

Unit Commitment in Electric Energy Systems

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Abstract

The unit commitment problem is a fundamental problem in the electric power industry. The objective of unit commitment is to determine an optimal schedule for each generating unit so that the demand for electricity is met at minimum cost for the system as a whole. This tutorial presents the most relevant mathematical optimization models for the unit commitment problem. It is intended as a starting point for learning about this important problem, and thus only the key technical details are included. Likewise, we point out selected references instead of providing a comprehensive literature review of the area.

1

Introduction to the Unit Commitment Problem

The unit commitment (UC) problem addresses a fundamental decision in the operation of a power system, namely determining the schedule of power production for each generating unit in the system so that the demand for electricity is met at minimum cost. The schedule must also ensure that each unit operates within its technical limits; these typically include ramping constraints and minimum uptime/downtime constraints. Units that are scheduled to produce electricity during a given time period are said to be *committed* for that period.

Various jurisdictions solve UC on a daily basis. In particular, it is the standard tool for clearing spot markets, and particularly day-ahead markets in the USA. In North American jurisdictions without markets, the system operators use UC to determine the day-ahead commitments and dispatches.

The UC problem can be formulated as a mixed-integer nonlinear optimization problem, and it is generally large-scale and nonconvex. It is NP-hard in general, but its practical importance has motivated a tremendous amount of research dedicated to techniques for computing global optimal solutions. This is both because of the significance of the operational costs and because in competitive market environments,

the nonconvexity of the UC problem allows the existence of multiple local optimal solutions that may lead to considerably different pricing and market settlement outcomes. Indeed, a mixed-integer linear (or nonlinear but convex) optimization model of the UC problem is among the few techniques that can provide provably global optimal solutions for the commitment decisions and corresponding financial settlements. At the same time, the time available to solve the problem is a hard constraint in practice. Hence, UC is an optimization problem that is both important and challenging.

Various important aspects of UC can be integrated in a mixed-integer nonlinear optimization approach, but the time required to solve the UC models is a hard practical limitation that restricts the size and scope of UC formulations. For this reason there is no single formulation of UC; instead it is a matter of designing a formulation that incorporates the important aspects of the problem for a given context while ensuring that the resulting optimization problem can be solved to optimality, or near-optimality, in a reasonable time.

With the increasing penetration of stochastic sources of electricity in modern power systems, most notably wind and solar generation, techniques for handling uncertainty are acquiring greater importance in UC modeling. We focus our presentation on two well-known techniques for modeling uncertainty in mathematical optimization, namely stochastic optimization and robust optimization. These are by no means the only mathematical optimization techniques for handling uncertainty, but we believe that they are the most relevant in the context of UC because power system operators will always prefer approaches that enforce constraints, rather than satisfying them with some probability, which is the basis of most other approaches.

All the formulations that we present here are mathematical optimization problems. The Introduction to Optimization of the NEOS Guide provides information about the different classes of mathematical optimization problems and the software available to solve them. Most of the state-of-the-art solvers, whether commercial or open source, can be accessed for free on the NEOS Server [Czyzyk et al., 1998, Dolan, 2001,

Gropp and Moré, 1997]. All the computations made in the preparation of this book were carried out on the NEOS Solver.

1.1 Outline of this Book

We introduce in the next six chapters a selection of formulations of UC that integrate different aspects of the problem. We discuss the motivation for and the detailed structure of each formulation and then recapitulate the mathematical model. Each chapter concludes with a small example, accompanied by a description of how the results illustrate the features of the corresponding formulation.

We begin in Chapter 2 with a basic formulation of UC that focuses on the modeling of the generating units and ensuring that generation meets demand (with spinning reserves). The next step is to integrate the impact of the power network; this can be done using power flow equations in either linear (DC) form (Chapter 3) or alternating current (AC) form (Chapter 5). The security of the system is a common concern. In Chapter 4 we integrate constraints to ensure that the system can cope with the failure of one of its major components.

The subsequent two chapters are concerned with modeling uncertainty in the data for UC. We consider two modeling approaches: Chapter 6 introduces a stochastic optimization approach that is based on the use of scenarios, and Chapter 7 presents a robust optimization approach that focuses on the worst-case operating conditions.

While we briefly comment in the presentation of each example on how the computational results were obtained, a detailed discussion of the computational aspects of solving each formulation is given in Chapter 8. Chapter 9 provides concluding remarks and discusses future research.

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