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Nonparametric Measurement of Productivity Growth and Technical Change

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Foundations and Trends® in Econometrics

Published, sold and distributed by:
now Publishers Inc.
PO Box 1024
Hanover, MA 02339
United States
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Outside North America:
now Publishers Inc.
PO Box 179
2600 AD Delft
The Netherlands
Tel. +31-6-51115274

The preferred citation for this publication is

S. C. Ray. *Nonparametric Measurement of Productivity Growth and Technical Change*. Foundations and Trends® in Econometrics, vol. 13, no. 2, pp. 67–169, 2024.

ISBN: 978-1-63828-411-6

Foundations and Trends® in Econometrics
Volume 13, Issue 2, 2024

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Foundations and Trends® in Econometrics, 2024, Volume 13, 4 issues. ISSN paper version 1551-3076. ISSN online version 1551-3084. Also available as a combined paper and online subscription.

Contents

Nonparametric Measurement of Productivity Growth and Technical Change

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ABSTRACT

The three main components of productivity change are technical change, change in technical efficiency, and returns to scale effects. Solow measured productivity change at the macroeconomic level as the difference between the growth rates of output and input which, under constant returns to scale and in the absence of any technical inefficiency, is a measure of technical change. The focus in this monograph is on the individual firm and both technical inefficiency and variable returns to scale are accommodated.

In neoclassical production economics, productivity change can be measured alternatively from the production, cost, profit, or distance functions. In continuous time analysis, one measures the *rates* of productivity and technical change. In discrete time, one measures *indexes* of productivity and technical change over time. This work describes the Tornqvist, Fisher, and Malmquist productivity indexes along with the Luenberger productivity indicator and a Geometric Young index and how they relate to one another. The relevant

Subhash C. Ray (2024), “Nonparametric Measurement of Productivity Growth and Technical Change”, Foundations and Trends® in Econometrics: Vol. 13, No. 2, pp 67–169. DOI: 10.1561/0800000045.

nonparametric DEA models for measuring the different productivity indexes are formulated for nonparametric analysis of productivity, technical change, and change in efficiency.

Keywords: Shephard distance function; directional distance function; data envelopment analysis; Neutral and biased technical change.
JEL Classification Codes: D24; C61.

1

Introduction

Productivity is by far the most widely used and easily understood criterion for comparing performance across firms. In the simplest case of a single output produced from a single input, it is merely the ratio of the output and input quantities. When multiple inputs and/or multiple outputs are involved, one needs aggregated measures of inputs and outputs. Nevertheless, in either case it is a descriptive measure and productivity can be a basis of performance evaluation only in a comparative sense. However, the simple descriptive productivity measure even in the 1-input 1-output case gives rise to a number of follow up questions.

Suppose that the firm produces output y_0 from input x_0. A natural question to ask is whether y_0 is the maximum output that can be produced from x_0. If not, what is the maximum output? A comparison of the actual output (y_0) with the maximum quantity (y_0^*) producible measures the technical efficiency of the firm. Clearly, productivity would increase if the firm produced y_0^* instead of y_0 from the same input x_0.

One may also ask whether productivity would be higher if it used a different input quantity x_1 and produced the corresponding maximum producible output y_1^*. Because there is no inefficiency, any productivity difference would be due to returns to scale effects.

Finally, suppose that the firm is using the same input quantity x_0 in two periods and in both periods, it is producing the maximum producible output from x_0. However, due to technical progress, the maximum producible output from the same input quantity has increased from y_0^* to y_0^{**}. Thus, productivity has increased between the two periods even though there is neither any change in scale nor any change in technical efficiency. In this case, productivity change is solely due to technical change.

In practice, all of these factors – changes in technical efficiency, technical change, and returns to scale effects – contribute to changes in productivity over time. Under appropriate assumptions about the producer's behavior, productivity change over time itself can be measured in alternative ways from the production, cost, profit, or distance function.

An appropriate starting point for any discussion of productivity growth and technical change in the neoclassical production economics framework is the seminal paper by Solow (1957) on technical progress and productivity change. The famous Solow Residual measuring the difference between the rates of growth in output and inputs is interpreted as the rate of technical progress. Solow assumed constant returns to scale, which is quite appropriate in the context of his macroeconomic model. When applied to an individual producer, one needs to allow variable returns to scale. Further, changes in technical efficiency may account in part for a higher or lower rate of growth in output. It is now generally accepted that in addition to technical progress, returns to scale effects of a change in inputs along with changes in technical efficiency may also contribute to the Solow Residual. The principal objective of this monograph is to explain how to isolate technical progress, scale effects, and efficiency change as three distinct components of productivity change measured empirically using the nonparametric method of Data Envelopment Analysis (DEA).[1]

[1]This monograph builds upon and extends Ray (2022). A comparable analysis of productivity change using the parametric method of Stochastic Frontier Analysis (SFA) may be found in Kumbhakar (2022). Zelenyuk (2023) focuses more on the

The rest of the monograph unfolds as follows. Section 2 provides a brief theoretical background starting from the production possibility set, lists the basic assumptions about the reference technology, defines the Shephard output and input distance functions (along with the corresponding Farrell technical efficiencies), and technical change as shifts in the frontier of the production possibility set. Section 3 shows how total factor productivity can be measured and decomposed into technical change, technical efficiency change, and scale effect alternatively from a parametrically specified production, cost, profit, or distance function. Section 4 explains the nonparametric method of Data Envelopment Analysis and formulates the appropriate models for measuring input- or output-oriented technical efficiency, cost efficiency, and profit efficiency. The non-convex method of Free Disposal Hull (FDH) analysis is also briefly explained. Section 5 considers productivity change in discrete time. The Malmquist index, which is a ratio of distance functions, is contrasted with the descriptive measures of total factor productivity change like Tornqvist and Fisher indexes (which can be directly computed from data without solving any optimization problem).

Alternative multiplicative decompositions of the Malmquist productivity index into factors measuring technical change, technical efficiency change, and scale efficiency change are explained and the corresponding DEA optimization problems are formulated. A comparable decomposition of the Fisher Productivity index is also provided. Non-neutrality of technical change and output, input, and scale bias are explained. A Geometric Young index of multi-factor productivity measured by the ratio of geometric distance functions and its decomposition into efficiency change and technical change is explained. This section ends with a discussion of the directional distance function, and the Luenberger productivity indicator. Section 6 explains the relation between alternative productivity indexes as well as the Luenberger productivity indicator. Finally, Section 7 concludes with a summing up and also an acknowledgement of a number of important topics related to nonparametric measurement of productivity change not covered in this

theoretical underpinnings of measurement of total factor productivity and labor productivity.

monograph. These include (but are not limited to) explicit accommodation of random noise in the DEA models, accounting for bad outputs (either as joint products or by-products), and aggregation of firm-level measures of productivity change for comparison across groups.

Appendices

Appendix A

If the output aggregation weights are some fixed value shares ($\alpha_r, r = 1, 2, \ldots, m$) and the input aggregation weights are fixed cost shares ($\beta_i{}'i = 1, 2, \ldots, n$), the TFP-GDF of PT(2006) become a Geometric Young index. Although the discussion is easily generalizable to m outputs and n inputs, the 2-input 2-output case is more helpful for simplicity of exposition.

Let $(x^0, y^0) = (x_{10}, x_{20}; y_{10}, y_{20})$ and $(x^1, y^1) = (x_{11}, x_{21}; y_{11}, y_{21})$ be the input-output bundles of the same firm in periods 0 and 1. Further, let the GDFs for (x^0, y^0) for the period 0 and period 1 reference technologies ($T^0$and T^1) be

$$G^0(x^0, y^0) = \frac{(\theta_{10}^0)^{\beta_1}(\theta_{20}^0)^{\beta_2}}{(\varphi_{10}^0)^{\alpha_1}(\varphi_{20}^0)^{\alpha_2}} \tag{A.1}$$

and

$$G^1(x^0, y^0) = \frac{(\theta_{10}^1)^{\beta_1}(\theta_{20}^1)^{\beta_2}}{(\varphi_{10}^1)^{\alpha_1}(\varphi_{20}^1)^{\alpha_2}}. \tag{A.2}$$

Similarly, let the GDFs for (x^1, y^1) be

$$G^0(x^1, y^1) = \frac{(\theta_{11}^0)^{\beta_1}(\theta_{21}^0)^{\beta_2}}{(\varphi_{11}^0)^{\alpha_1}(\varphi_{21}^0)^{\alpha_2}} \tag{A.3}$$

and

$$G^1(x^1, y^1) = \frac{(\theta_{11}^1)^{\beta_1}(\theta_{21}^1)^{\beta_2}}{(\varphi_{11}^1)^{\alpha_1}(\varphi_{21}^1)^{\alpha_2}}. \tag{A.4}$$

Now define

$$(x^{*0}, y^{*0}) = (\theta_{10}^0 x_{10}, \theta_{20}^0 x_{20}; \varphi_{10}^0 y_{10}, \varphi_{20}^0 y_{20}). \tag{A.5}$$

Then,

$$G^0(x^0, y^0) = \frac{(\theta_{10}^0)^{\beta_1}(\theta_{20}^0)^{\beta_2}}{(\varphi_{10}^0)^{\alpha_1}(\varphi_{20}^0)^{\alpha_2}} = \frac{(\frac{x_{10}^*}{x_{10}})^{\beta_1}\left(\frac{x_{20}^*}{x_{20}}\right)^{\beta_2}}{(\frac{y_{10}^*}{y_{10}})^{\alpha_1}(\frac{y_{20}^*}{y_{20}})^{\alpha_2}} = \frac{(\frac{y_{10}}{y_{10}^*})^{\alpha_1}(\frac{y_{20}}{y_{20}^*})^{\alpha_2}}{(\frac{x_{10}}{x_{10}^*})^{\beta_1}(\frac{x_{20}}{x_{20}^*})^{\beta_2}} \tag{A.6}$$

Also, for

$$(x^{*1}, y^{*1}) = (\theta_{11}^0 x_{11}, \theta_{21}^0 x_{21}; \varphi_{11}^0 y_{11}, \varphi_{21}^0 y_{21}), \tag{A.7}$$

$$G^0(x^1, y^1) = \frac{(\theta_{11}^0)^{\beta_1}(\theta_{21}^0)^{\beta_2}}{(\varphi_{11}^0)^{\alpha_1}(\varphi_{21}^0)^{\alpha_2}} = \frac{(\frac{x_{11}^*}{x_{11}})^{\beta_1}(\frac{x_{21}^*}{x_{21}})^{\beta_2}}{(\frac{y_{11}^*}{y_{11}})^{\alpha_1}(\frac{y_{21}^*}{y_{21}})^{\alpha_2}} = \frac{(\frac{y_{11}}{y_{11}^*})^{\alpha_1}(\frac{y_{21}}{y_{21}^*})^{\alpha_2}}{(\frac{x_{11}}{x_{11}^*})^{\beta_1}(\frac{x_{21}}{x_{21}^*})^{\beta_2}}. \tag{A.8}$$

Hence,

$$\frac{G^0(x^1, y^1)}{G^0(x^0, y^0)} = \left[\frac{(\frac{y_{11}}{y_{10}})^{\alpha_1}(\frac{y_{21}}{y_{20}})^{\alpha_2}}{(\frac{x_{11}}{x_{10}})^{\beta_1}(\frac{x_{21}}{x_{20}})^{\beta_2}}\right]\left[\frac{(\frac{x_{11}^*}{x_{10}^*})^{\beta_1}(\frac{x_{21}^*}{x_{20}^*})^{\beta_2}}{(\frac{y_{11}^*}{y_{10}^*})^{\alpha_1}(\frac{y_{21}^*}{y_{20}^*})^{\alpha_2}}\right]. \tag{A.9}$$

Next, we show that

$$\frac{(\frac{x_{11}^*}{x_{10}^*})^{\beta_1}(\frac{x_{21}^*}{x_{20}^*})^{\beta_2}}{(\frac{y_{11}^*}{y_{10}^*})^{\alpha_1}(\frac{y_{21}^*}{y_{20}^*})^{\alpha_2}} = 1.$$

Suppose this is not true and assume arbitrarily that.

$$\left(\frac{x_{11}^*}{x_{10}^*}\right)^{\beta_1}\left(\frac{x_{21}^*}{x_{20}^*}\right)^{\beta_2} < \left(\frac{y_{11}^*}{y_{10}^*}\right)^{\alpha_1}\left(\frac{y_{21}^*}{y_{20}^*}\right)^{\alpha_2}.$$

At this point, define,

$$\psi_{11}^0 = \frac{x_{11}^*}{x_{10}}, \quad \psi_{21}^0 = \frac{x_{21}^*}{x_{20}}, \quad \kappa_{11}^0 = \frac{y_{11}^*}{y_{10}}, \quad \text{and} \quad \kappa_{21}^0 = \frac{y_{21}^*}{y_{20}}.$$

Then, $\frac{(\psi_{10}^0)^{\beta_1}(\psi_{20}^0)^{\beta_2}}{(\kappa_{10}^0)^{\alpha_1}(\kappa_{20}^0)^{\alpha_2}} < \frac{(\theta_{10}^0)^{\beta_1}(\theta_{20}^0)^{\beta_2}}{(\varphi_{10}^0)^{\alpha_1}(\varphi_{20}^0)^{\alpha_2}}$ and $\frac{(\theta_{10}^0)^{\beta_1}(\theta_{20}^0)^{\beta_2}}{(\varphi_{10}^0)^{\alpha_1}(\varphi_{20}^0)^{\alpha_2}}$ could not be the $GDF^0(x^0, y^0)$.

This proves that

$$\frac{(\frac{x_{11}^*}{x_{10}^*})^{\beta_1}(\frac{x_{21}^*}{x_{20}^*})^{\beta_2}}{(\frac{y_{11}^*}{y_{10}^*})^{\alpha_1}(\frac{y_{21}^*}{y_{20}^*})^{\alpha_2}} = 1. \tag{A.10}$$

Appendix B

For simplicity, we show this with the 2-input 2-output example although the result holds for any number of outputs and inputs. Consider two firms A and B. Suppose that their input-output bundles in period 0 are $(x_A^0, y_A^0) = (x_{1A}^0, x_{2A}^0; y_{1A}^0, y_{1A}^0)$ and $(x_B^0, y_B^0) = (x_{1B}^0, x_{2B}^0; y_{1B}^0, y_{1B}^0)$.

Further, let their corresponding period-0 GDF-efficient projections be

$$(x_A^{*0}, y_A^{*0}) = (\theta_{1A}^0 x_{1A}^0, \theta_{2A}^0 x_{2A}^0; \varphi_{1A}^0 y_{1A}^0, \varphi_{2A}^0 y_{2A}^0)$$

and

$$(x_B^{*0}, y_B^{*0}) = (\theta_{1B}^0 x_{1B}^0, \theta_{2B}^0 x_{2B}^0; \varphi_{1B}^0 y_{1B}^0, \varphi_{2B}^0 y_{2B}^0)$$

Then,

$$G^0(x_A^0, y_A^0) = \frac{(\theta_{1A}^0)^{\beta_1}(\theta_{2A}^0)^{\beta_2}}{(\varphi_{1A}^0)^{\alpha_1}(\varphi_{2A}^0)^{\alpha_2}} = \frac{(\frac{x_{1A}^{*0}}{x_{1A}^0})^{\beta_1}(\frac{x_{2A}^{*0}}{x_{2A}^0})^{\beta_2}}{(\frac{y_{1A}^{*0}}{y_{1A}^0})^{\alpha_1}(\frac{y_{2A}^{*0}}{y_{2A}^0})^{\alpha_2}} \tag{B.1}$$

and

$$G^0(x_B^0, y_B^0) = \frac{(\theta_{1B}^0)^{\beta_1}(\theta_{2B}^0)^{\beta_2}}{(\varphi_{1B}^0)^{\alpha_1}(\varphi_{2B}^0)^{\alpha_2}} = \frac{(\frac{x_{1B}^{*0}}{x_{1B}^0})^{\beta_1}(\frac{x_{2B}^{*0}}{x_{2B}^0})^{\beta_2}}{(\frac{y_{1B}^{*0}}{y_{1B}^0})^{\alpha_1}(\frac{y_{2B}^{*0}}{y_{2B}^0})^{\alpha_2}} \tag{B.2}$$

In an analogous way, we consider the period-1 GDF-efficient projections of the same two input bundles as

$$(x_A^{*1}, y_A^{*1}) = (\theta_{1A}^1 x_{1A}^0, \theta_{2A}^1 x_{2A}^1; \varphi_{1A}^1 y_{1A}^1, \varphi_{2A}^1 y_{2A}^1)$$

and

$$(x_B^{*1}, y_B^{*1}) = (\theta_{1B}^1 x_{1B}^0, \theta_{2B}^1 x_{2B}^0; \varphi_{1B}^1 y_{1B}^0, \varphi_{2B}^1 y_{2B}^0)$$

Then,

$$G^1(x_A^0, y_A^0) = \frac{(\theta_{1A}^1)^{\beta_1}(\theta_{2A}^1)^{\beta_2}}{(\varphi_{1A}^1)^{\alpha_1}(\varphi_{2A}^1)^{\alpha_2}} = \frac{(\frac{x_{1A}^{*1}}{x_{1A}^0})^{\beta_1}(\frac{x_{2A}^{*1}}{x_{2A}^0})^{\beta_2}}{(\frac{y_{1A}^{*1}}{y_{1A}^0})^{\alpha_1}(\frac{y_{2A}^{*1}}{y_{2A}^0})^{\alpha_2}} \tag{B.3}$$

and

$$G^1(x_B^0, y_B^0) = \frac{(\theta_{1B}^1)^{\beta_1}(\theta_{2B}^1)^{\beta_2}}{(\varphi_{1B}^1)^{\alpha_1}(\varphi_{2B}^1)^{\alpha_2}} = \frac{(\frac{x_{1B}^{*1}}{x_{1B}^0})^{\beta_1}(\frac{x_{2B}^{*1}}{x_{2B}^0})^{\beta_2}}{(\frac{y_{1B}^{*1}}{y_{1B}^1})^{\alpha_1}(\frac{y_{2B}^{*1}}{y_{2B}^1})^{\alpha_2}} \tag{B.4}$$

Therefore,

$$\frac{G^1(x_A^0, y_A^0)}{G^0(x_A^0, y_A^0)} = \frac{\left(\frac{x_{1A}^{*1}}{x_{1A}^{*0}}\right)^{\beta_1}\left(\frac{x_{2A}^{*1}}{x*_{2A}^0}\right)^{\beta_2}}{\left(\frac{y_{1A}^{*1}}{y_{1A}^{*0}}\right)^{\alpha_1}\left(\frac{y_{2A}^{*1}}{y_{2A}^{*0}}\right)^{\alpha_2}} \tag{B.5}$$

and

$$\frac{G^1(x_B^0, y_B^0)}{G^0(x_b^0, y_b^0)} = \frac{(\frac{x_{1b}^{*1}}{x_{1b}^{*0}})^{\beta_1}(\frac{x_{2B}^{*1}}{x*_{2B}^0})^{\beta_2}}{(\frac{y_{1B}^{*1}}{y_{1B}^{*0}})^{\alpha_1}(\frac{y_{2B}^{*1}}{y_{2B}^{*0}})^{\alpha_2}} \tag{B.6}$$

Finally, by virtue of (A.10)

$$\frac{G^1(x_A^0, y_B^0)}{G^0(x_A^0, y_A^0)} = \frac{G^1(x_B^0, y_B^0)}{G^0(x_B^0, y_B^0)} \tag{B.7}$$

Similarly, for the input-output bundles from period 1,

$$\frac{G^1(x_A^1, y_B^1)}{G^0(x_A^1, y_A^1)} = \frac{G^1(x_B^1, y_B^1)}{G^0(x_B^1, y_B^1)} \tag{B.8}$$

This shows that the technical change measure between the periods 0 and 1 will be identical for both firms A and B.

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