Rivers and Electric Networks: Crossing Disciplines in Modeling and Simulation

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Contents

Abstract

Electric circuits and river networks share similarities in both their network structure and derivation from conservation principals. However, the disciplines have evolved separately and developed methods and models. This paper presents the foundations for network analysis for both disciplines and shows how numerical methods developed for circuit simulations can significantly improve river network models. The equations, models, and jargon are described, providing a reference for future studies to transfer knowledge across disciplinary boundaries.

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1

Introduction

1.1 Crossing disciplines is *not* always interdisciplinary

Interdisciplinary studies typically occur at the boundary between closely related areas, for example VLSI(Very Large Scale Integration) design meets at the boundaries of material science, electrical engineering, and computer science; similarly, water quality modeling depends on chemistry, biology, and physical transport processes. The importance of breaking through disciplinary isolation has been well-recognized for the purpose of addressing such problems. Less recognized are the opportunities to apply established ideas from one discipline in another, seemingly unrelated, discipline. Indeed, there is a significant challenge here: how do you move ideas across a boundary when there is no problem at the boundary and no link between the disciplines? We might call this the "cross-disciplinary" problem. In the present work we are interested in river networks – at the largest scale covering continents – that do not share any interdisciplinary boundaries with microscopic electronic circuits in semiconductors. The only connection is that both are network problems.

As opposed to interdisciplinary knowledge transfer, cross-disciplinary transfer tends to be serendipitous rather than organized, typically an unexpected result of curiosity. Indeed, unlike interdisciplinary studies, there do not appear to be any formal programs for fostering cross-disciplinary knowledge transfer. It seems that "thinking outside the box" mostly means working at the boundaries of your box, rather than stepping into another completely different box. Cross-disciplinary transfer can also be a one-way affair; i.e. the methods/ideas from one discipline may provide immediate improvement or advancement in another, but reverse transfers might be obscure or entirely non-existent. From the authors' experience there are three key challenges to crossing disciplines: (1) identifying the opportunities where one discipline has advances that might be useful to another, (2) communicating across the jargon-laced literature of different disciplines, and (3) clearly articulating and demonstrating the benefits and value to broader research communities.

To date, our efforts in crossing disciplines have been one-way: using insights from electric circuits to improve river models. Because of nonlinear complexities in governing equations, the development path for river modeling diverged from that of electric circuit modeling. In river modeling, approximations were made to fit computer and numerical capabilities in the 1970s and 1980s and were never significantly revisited – despite increasing computer power and improved numerical methods. Numerical methods developed in the 1990s and 2000s to solve microchip circuit simulation problems in VLSI simply never reached river network modelers. The underlying cause is likely the disparity in commercial markets, which are limited for river models and give little encouragement to investing in model updates; indeed, some of the most commonly used models are built on numerical frameworks developed in government-funded projects in the 1970s. Remedying these problems was the focus of [52], which provided a cross-disciplinary experience that inspired the present paper. Although we are writing from the foundations of our electric-to-river experience, it is hoped we might inspire future synergies that cross boundaries back in the other direction.

This paper provides the foundations for communication between electrical and hydraulic engineers/scientists who work on network problems. The audience is intended to be from both disciplines, so we endeavor to explain the physics of both systems and draw out their similarities and differences. In the process, we present an example of how numerical methods from microchip circuit analysis have been used in modeling a river network to a level of detail that previously was not possible. We are not attempting to provide a rigorous survey of all the work in these two areas as the background literature is simply too vast; the citations herein should be considered examples rather than exhaustive. Indeed, our focus is on putting forward what are mostly textbook ideas in a format that is accessible across boundaries.

Our introduction begins with an overview of the key similarities and differences between rivers and electric circuits (§1.2 – 1.4), which introduces the reader to the jargon and relationships between disciplines. Interestingly, because today's general physics curriculum typically introduces electric circuits but not open-channel flow, hydraulic engineers might have a better foundation in electrical jargon than vice-versa. Unfortunately, a major impediment to quickly understanding hydraulics is that the jargon is rooted in history. Because civil engineering projects are long-lived and costly the discipline is slow to change; there are curiosities in the jargon and equations that must be accepted simply as echoes of history. With this problem in mind, we provide a short introduction on how river hydraulics developed (§1.5), which will help the electrical engineering reader understand not just what the hydraulic equation are, but why they take their peculiar forms.

In keeping with our goal of introducing the disciplines across boundaries, beyond this introduction the reader will find a brief review of electric theory (Chapter 2), which is intended as both a primer for hydraulic engineers and to provide analogies between circuits and rivers to guide electrical engineers through the in-depth discussion of the governing equations for river flow in a single channel (Chapter 3) and the peculiarities of river networks (Chapter 4). These sections on theory are followed by a discussion of numerical approaches that have been applied crossing disciplines (Chapter 5). We close with some final thoughts on the similarities and contrasts between these disciplines, and possible areas for future work (Chapter 6).

1.2 Networks

The common feature of large-scale river networks, e.g. Fig. 1.1, and VLSI networks, e.g. Fig. 1.2, is the complexity of their interconnections. At the most basic level, river networks can be treated as directed acyclic graphs (DAG) connecting tributary junctions with differential equations representing the evolution of water surface elevation and flow rate. River systems are inherently acyclic because water must flow downhill; i.e. without a pump it is impossible for water to return to an uphill point to form a cyclic graph. In contrast, the network structure for a VLSI can include cyclic connections. Furthermore, rivers are *mostly* simple tree structures as upstream tributary branches join to downstream main-stem channels in the classic root-branch pattern (Fig. 1.1). However, important exceptions occur – particularly where water flows around a mid-channel island (Figs. 1.3, 1.4), where man-made canals create acyclic paths (Fig. 1.5), or the distributaries where a river debouches into the ocean in a network of channels that branch and join through a complex maze of marshes (Fig. 1.6).

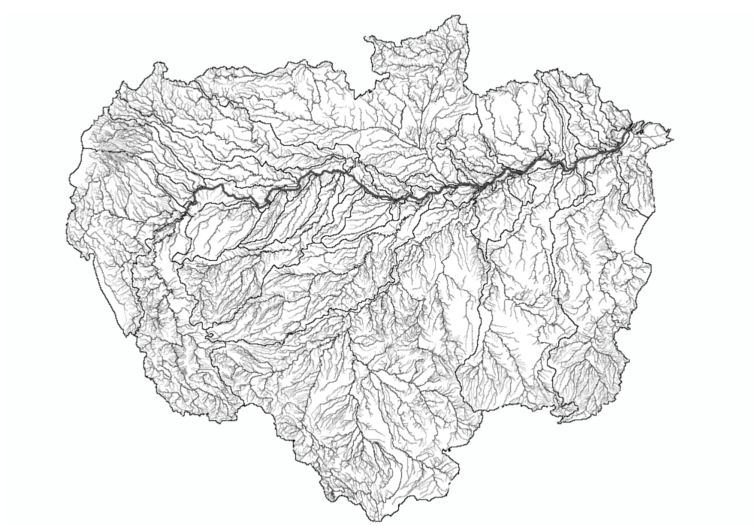

Figure 1.1: Amazon River basin. URL:
http://daac.ornl.gov/LBA/guides/CD06_CAMREX.html

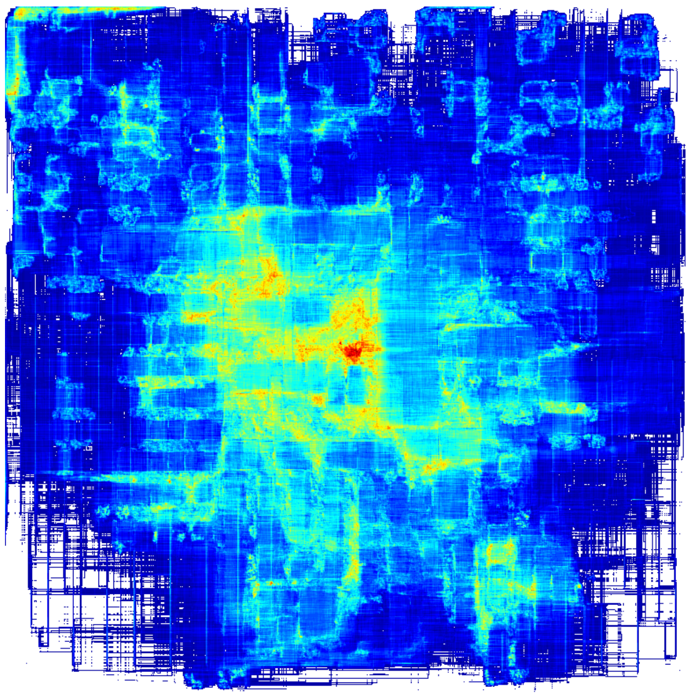

Figure 1.2: Connection network of a VLSI chip. URL: http://vlsicad.eecs.umich.edu/BK/FGR/. Reproduced with permission

Figure 1.3: Charley River at Yukon showing mid-channel islands (Photo by USGS) URL: http://ak.water.usgs.gov/yukon/index.php

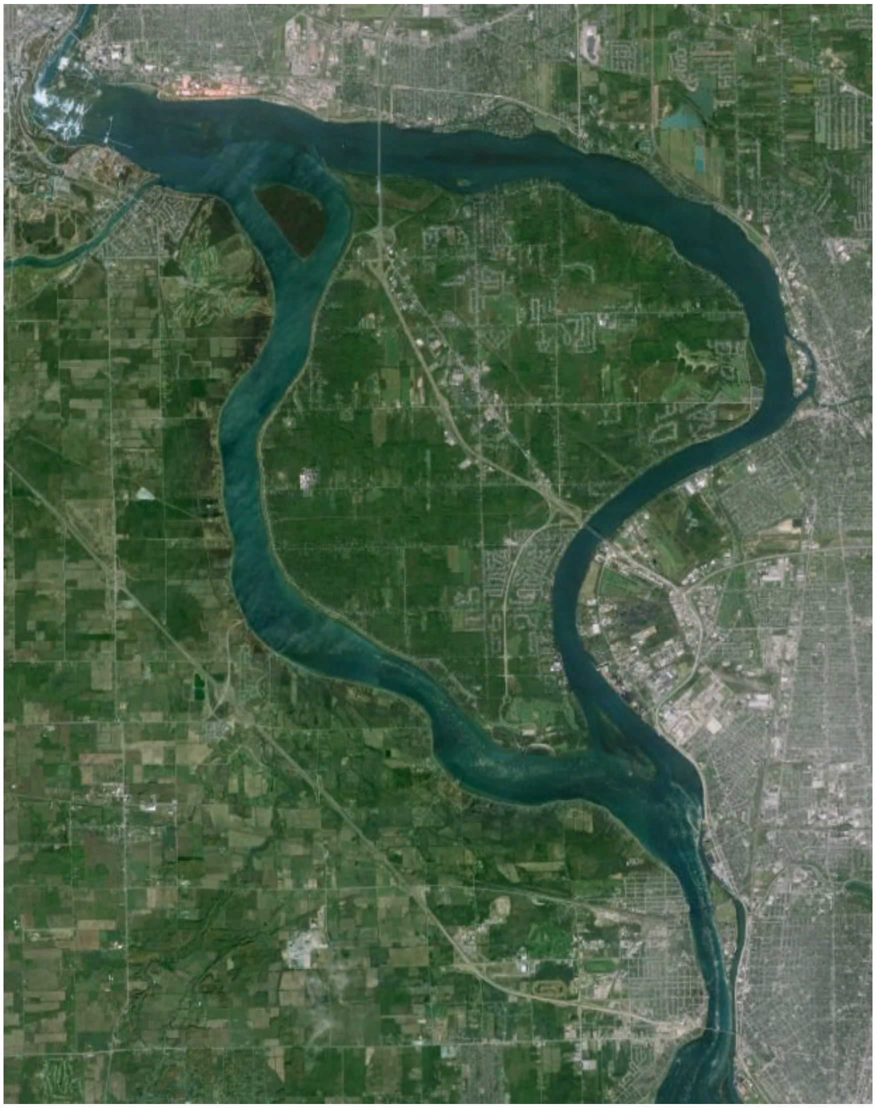

Figure 1.4: Niagara River and Grand Island from Lake Erie (south) to Niagara Falls (USA/Canada) illustrating river split and rejoining. GoogleEarth, image ©USGS and DigitalGlobe

Figure 1.5: Atchafalaya River and canals near Morgan City (Louisiana, USA). GoogleEarth, image ©TerraMetrics

Figure 1.6: Wax Lake Delta (Louisiana, USA). GoogleEarth, image ©TerraMetrics

1.3 Notation

Cross-disciplinary research in any area is particularly challenging due to the difference in jargon and notation. To facilitate the readers from diverse backgrounds, Table 1.1 provides some of the common variables in river hydraulics, and Table 1.2 provides the same for electric circuits.

Symbol	Meaning	Units
β	velocity non-uniformity coefficient	–
η	free surface elevation	m
ρ	density	kg/m^3
ν	kinematic viscosity	m^2/s
σ	cross-section breadth	m^2/s
a	wetted cross-sectional area	m^2
g	gravity	m/s^2
h	depth	m
L	length	m
n	Manning's n	see §1.5.2
p	pressure	Pa
P	time-averaged pressure	Pa
q	inflow rate per unit length	m^2/s
Q	volume flow rate	m^3/s
R_h	hydraulic radius	m
S	water surface slope	–
S_f	friction slope	–
S_0	slope of channel bottom	–
u	flow velocity	m/s
U	time-averaged flow velocity	m/s
z	vertical elevation	–

Table 1.1: Common variables in river hydraulics

Symbol	Meaning	Unit
G	conductance	siemens (S)
I	electric current	ampere (A)
L	inductance	henry (H)
R	resistance	ohm (Ω)
Q	electric charge	coulomb (C)
V	voltage	volt (V)
σ	conductivity	$\mathrm{S} \cdot \mathrm{m}^{-1}$

Table 1.2: Common variables in electric circuits

1.4 Rivers and electric circuits

River networks develop because water flows downhill; more precisely, the potential energy difference between higher and lower elevations causes water to seek a lower energy state. The elevation gradients of a river bed give rise to hydrostatic pressure differences that are effectively the voltages driving the river current. However as the landscape elevations cannot be reversed in the same way as an electric potential, a river flow is (for the most part) a unidirectional process, which makes river networks topologically simpler than electric circuits. With a clear path in an open channel, water seeks the lower potential energy state at a flow rate balancing the driving pressure gradient against frictional resistance. Dynamic river network models represent the flow rate and water surface elevation as functions of space and time.

The fundamental analogy for our work is that the water flow rate Q, in m^2s^{-1}, is similar to electric current (I); the local water surface gradient $\partial\eta/\partial x$, provides a driving pressure gradient similar to a voltage between two terminals V; and water flow, like charge flow, is moderated by resistance. To illustrate this point, consider a simple resistive conductor versus a simple river channel, as shown in Fig. 1.7. For this conceptual model, the current (I) is caused by the voltage (or potential difference in the electric field) between two terminals of the conductor and is moderated by the resistance (R) in Ohm's law as

$I = V/R$. Similarly, for the river channel the water flow is caused by the potential difference in earth's gravitational field associated with the different water elevations ($\Delta\eta$), and moderated by frictional resistance $\mathcal{R}$. This idea can be written as an idealized energy law in the form $Q^2 = g\Delta\eta/\mathcal{R}$, where g is gravitational acceleration.

Except in the superconductivity state, all conductors have resistivity. The resistance to charge flow (or its inverse, conductance) is a function of the cross-section area, material, and the element length. Similarly, the water flow resistance of some section of river channel depends on its physical characteristics: the cross-section area, the length of "wetted" perimeter where water contacts the river bed, the material of the river bed (e.g. sand, gravel, or boulders), the turbulence in the flow, and how all these physical characteristics change over the length.

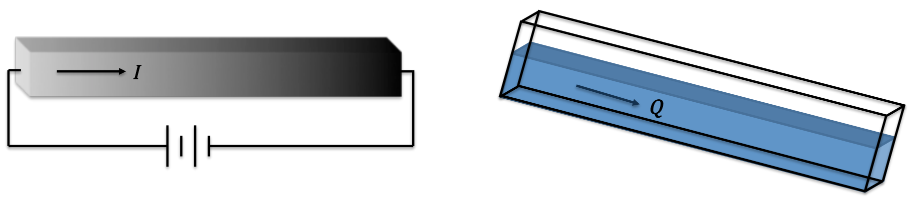

Figure 1.7: Current flow in an ideal resistor with rectangular cross section versus water flow in a rectangular channel

This simplistic river-circuit analogy is useful for an intuitive understanding of relationships between the disciplines. However, for quantitative analysis of more realistic problems, we must face several confounding factors. Firstly, unlike electrons, water has significant mass and therefore inertia. Inertia provides flow "memory," such that the recent past affects the near future flow evolution. As a result, simple models using proportional and instantaneous response to changing conditions (which are common in electric circuits) are poor representations of flow physics. Inertia is governed by Newton's second law, $\sum F = ma$, so sudden introduction of an adverse pressure gradient (the equivalent of reversing V) requires time and distance to slow, stop and reverse the flow. The equivalent behavior in an electric circuit requires a component in which $\partial I/\partial t = f(I, V)$, where f may be a nonlinear in the independent variables. To the first order, this behavior is similar to an inductive element in a circuit, albeit more complicated since the underlying function can be nonlinear. This topic is discussed more fully in §2.4.

A second confounding factor is the complicated relationship between the water surface elevation and the driving pressure gradient in a river. To the first order, the pressure gradient is proportional to the gradient of the water surface elevation, i.e. $\partial\eta/\partial x$, where η is the vertical distance of the water surface from a $z = 0$ baseline. To make another river-circuit analogy, we can imagine a circuit where a local voltage V is generated as a gradient of some auxiliary variable (η), which is itself a function of current I and another auxiliary variable representing effects of the conductor's geometry (Γ). As another layer of complication, a change in water elevation will change a river's cross-sectional area, so to continue the analogy we imagine an electric conductor that also changes geometry Γ as a function of auxiliary variable η.

A third confounding factor is the relationship between resistance and flow in a river. The frictional resistance of a river is a strong function of both flow rate and geometry, whereas resistance in an electric conductor is strongly affected by geometry but only weakly affected by current – the latter typically through changes in material temperature when carrying large currents. Therefore, our river-circuit analogy requires a resistance R that is a function of both I and Γ.

To summarize, an electric circuit that has functional dependencies analogous to a river requires:

$$\Gamma = f_1\left(\eta\right) \tag{1.1}$$

$$V = f_2\left(\eta, I, \Gamma\right) \tag{1.2}$$

$$R = f_3\left(I, \Gamma\right) \tag{1.3}$$

$$I = f_4\left(V, R, \Gamma\right) \tag{1.4}$$

where V, R, and I are the traditional electric circuit variables, Γ is geometry, and η is an auxiliary function that couples geometry and potential. For rivers, the equivalent functions are typically nonlinear and require empirical coefficients that vary in different rivers – and often vary in different reaches of a single river. In contrast, simple electric circuit relationships typically follow:

$$I = \frac{V}{R} \tag{1.5}$$

$$R = f(\Gamma) \tag{1.6}$$

$$\Gamma = \text{constant.} \tag{1.7}$$

where empirical coefficients are generally associated with material properties. Thus, key differences between electric circuits and river networks are in the complicated nonlinearities and dependencies between flow, the driving potential, and geometry associated with rivers.

The above analogy glosses over the central problem of fluid mechanics: uncertainties associated with turbulence. The functional dependence of turbulence on both flow rate and geometry adds empirical complexity to quantifying frictional losses (i.e. the R in an analogous electric circuit). In a river, the resistance depends on both the shape of the channel and the bed material in contact with the flow. Typically, we have incomplete data on both river geometry and bed material, which makes it difficult to parameterize flow resistance. Detailed bathymetric surveys are expensive and, to date, the only remote sensing technique that can directly measure river geometry is blue-green lidar, which requires clear water for effectiveness [40]. Furthermore, river channels are continually undergoing both erosion and aggradation, so even a perfect survey has only ephemeral validity. From satellite photogrammetry it is possible to define river widths and monitor water level changes with time [2, 31], but routinely quantifying the shape and composition of the channel bottom for a continental-scale network of more than 10^6 km of river requires surveying resources that are simply beyond practical capabilities. Surprisingly, lack of adequate data is also a problem in semiconductor circuits, where the average width of the conducting wires is less one micrometer (10^{-6} m). However, this particular issue is partially addressed by macromodeling in electric circuit analysis, which we revisit in §2.6.

In summary, river networks are topologically simpler than electric circuit networks, but are governed by coupled nonlinear equations that are more complicated than the simplest electrical relationships. However, the more sophisticated devices that are common in semiconductor circuitry allow a more complete, albeit more complicated, analogy between systems (see Chapter 2).

1.5 The development of river hydraulics

1.5.1 A brief history

Organized hydraulics can be traced back to the digging of irrigation channels in pre-historic Mesopotamia. As cities grew throughout the ancient world, the need for effective water supply, sanitation, and transportation resulted in dominant cities appearing on major rivers. For millennia, hydraulics principally advanced through slow accumulation of empirical knowledge, whose apex is arguably seen in the viaducts of the Roman Empire. An engineer from that era would have been unsurprised by the hydraulics of medieval Europe. This slow advance changed in the 18th century with the widespread dissemination of the calculus along with theoretical and experimental advances in fluid mechanics. A short history of the foundations of modern hydraulics from this era can be found in [24], from which we will extract some high points to provide context for some of the curious equation forms that are still used in river engineering.

Arguably, modern hydraulics begins with Antoine Chézy (1718-1798) and Pierre-Georges-Louis du Buat, who observed a relationship between flow velocity (u) and the channel slope (S) now known as the Chézy equation: $u = C\sqrt{Sa/P_w}$, where a is the cross-sectional area, P_w is the wetted perimeter of the channel (i.e. where water contacts the bed, see Fig. 1.8.), and C is an empirical coefficient that varies in different rivers, now known as the Chézy coefficient. Note that the C is a "conveyance" coefficient in that increasing values result in increasing velocity (and hence flow rate for a given a). Most modern hydraulic equations use a "roughness" coefficient – essentially an inversion of C – such that increasing values cause decreasing flow rates. Roughness and conveyance are similar to inverse concepts of resistivity and conductivity in electric circuits. In later notation, the "hydraulic radius"

(R_h) was introduced into the Chézy equation as

$$R_h \equiv \frac{a}{P_w} \tag{1.8}$$

so $u = C\sqrt{R_h S}$. Although the 18^{th} century Chézy equation is rarely used today, the hydraulic radius is still encountered as a key geometry definition in many 20^{th} and 21^{st} century hydraulic models.

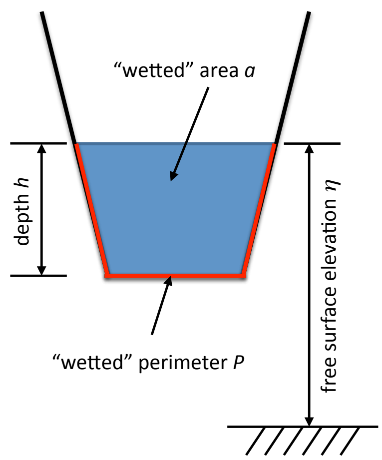

Figure 1.8: Illustrative example of a trapezoidal cross section, as often used by hydraulic engineers in viaducts for water transport. A natural river cross section is much more complicated.

In the 19^{th} and early 20^{th} centuries, the Chézy approach was modified and eventually supplanted by Manning's equation (§1.5.2) for steady flow, while on a separate path Newton's equations of motion were developed into the three-dimensional (3D) Navier-Stokes equations (§3.2) and the 1D Saint-Venant equations (§1.5.3).

1.5.2 Chezy-Manning equation

In the mid 1800s, experimentalists Darcy and Bazin provided data from which Phillippe Gauckler (1826-1905) observed that Chézy's C could be further refined by using $u = \alpha R_h^{2/3} S^{1/2}$ for $S > 7 \times 10^{-4}$ or $u = \beta R_h^{3/4} S$ for $S < 7 \times 10^{-4}$; where α, β are empirical coefficients. Gauckler put forward his formula in 1868 [33], but his place in history was supplanted by Robert Manning, who published a more widely-read version of a similar equation in 1889 that usually bears his name. This formula is now generally written as

$$u = \frac{1}{n} R_h^{2/3} S^{1/2} \tag{1.9}$$

where n has a reciprocal relationship with Chézy's C and Gauckler's $[\alpha, \beta]$. Commonly known as "Manning's n" in the English-speaking world, this coefficient is also misnamed: n was not in the original formula of Manning, but became a common adaptation to match the roughness n proposed by Wilhelm R. Kutter [24]. In the U.S. eq. (1.9) is often called the Chézy-Manning equation to acknowledge the contribution of Chézy in the 1/2 power relationship with the channel slope. In Europe the equation is known as Manning-Strickler or Gauckler-Manning-Strickler to acknowledge Albert Strickler, whose early 20th century work provided a rigorous approach to quantifying roughness [70].

Note that eq. (1.9) implies that n has units of $TL^{-1/3}$, so that a common non-dimensional reformulation is to replace n^{-1} with Kn^{-1}, so that n can be considered non-dimensional. The coefficient K takes on a value of unity for $[m, s]$ and $K = 1.486$ for units of $[ft, s]$. The K form is not consistently used in the literature, but implicitly hydraulic engineers treat n as non-dimensional and use either $1.486/n$ or $1/n$ along with consistent units for u and R_h. Because n is generally not known with great precision, $K = 1.49$ or $K = 1.5$ are often seen for $[ft, s]$ units.

In river modeling, Manning's equation is typically written in terms of the cross-section integrated volumetric flow rate, Q with units of $m^3 s^{-1}$ or $ft^3 s^{-1}$. Treating the u in eq. (1.9) as the average velocity, the flow rate for cross-sectional area of a is given by $Q = ua$ so that

$$Q = \frac{1}{n} a R_h^{2/3} S^{1/2} \tag{1.10}$$

The above provides a simple approach for quantifying steady flow in rivers that are either "uniform" or "gradually varying" in space. In hydraulics jargon, a uniform flow does not have spatial gradients in geometry, S, or n, whereas a gradually-varying flow allows weak gradients of independent variables. Where sharp spatial changes in any variable occur, Manning's equation is invalid and the flow is considered "rapidly varying." The use of "gradually" and "rapidly" to describe spatial gradients rather than temporal gradients is an unfortunate legacy in hydraulic engineering jargon that can only be defended by resorting to a Lagrangian viewpoint: when following a particle in a gradually- or rapidly-varying flow the observed temporal changes will indeed be either gradual or rapid, respectively.

Strictly speaking, the slope S in the Chézy and Manning equations is the channel bottom slope, which is typically designated as S_0. If a long section of river is spatially uniform in both S_0 and channel geometry, then for steady flow (constant in time) the water surface slope will be exactly parallel to the channel bottom implying an identical slope. Such flows are said to be uniform and the "normal" flow (§4.3) will approximately satisfy eq. (1.10) using S_0. A more general form of Manning's equation uses the "friction slope," S_f for S. The friction slope is defined in §4.5 and discussed in more detail in §3.8, but for now we simply note that Manning's equation with S_f is generally valid for any quasi-steady gradually-varying flow, which has made it a common model equation to relate Q to frictional losses, even when modeling river physics with more complex unsteady-flow equations.

1.5.3 Saint-Venant's equation

The approximations used in Manning's equation are degraded when encountering time-varying flow rates or spatial gradients of the water surface slope, river geometry, or channel bed roughness. Alexandre de Saint-Venant derived more general differential conservation equations for mass and momentum [19], which may be written for a uniform density fluid as

$$\frac{\partial a}{\partial t} + \frac{\partial}{\partial x}(au) = 0 \tag{1.11}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -g\frac{\partial \eta}{\partial x} - \frac{1}{R_h}f \tag{1.12}$$

where g is gravity, η is the local elevation of the fluid free surface, f is a dimensional model of frictional losses, x is an along-channel coordinate that increases going downstream, and R_h is the "hydraulic radius," of eq. (1.8).

The Saint-Venant equations did not become a practical tool for river analysis until the advent of digital computers (e.g. [62]). However, computers were not a panacea: the Saint-Venant equations are difficult to solve for a natural river due to nonlinear relationships between η and a, as well as f being cast typically as a nonlinear function of u, R_h, and n. More than a half century of literature is replete with simplifications of the Saint-Venant equations, typically falling into the categories of "diffusive wave" or "kinematic wave" approximations [28]. These approximate models have some advantages in simplicity, numerical stability, and amenity to calibration. Although such models are likely to remain in practical engineering use for the near future, it seems likely that increases in computational power and data availability will render them obsolete. The Saint-Venant equations are the fundamental equations for river network analysis and are discussed in detail in §3.7.

1.6 Summary

In the same way that electric circuits have their idealized equations such as Ohm's law, $I = V/R$, hydraulics has a suite of equations with varying levels of idealization. These were developed over the past 250 years by numerous scientists and engineers, but only some are commonly remembered in equation names: Chézy, Manning, and Saint-Venant. The key complicating factor for river hydraulics is the nonlinear relationship between the free-surface slope, i.e. $\partial\eta/\partial x$ in eq. (1.12), and the flow velocity. This relationship is further strained by the difficulty in accurately estimating the river geometry and bed composition, which affects flow resistance and has a nonlinear feedback into flow rates and free-surface gradients.

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