-dea(X1, Y1, RTS="crs", ORIENTATION="out", XREF=X1, YREF=Y1)$eff
dea10<-dea(X0, Y0, RTS="crs", ORIENTATION="out", XREF=X1, YREF=Y1)$eff
dea01<-dea(X1, Y1, RTS="crs", ORIENTATION="out", XREF=X0, YREF=Y0)$eff

mpi.dea<-sqrt(dea01/dea00*dea11/dea10)
summary(mpi.dea) # Same results as above
summary(mpi.crs.out)

# Heatmap for MPI
IDs = cbind(data$id, data$NetworkID, rep(1,nrow(data)))[1:95,]
X0 <- as.matrix(X[Year==1,])
X1 <- as.matrix(X[Year==2,])
Y0 <- as.matrix(Y[Year==1,])
Y1 <- as.matrix(Y[Year==2,])
ID0 <- as.matrix(id[Year==1])
ID1 <- as.matrix(id[Year==2])
mpi.crs.in.1 = malmq(X0, Y0, ID0, X1, Y1, ID1, RTS = "crs", ORIENTATION = "in")$m
mpi.crs.in.1 = cbind(mpi.crs.in.1, IDs)

```

```

IDs = cbind(data$id, data$NetworkID, rep(2,nrow(data)))[1:95,]
X0 <- as.matrix(X[Year==2,])
X1 <- as.matrix(X[Year==3,])
Y0 <- as.matrix(Y[Year==2,])
Y1 <- as.matrix(Y[Year==3,])
ID0 <- as.matrix(id[Year==2])
ID1 <- as.matrix(id[Year==3])
mpi.crs.in.2 = malmq(X0, Y0, ID0, X1, Y1, ID1, RTS = "crs", ORIENTATION = "
in")$m
mpi.crs.in.2 = cbind(mpi.crs.in.2, IDs)

IDs = cbind(data$id, data$NetworkID, rep(3,nrow(data)))[1:95,]
X0 <- as.matrix(X[Year==3,])
X1 <- as.matrix(X[Year==4,])
Y0 <- as.matrix(Y[Year==3,])
Y1 <- as.matrix(Y[Year==4,])
ID0 <- as.matrix(id[Year==3])
ID1 <- as.matrix(id[Year==4])
mpi.crs.in.3 = malmq(X0, Y0, ID0, X1, Y1, ID1, RTS = "crs", ORIENTATION = "
in")$m
mpi.crs.in.3 = cbind(mpi.crs.in.3, IDs)

mpi.heat = as.data.frame(rbind(mpi.crs.in.1, mpi.crs.in.2, mpi.crs.in.3))
colnames(mpi.heat) <- c("mpi.crs.in", "id", "HHS", "Period")

require("ggplot2")
require("hrbrthemes")
ggplot(mpi.heat, aes(as.character(HHS), Period, fill= mpi.crs.in)) +
  geom_tile() +
  xlab("Local Hospital Networks in Queensland")+
  ylab("Period")+
  scale_fill_distiller(palette = "GnBu")+
  scale_y_discrete(limit = c("12/13-13/14","13/14-14/5","14/15-15/16"))+
  guides(fill=guide_legend(title="MPI (CRS, input-oriented)")+
  theme_bw(base_size = 16)+
  theme(panel.border = element_blank(), panel.grid.major = element_blank(),
        panel.grid.minor = element_blank(), axis.line = element_line(colour
        = "white"))

##### Bias-correction #####

attach(data)

# Use the functions by Wilson (2020)
library(FEAR)

# Bias-corrected efficiency (CRS-Output-oriented)
Bootstrap.fear = boot.sw98(Xt, Yt, NREP = 2000, RTS = 3, ORIENTATION = 2,
  alpha = 0.05, CI.TYPE=2)
cdea = 1/Bootstrap.fear$dhat.bc

# Compare the original and bias-corrected estimates
dea = dea(Xt, Yt, RTS=3, ORIENTATION=2)
cdea = cbind(cdea,rep(1,380))
dea = cbind(dea,rep(0,380))

```

```

correction = as.data.frame(rbind(cdea, dea))
colnames(correction) = c("DEA", "method")

correction$Estimates[correction$method==1]='Bias-corrected'
correction$Estimates[correction$method==0]='Original'

##### Kernel density plot #####

attach(correction)
require("ggplot2")
.df <- na.omit(data.frame(x = correction$DEA))
.nbins <- pretty(range(.df$x), n = nclass.FD(.df$x), min.n = 1)
.dea <- ggplot(data = .df, aes(x = x, y = ..density..)) +
  # Epanechnikov kernel and CV bandwidth
  geom_density(
    kernel = "gaussian",
    bw = "ucv",
    alpha = 0.5,
    aes(color = Estimates, fill = Estimates)
  ) +
  scale_y_continuous(expand = c(0.01, 0)) +
  xlab("Estimated inefficiency") +
  ylab("Estimated Density") +
  labs(colour = "Estimates",
       shape = "Estimates",
       fill = "Estimates") +
  RcmdrPlugin.KMggplot2::theme_simple(base_size = 14, base_family = "sans")
print(.dea)
rm(.df, .nbins)

# Use the functions by Simm and Besstremyannaya (2020)
library(rDEA)
Bootstrap.rDEA = dea.robust(X, Y, W=NULL, model="output", RTS="constant", B
  =2000, alpha=0.05, bw="bw.ucv")

##### Aggregate #####

# Generate artificial matrix of prices
p <- as.matrix(4)

# Calculate weight
weight = p%*%Yt/sum(p%*%rowSums(Yt))

# Aggregate
aggregate = sum(dea.crs.out.bench%*%t(weight))
aggregate
mean(dea.crs.out.bench)

##### SW07 #####

attach(data)

# Use the functions by Simm and Besstremyannaya (2020)
library(rDEA)

```

```
# Define environmental variables
Z = as.matrix(cbind(TEACH, Small, Remote))

# Output-oriented & CRS
sw07 = dea.env.robust(X, Y, W=NULL, Z, "output", RTS="constant", L1=100, L2
=2000, alpha=0.05)
sw07$beta_hat_hat
sw07$beta_ci
```


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