

Distributed Optimization for the DER-Rich Electric Power Grid

Other titles in Foundations and Trends® in Electric Energy Systems

Peer-to-Peer Energy Sharing: A Comprehensive Review

Wayes Tushar, Sohrab Nizami, M. Imran Azim, Chau Yuen, David B. Smith, Tapan Saha and H. Vincent Poor

ISBN: 978-1-63828-156-6

Distribution System Optimization to Manage Distributed Energy Resources (DERs) for Grid Services

Anamika Dubey and Sumit Paudyal

ISBN: 978-1-63828-188-7

LLC Resonant Converters: An Overview of Modeling, Control and Design Methods and Challenges

Claudio Adragna

ISBN: 978-1-63828-066-8

The Role of Power Electronics in Modern Energy System Integration

Saeed Peyghami, Subham Sahoo, Huai Wang, Xiongfei Wang and Frede Blaabjerg

ISBN: 978-1-63828-008-8

Cyber-Physical System Security of Distribution Systems

Chen-Ching Liu, Juan C. Bedoya, Nitasha Sahani, Alexandru Stefanov, Jennifer Appiah-Kubi, Chih-Che Sun, Jin Young Lee and Ruoxi Zhu

ISBN: 978-1-68083-852-7

Distributed Optimization for the DER-Rich Electric Power Grid

Jannatul Adan

West Virginia University
ja00060@mix.wvu.edu

Anurag K. Srivastava

West Virginia University
anurag.srivastava@mail.wvu.edu

now

the essence of knowledge

Boston — Delft

Foundations and Trends® in Electric Energy Systems

Published, sold and distributed by:

now Publishers Inc.
PO Box 1024
Hanover, MA 02339
United States
Tel. +1-781-985-4510
www.nowpublishers.com
sales@nowpublishers.com

Outside North America:

now Publishers Inc.
PO Box 179
2600 AD Delft
The Netherlands
Tel. +31-6-51115274

The preferred citation for this publication is

J. Adan and A. K. Srivastava. *Distributed Optimization for the DER-Rich Electric Power Grid*. Foundations and Trends® in Electric Energy Systems, vol. 7, no. 1, pp. 1–62, 2023.

ISBN: 978-1-63828-293-8

© 2023 J. Adan and A. K. Srivastava

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, mechanical, photocopying, recording or otherwise, without prior written permission of the publishers.

Photocopying. In the USA: This journal is registered at the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923. Authorization to photocopy items for internal or personal use, or the internal or personal use of specific clients, is granted by now Publishers Inc for users registered with the Copyright Clearance Center (CCC). The 'services' for users can be found on the internet at: www.copyright.com

For those organizations that have been granted a photocopy license, a separate system of payment has been arranged. Authorization does not extend to other kinds of copying, such as that for general distribution, for advertising or promotional purposes, for creating new collective works, or for resale. In the rest of the world: Permission to photocopy must be obtained from the copyright owner. Please apply to now Publishers Inc., PO Box 1024, Hanover, MA 02339, USA; Tel. +1 781 871 0245; www.nowpublishers.com; sales@nowpublishers.com

now Publishers Inc. has an exclusive license to publish this material worldwide. Permission to use this content must be obtained from the copyright license holder. Please apply to now Publishers, PO Box 179, 2600 AD Delft, The Netherlands, www.nowpublishers.com; e-mail: sales@nowpublishers.com

Foundations and Trends[®] in Electric Energy Systems

Volume 7, Issue 1, 2023

Editorial Board

Editor-in-Chief

Marija D. Ilić

MIT and Carnegie Mellon University
United States

Editors

David Hill

University of Hong Kong and University of Sydney

Rupamathi Jaddivadal

SmartGridz

Daniel Kirschen

University of Washington

J. Zico Kolter

Carnegie Mellon University

Chao Lu

Tsinghua University

Steven Low

California Institute of Technology

Masoud H. Nazari

Wayne State University

Ram Rajagopa

Stanford University

Lou van der Sluis

TU Delft

Goran Strbac

Imperial College London

Robert J. Thomas

Cornell University

David Tse

University of California, Berkeley

Le Xie

Texas A&M University

Editorial Scope

Topics

Foundations and Trends® in Electric Energy Systems publishes survey and tutorial articles in the following topics:

- Advances in power dispatch
- Demand-side and grid scale data analytics
- Design and optimization of electric services
- Distributed control and optimization of distribution networks
- Distributed sensing for the grid
- Distribution systems
- Fault location and service restoration
- Integration of physics-based and data-driven modeling of future electric energy systems
- Integration of Power electronics, Networked FACTS
- Integration of renewable energy sources
- Interdependence of power system operations and planning and the electricity markets
- Microgrids
- Modern grid architecture
- Power system analysis and computing
- Power system dynamics
- Power system operation
- Power system planning
- Power system reliability
- Power system transients
- Security and privacy
- Stability and control for the whole multi-layer (granulated) network with new load models (to include storage, DR, EVs) and new generation
- System protection and control
- The new stability guidelines and control structures for supporting high penetration of renewables (>50%)
- Uncertainty quantification for the grid
- System impacts of HVDC

Information for Librarians

Foundations and Trends® in Electric Energy Systems, 2023, Volume 7, 4 issues. ISSN paper version 2332-6557. ISSN online version 2332-6565. Also available as a combined paper and online subscription.

Contents

| | | |
|----------|---|-----------|
| 1 | Introduction to Distributed Optimization in Power System | 3 |
| 1.1 | Optimization Requirements in Power System | 3 |
| 1.2 | Limitations of Centralized Optimization | 4 |
| 1.3 | Addressing the Limitations by Distributed Optimization | 5 |
| 1.4 | Example Applications of Distributed Optimization | 7 |
| 2 | Classification of Distributed Optimization | 9 |
| 2.1 | Types of Optimization Problems | 9 |
| 2.2 | Taxonomy of the Distributed Approaches | 15 |
| 3 | Distributed Approaches with Continuous Decision Variables | 18 |
| 3.1 | Offline Approaches with One-time Snapshot Measurements | 18 |
| 3.2 | Online Distributed Approaches with Continuous Measurement and Feedback | 24 |
| 3.3 | Machine Learning for Distributed Approaches with Continuous Variables | 25 |
| 4 | Distributed Approaches with Integer Decision Variables | 30 |
| 4.1 | Power System Legacy Devices Operating in Integer Domain | 30 |
| 4.2 | Challenges of Distributed Optimization Techniques with Discrete Decision Variables | 32 |

| | | |
|----------|---|-----------|
| 4.3 | Proposed Distributed Approaches for Integer Decision Variables | 33 |
| 4.4 | Machine Learning for Distributed Approaches with Discrete Variables | 42 |
| 5 | Summary, Limitations and Path Forward | 46 |
| 5.1 | Summary and Limitations of the Existing Distributed Optimization Algorithms | 46 |
| 5.2 | Path Forward | 47 |
| | Acknowledgements | 50 |
| | References | 51 |

Distributed Optimization for the DER-Rich Electric Power Grid

Jannatul Adan and Anurag K. Srivastava

*West Virginia University, USA; ja00060@mix.wvu.edu,
anurag.srivastava@mail.wvu.edu*

ABSTRACT

Centralized algorithms are widely used for optimization and control in power system applications. These algorithms require all the measurements and data to be accumulated at central location and hence suffer from single-point-of-failure. Additionally, these algorithms lack scalability with an increasing number of sensors and actuators, specially with the increasing integration of distributed energy resources (DERs). As the power system becomes a confluence of a diverse set of decision-making entities with a multitude of objectives, the preservation of privacy and operation of the system with limited information has been a growing concern. Distributed optimization techniques solve these challenges while also ensuring resilient computational solutions for the power system operation in the presence of both natural and man-made adversaries. A detailed discussion of possible applications of distributed optimization in power systems is provided in this work. However, there exist multiple challenges for accurate and computationally efficient distributed solutions.

Commonly-used distributed optimization approaches include Lagrange relaxation, augmented Lagrangian, approximate

Jannatul Adan and Anurag K. Srivastava (2023), "Distributed Optimization for the DER-Rich Electric Power Grid", Foundations and Trends[®] in Electric Energy Systems: Vol. 7, No. 1, pp 1–62. DOI: 10.1561/3100000036.

©2023 J. Adan and A. K. Srivastava

network directions in conjunction with standard Lagrangian, auxiliary problem principle, alternating directions method of multipliers, optimality condition decomposition, proximal atomic coordination, and optimal feedback-based voltage control. A comprehensive classification of the distributed optimization problems has been discussed and detailed in this work. All of these algorithms have displayed efficient identification of global optimum solutions for convex continuous distributed optimization problems. The algorithms discussed so far are predominantly used to manage continuous state variables. Inclusion of integer variables in the decision support are needed for specific power system problems.

Mixed integer programming (MIP) problem arises in a power system operation and control due to tap changing transformers, capacitors and switches. The global optimization techniques for MIPs are Branch and Bound, Branch and Cut, Cutting planes, Adaptive coordinate search, Nelder-Mead, Genetic algorithm etc. Although the above optimization techniques are able to solve NP-hard convexified MIP problems centrally, but are time consuming and do not scale well for large scale distributed problems. Decomposition and solution approach of distributed coordination can resolve the scalability issue. Despite the fact that a large body of work is present on the centralized solution methods for convexified MIP problems, the literature on distributed MIPs is relatively limited. The distributed optimization algorithms applied in power network to solve MIPs are reported here. ML based solutions can help to get faster convergence for distributed optimization or can replace optimization techniques depending on the problem as discussed in this work. Finally, a summary and path forward are provided, and the advancement needed in distributed optimization for the power grid is also presented.

1

Introduction to Distributed Optimization in Power System

1.1 Optimization Requirements in Power System

Majority of the decision-making tools used in the power system can be classified into (i) rule/heuristic-based approaches, and (ii) optimization-based approaches. Therefore, the scope of power system resource optimization problems, also known as mathematical programming, ranges from tools deployed within the energy management system (EMS) of the power transmission system control center, advanced distribution management system (ADMS), outage management systems, energy market operational problems (economic dispatch and unit commitment) to enterprise asset management. Consequently, optimal power flow (OPF) problems, where power flow equations are considered to be constraints of the optimization problem, is one of the well studied problems in the power engineering literature, since it was introduced by Carpentier (Carpentier, 1962). Typically, power system optimization problems deal with steady-state system operation subject to satisfying system operating conditions, independent of having to worry about how these states would be reached. In this regard, model predictive control (MPC) has also been used in the power engineering context to determine the control action for a dynamical system over a finite,

receding time horizon. In this case, ‘dynamics’ of the system is typically captured by the load and generation variability, and hence, it would be wise to classify them as multi-period rolling horizon optimization problems. Additionally, the research community is increasingly concerned about security-/chance-constrained OPF so that the system performs desirably under varying operating conditions. Therefore, the scope of OPF problems is indeed vast, with each of the optimization algorithms having widely varying operational, and infrastructural requirements, depicting their performance.

1.2 Limitations of Centralized Optimization

The underlying physics and system behaviour of a DER-Rich electric grid is substantially different compared to a traditional one. On one hand, DERs including micro turbines, diesel generators or inverter based renewable energy resources have inherent capability of enhancing the reliability of the system in conjunction with providing crucial self-healing support during natural disasters. On the other hand, if not carefully monitored and controlled, they can significantly hamper the stability of the grid. The diverse set of unprecedented as well as unknown possible scenarios emphasizes on strengthening the situational cognizance of the system which has led to advancing sophisticated sensor arrangement, data accumulation and processing technology. But the high volume of data as well as the huge number of control variables drag some serious concerns with the Centralized Controller. As the number of variables increase, the computational burden and the time to find a feasible solution increase exponentially instead of linearly which limit the scalability of the centralized optimization solver. Also, centralized controller suffer from the risk of single point of failure and increased cyber vulnerability. A technical failure or a cyber attack at the central controller can compromise the security of the whole system putting all the decision making at hold. Apart from that, centralized controllers requiring all information to be shared at a central location fall behind in ensuring privacy preferred by many utilities, especially independently owned DERs. Distributed approaches can potentially circumvent the aforementioned challenges and act as a successful alternative with satisfactory performance especially for DER-Rich Electric power grid.

1.3 Addressing the Limitations by Distributed Optimization

In distributed optimization approach, the control system is divided into multiple local agents each solving its own sub-problem and handling its control variables. The local controllers share limited information only to their neighboring agents and are expected to reach the global optimal solution as would be determined by the central controller. Various aspects of distributed approaches and their impact especially on solving OPF for DER-Rich distribution systems are briefly discussed below.

1. Scalability:

Through the decomposition of the root optimization problem, distributed approaches have a notable effect on the scalability of the system. Since each controller deals with a subset of the original set of decision variables, their computational requirement become way less than the central controller. Furthermore, if a new component such as a new DER or regulator is added to the system, it would necessitate only the corresponding local controller to reorganize its sub-problem to incorporate the new set of variables associated with it. As with the increasing penetration of DERs, number of decision variables to be optimized multiplies substantially. In such a case, the development and implementation of suitable distributed approaches can be a timely adaptation to improve the scalability for DER-Rich distribution systems.

2. Privacy:

Distributed approaches do not require all the information from local agents including critical data related to the privately owned DERs to be sent to the central coordinator ensuring much needed privacy to the private utilities.

3. Computation Requirement:

In a centralized approach, the central controller handles all the variables which are part of the non-convex OPF problem requiring the central controller to have a sophisticated computation capability. But with distributed controller, each agent handles a subset of variables which can reduce their computational requirements.

4. Communication Requirement:

In centralized manner, every local agent communicates with the central controller via a communication link. So if there are n local agents, there will at least be n communication links from each agent to the central controller which may increase with presence of backup communication links. For distributed approach, n agents will at least need $n-1$ number of communication links which usually increases if agents are more densely connected. To decrease the number of communication links and hence communication burden, some researchers propose suitable partitioning techniques to ensure weak coupling requirements (Wang *et al.*, 2017; Guo *et al.*, 2017). Furthermore, some researchers share concerns about an agent participating in a distributed optimization algorithm failing to share the data at the end of an iteration will cause other agents to wait and not move to the next iteration. Hence, some works have been reported to enable asynchronous update among the subsystems to address the aforementioned problem (Mohammadi *et al.*, 2018; Mohammadi and Kargarian, 2022).

5. Cyber Resiliency:

One concern with distributed approaches is that agents repetitively share data during the iterative process of distributed algorithms, and in case of unauthorized access to any controller, the shared data can be used to infer information about the local systems. But distributed approaches have provisions to improve the privacy by choosing a modified set of variables to be broadcasted to neighbors rather than the raw measurements which prevents the adversaries to directly extract useful information even if they get access to the shared data. The work proposed in Wu *et al.* (2021), Dvorkin *et al.* (2021), and Ryu and Kim (2022) discusses different encryption or modification techniques to ensure the privacy of the shared variables. Furthermore, the impact of distributed optimization on the propagation of a cyber attack is another important parameter to assess the cyber resiliency. An adversary getting access to one agent may result in compromising itself and its neighbors. But in the case of centralized approach, since every local agent directly

communicates with the central controller, the central controller may directly get exposed through compromising a local agent which increases the vulnerability of the controller. In addition to that, distributed approaches are more robust to single point of failure. Even if one local controller undergoes some technical problem and fails to operate, the rest of the system can continue their decision-making, keeping the rest of the system unaffected. But a centralized controller will put all the decision making at a halt if it fails to operate. Vosughi *et al.* (2022) provides a detailed comparative analysis on the performance and characteristics of centralized and distributed approaches along with local and decentralized techniques. Nonetheless, Augmenting the robustness and resiliency of distributed approaches is an imperative field of research. Research works including Alkhrajah *et al.* (2022a) and Zhao *et al.* (2017) analyze the effect of cyber attacks in the operation of distributed optimization and provides insight for improving it. Further discussion on this topic is added in Section 5.

Observing the aforementioned advantages, distributed optimization techniques have begun to gain peak attention from the researchers and essentially proving to achieve much importance onto solving power system optimization problems specially for DER-Rich environments.

1.4 Example Applications of Distributed Optimization

OPF is a fundamental problem in power system operation which searches for an operating point that optimizes a certain cost while ensuring various security constraints of the network and satisfying network physics. OPF problems can be single or multi-objective constrained optimization problem. Some common use-cases of the OPF problem include:

- **Voltage Regulation** seeks to minimize the voltage deviation of the overall system usually by utilizing VAR support from inverter based DERs or through Volt-Watt optimization.
- **Generation Cost Minimization** decides optimal generation from traditional and distributed energy resources that minimizes the cost of generation while ensuring power balance between loads and generation.

- **Loss Minimization** decides optimal power flow through the lines and from generators so that total network loss is minimized.
- **Unit Commitment** as part of OPF tries to determine when and which power plants at each generating station should be shut down or started up so that cumulative generating cost is minimized whilst generation-load demand equilibrium is met.
- **Service Restoration** is the process of gradually restoring the network after partial or complete black-out. DERs can help in providing the emergency power supply to critical loads. Service restoration often entails network reconfiguration and co-ordination of distributed generators to maximize the restored load.
- **Market Pricing and Social Welfare** is, on the contrary to economic load dispatch, enabling load and generation to get matched through a competitive electricity market to determine the marginal price of electricity. Market clearing involves both buyers and sellers to provide bids to be cleared, ensuring maximization of social welfare.
- **Active Power Curtailment** cost of a generating unit represents the opportunity cost of supplying real power that is lost due to allocating reactive power from that unit. Active power curtailment may also include minimizing the active power to meet the demand or to reduce loading on distribution lines during peak demand periods.

References

- Adan, J., S. Majumder, and A. K. Srivastava. (2022). “Distributed Optimization Approaches with Discrete Variables in the Power Distribution Systems”. In: *2022 54th North American Power Symposium (NAPS)*. 1–6.
- Ahmadi, H. and J. R. Martí. (2015). “Distribution System Optimization Based on a Linear Power-Flow Formulation”. *IEEE Transactions on Power Delivery*. 30(1): 25–33. DOI: [10.1109/TPWRD.2014.2300854](https://doi.org/10.1109/TPWRD.2014.2300854).
- Alkhraijah, M., M. Alowaifeer, S. Grijalva, and D. K. Molzahn. (2021). “Distributed Multi-Period DCOPF via an Auxiliary Principle Problem Algorithm”. In: *2021 IEEE Texas Power and Energy Conference (TPEC)*. 1–6. DOI: [10.1109/TPEC51183.2021.9384964](https://doi.org/10.1109/TPEC51183.2021.9384964).
- Alkhraijah, M., R. Harris, S. Litchfield, D. Huggins, and D. K. Molzahn. (2022a). “Analyzing Malicious Data Injection Attacks on Distributed Optimal Power Flow Algorithms”. In: *2022 North American Power Symposium (NAPS)*. 1–6. DOI: [10.1109/NAPS56150.2022.10012244](https://doi.org/10.1109/NAPS56150.2022.10012244).
- Alkhraijah, M., C. Menendez, and D. K. Molzahn. (2022b). “Assessing the impacts of nonideal communications on distributed optimal power flow algorithms”. *Electric Power Systems Research*. 212: 108297. URL: <https://www.sciencedirect.com/science/article/pii/S0378779622004801>.

- Bai, X., H. Wei, K. Fujisawa, and Y. Wang. (2008). “Semidefinite programming for optimal power flow problems”. *International Journal of Electrical Power & Energy Systems*. 30(6): 383–392. URL: <https://www.sciencedirect.com/science/article/pii/S0142061507001378>.
- Baran, M. and F. Wu. (1989a). “Optimal sizing of capacitors placed on a radial distribution system”. *IEEE Transactions on Power Delivery*. 4(1): 735–743. DOI: [10.1109/61.19266](https://doi.org/10.1109/61.19266).
- Baran, M. and F. Wu. (1989b). “Network reconfiguration in distribution systems for loss reduction and load balancing”. *IEEE Transactions on Power Delivery*. 4(2): 1401–1407. DOI: [10.1109/61.25627](https://doi.org/10.1109/61.25627).
- Biagioni, D., P. Graf, X. Zhang, A. S. Zamzam, K. Baker, and J. King. (2022). “Learning-Accelerated ADMM for Distributed DC Optimal Power Flow”. *IEEE Control Systems Letters*. 6: 1–6. DOI: [10.1109/LCSYS.2020.3044839](https://doi.org/10.1109/LCSYS.2020.3044839).
- Bobo, L., A. Venzke, and S. Chatzivasilieiadis. (2021). “Second-order cone relaxations of the optimal power flow for active distribution grids: Comparison of methods”. *International Journal of Electrical Power & Energy Systems*. 127: 106625. URL: <https://www.sciencedirect.com/science/article/pii/S0142061520341703>.
- Bose, S., S. H. Low, T. Teeraratkul, and B. Hassibi. (2015). “Equivalent Relaxations of Optimal Power Flow”. *IEEE Transactions on Automatic Control*. 60(3): 729–742. DOI: [10.1109/TAC.2014.2357112](https://doi.org/10.1109/TAC.2014.2357112).
- Boyd, S., N. Parikh, E. Chu, B. Peleato, and J. Eckstein. (2011). “Distributed Optimization and Statistical Learning via the Alternating Direction Method of Multipliers”. *Foundations and Trends in Machine Learning*. 3(1): 1–122. DOI: [10.1561/22000000016](https://doi.org/10.1561/22000000016).
- Cao, D., J. Zhao, W. Hu, F. Ding, Q. Huang, Z. Chen, and F. Blaabjerg. (2021). “Data-Driven Multi-Agent Deep Reinforcement Learning for Distribution System Decentralized Voltage Control With High Penetration of PVs”. *IEEE Transactions on Smart Grid*. 12(5): 4137–4150. DOI: [10.1109/TSG.2021.3072251](https://doi.org/10.1109/TSG.2021.3072251).
- Carpentier, J. (1962). “Contribution a l’étude du dispatching économique”. *Bulletin de la Société Française des électriciens*. 3(1): 431–447.

- Chen, C., B. He, Y. Ye, and X. Yuan. (2016). “The direct extension of ADMM for multi-block convex minimization problems is not necessarily convergent”. *Mathematical Programming*. DOI: [10.1007/s10107-014-0826-5](https://doi.org/10.1007/s10107-014-0826-5).
- Dall’Anese, E., H. Zhu, and G. B. Giannakis. (2013). “Distributed Optimal Power Flow for Smart Microgrids”. *IEEE Transactions on Smart Grid*. 4(3): 1464–1475. DOI: [10.1109/TSG.2013.2248175](https://doi.org/10.1109/TSG.2013.2248175).
- Diamond, S., R. Takapoui, and S. Boyd. (2017). “A general system for heuristic minimization of convex functions over non-convex sets”. *Optimization Methods and Software*. 33(Apr.): 1–29. DOI: [10.1080/10556788.2017.1304548](https://doi.org/10.1080/10556788.2017.1304548).
- Dvorkin, V., F. Fioretto, P. Van Hentenryck, P. Pinson, and J. Kazempour. (2021). “Differentially Private Optimal Power Flow for Distribution Grids”. *IEEE Transactions on Power Systems*. 36(3): 2186–2196. DOI: [10.1109/TPWRS.2020.3031314](https://doi.org/10.1109/TPWRS.2020.3031314).
- Engelmann, A., Y. Jiang, T. Mühlfordt, B. Houska, and T. Faulwasser. (2019). “Toward Distributed OPF Using ALADIN”. *IEEE Transactions on Power Systems*. 34(1): 584–594. DOI: [10.1109/TPWRS.2018.2867682](https://doi.org/10.1109/TPWRS.2018.2867682).
- Farahani, V., B. Vahidi, and H. A. Abyaneh. (2012). “Reconfiguration and Capacitor Placement Simultaneously for Energy Loss Reduction Based on an Improved Reconfiguration Method”. *IEEE Transactions on Power Systems*. 27(2): 587–595. DOI: [10.1109/TPWRS.2011.2167688](https://doi.org/10.1109/TPWRS.2011.2167688).
- Gallego, L. A., J. M. López-Lezama, and O. G. Carmona. (2022). “A Mixed-Integer Linear Programming Model for Simultaneous Optimal Reconfiguration and Optimal Placement of Capacitor Banks in Distribution Networks”. *IEEE Access*. 10: 52655–52673. DOI: [10.1109/ACCESS.2022.3175189](https://doi.org/10.1109/ACCESS.2022.3175189).
- Gao, Y., W. Wang, and N. Yu. (2021). “Consensus Multi-Agent Reinforcement Learning for Volt-VAR Control in Power Distribution Networks”. *IEEE Transactions on Smart Grid*. 12(4): 3594–3604. DOI: [10.1109/TSG.2021.3058996](https://doi.org/10.1109/TSG.2021.3058996).

- Guo, J., G. Hug, and O. K. Tonguz. (2017). “A Case for Nonconvex Distributed Optimization in Large-Scale Power SystemsF”. *IEEE Transactions on Power Systems*. 32(5): 3842–3851. DOI: [10.1109/TPWRS.2016.2636811](https://doi.org/10.1109/TPWRS.2016.2636811).
- Guo, J., G. Hug, and O. K. Tonguz. (2018). “On the Role of Communications Plane in Distributed Optimization of Power Systems”. *IEEE Transactions on Industrial Informatics*. 14(7): 2903–2913. DOI: [10.1109/TII.2017.2774243](https://doi.org/10.1109/TII.2017.2774243).
- Haider, R., R. Jaddivada, A. M. Annaswamy, and M. D. Ilic. (2021). “Distributed Backward/Forward Sweep Algorithm for Economic Dispatch in Modern Distribution Grids”. In: *2020 52nd North American Power Symposium (NAPS)*. 1–6. DOI: [10.1109/NAPS50074.2021.9449801](https://doi.org/10.1109/NAPS50074.2021.9449801).
- Hu, D., Z. Ye, Y. Gao, Z. Ye, Y. Peng, and N. Yu. (2022). “Multi-Agent Deep Reinforcement Learning for Voltage Control With Coordinated Active and Reactive Power Optimization”. *IEEE Transactions on Smart Grid*. 13(6): 4873–4886. DOI: [10.1109/TSG.2022.3185975](https://doi.org/10.1109/TSG.2022.3185975).
- Kargarian, A., J. Mohammadi, J. Guo, S. Chakrabarti, M. Barati, G. Hug, S. Kar, and R. Baldick. (2018). “Toward Distributed/Decentralized DC Optimal Power Flow Implementation in Future Electric Power Systems”. *IEEE Transactions on Smart Grid*. 9(4): 2574–2594. DOI: [10.1109/TSG.2016.2614904](https://doi.org/10.1109/TSG.2016.2614904).
- Kekatos, V. and G. B. Giannakis. (2013). “Distributed Robust Power System State Estimation”. *IEEE Transactions on Power Systems*. 28(2): 1617–1626. DOI: [10.1109/TPWRS.2012.2219629](https://doi.org/10.1109/TPWRS.2012.2219629).
- Kim, B. and R. Baldick. (1997). “Coarse-grained distributed optimal power flow”. *IEEE Transactions on Power Systems*. 12(2): 932–939. DOI: [10.1109/59.589777](https://doi.org/10.1109/59.589777).
- Kim, Y. and K. Kim. (2022). “Accelerated computation and tracking of AC optimal power flow solutions using GPUs”. In: *Workshop Proceedings of the 51st International Conference on Parallel Processing*. 1–8.
- Klemets, J. R. A. and M. Z. Degefa. (2023). “A Distributed Algorithm for Controlling Continuous and Discrete Variables in a Radial Distribution Grid”. *IEEE Access*. 11: 2488–2499. DOI: [10.1109/ACCESS.2023.3234102](https://doi.org/10.1109/ACCESS.2023.3234102).

- Lam, A. Y., B. Zhang, and N. T. David. (2012). “Distributed algorithms for optimal power flow problem”. In: *2012 IEEE 51st IEEE conference on decision and control (CDC)*. IEEE. 430–437.
- Lee, H., A. K. Srivastava, V. V. G. Krishnan, S. Niddodi, and D. E. Bakken. (2022). “Decentralized Voltage Stability Monitoring and Control With Distributed Computing Coordination”. *IEEE Systems Journal*. 16(2): 2251–2260. DOI: [10.1109/JSYST.2021.3057614](https://doi.org/10.1109/JSYST.2021.3057614).
- Lei, J., H.-F. Chen, and H. Fang. (2016). “Primal-dual algorithm for distributed constrained optimization”. *Syst. Control. Lett.* 96: 110–117.
- Li, H. and H. He. (2022). “Learning to Operate Distribution Networks With Safe Deep Reinforcement Learning”. *IEEE Transactions on Smart Grid*. 13(3): 1860–1872. DOI: [10.1109/TSG.2022.3142961](https://doi.org/10.1109/TSG.2022.3142961).
- Li, H., Z. Wang, and H. He. (2021). “Distributed Volt-VAR Optimization based on Multi-Agent Deep Reinforcement Learning”. In: *2021 International Joint Conference on Neural Networks (IJCNN)*. 1–7. DOI: [10.1109/IJCNN52387.2021.9534348](https://doi.org/10.1109/IJCNN52387.2021.9534348).
- Li, Z., Q. Wu, J. Chen, S. Huang, and F. Shen. (2023). “Double-time-scale distributed voltage control for unbalanced distribution networks based on MPC and ADMM”. *International Journal of Electrical Power & Energy Systems*. 145: 108665. URL: <https://www.sciencedirect.com/science/article/pii/S0142061522006615>.
- Lin, C.-H. and S.-Y. Lin. (2008). “Distributed Optimal Power Flow With Discrete Control Variables of Large Distributed Power Systems”. *IEEE Transactions on Power Systems*. 23(3): 1383–1392. DOI: [10.1109/TPWRS.2008.926695](https://doi.org/10.1109/TPWRS.2008.926695).
- Liu, M., S. Tso, and Y. Cheng. (2002). “An extended nonlinear primal-dual interior-point algorithm for reactive-power optimization of large-scale power systems with discrete control variables”. *IEEE Transactions on Power Systems*. 17(4): 982–991. DOI: [10.1109/TPWRS.2002.804922](https://doi.org/10.1109/TPWRS.2002.804922).
- Liu, Q. and J. Wang. (2015). “A Second-Order Multi-Agent Network for Bound-Constrained Distributed Optimization”. *IEEE Transactions on Automatic Control*. 60(12): 3310–3315. DOI: [10.1109/TAC.2015.2416927](https://doi.org/10.1109/TAC.2015.2416927).

- Liu, Y., L. Guo, C. Lu, Y. Chai, S. Gao, and B. Xu. (2019). “A Fully Distributed Voltage Optimization Method for Distribution Networks Considering Integer Constraints of Step Voltage Regulators”. *IEEE Access*. 7: 60055–60066. DOI: [10.1109/ACCESS.2019.2912004](https://doi.org/10.1109/ACCESS.2019.2912004).
- Lobel, I. and A. Ozdaglar. (2011). “Distributed Subgradient Methods for Convex Optimization Over Random Networks”. *IEEE Transactions on Automatic Control*. 56(6): 1291–1306. DOI: [10.1109/TAC.2010.2091295](https://doi.org/10.1109/TAC.2010.2091295).
- Lotfi, M., G. J. Osório, M. S. Javadi, M. S. El Moursi, C. Monteiro, and J. P. Catalão. (2022). “A fully decentralized machine learning algorithm for optimal power flow with cooperative information exchange”. *International Journal of Electrical Power & Energy Systems*. 139: 107990. URL: <https://doi.org/10.1016/j.ijepes.2022.107990>.
- Lou, Y., G. Shi, K. H. Johansson, and Y. Hong. (2014). “Approximate Projected Consensus for Convex Intersection Computation: Convergence Analysis and Critical Error Angle”. *IEEE Transactions on Automatic Control*. 59(7): 1722–1736. DOI: [10.1109/TAC.2014.2309261](https://doi.org/10.1109/TAC.2014.2309261).
- Low, S. H. (2014). “Convex Relaxation of Optimal Power Flow—Part I: Formulations and Equivalence”. *IEEE Transactions on Control of Network Systems*. 1(1): 15–27. DOI: [10.1109/TCNS.2014.2309732](https://doi.org/10.1109/TCNS.2014.2309732).
- Lu, W., M. Liu, S. Lin, and L. Li. (2019). “Incremental-oriented ADMM for distributed optimal power flow with discrete variables in distribution networks”. *IEEE Transactions on Smart Grid*. 10(6): 6320–6331.
- Macfie, P. J., G. A. Taylor, M. R. Irving, P. Hurlock, and H.-B. Wan. (2010). “Proposed Shunt Rounding Technique for Large-Scale Security Constrained Loss Minimization”. *IEEE Transactions on Power Systems*. 25(3): 1478–1485. DOI: [10.1109/TPWRS.2010.2041675](https://doi.org/10.1109/TPWRS.2010.2041675).
- Magnússon, S., P. C. Weeraddana, and C. Fischione. (2015). “A Distributed Approach for the Optimal Power-Flow Problem Based on ADMM and Sequential Convex Approximations”. *IEEE Transactions on Control of Network Systems*. 2(3): 238–253. DOI: [10.1109/TCNS.2015.2399192](https://doi.org/10.1109/TCNS.2015.2399192).

- Mohammadi, A. and A. Kargarian. (2020). “Accelerated and Robust Analytical Target Cascading for Distributed Optimal Power Flow”. *IEEE Transactions on Industrial Informatics*. 16(12): 7521–7531. DOI: [10.1109/TII.2020.2973213](https://doi.org/10.1109/TII.2020.2973213).
- Mohammadi, A. and A. Kargarian. (2022). “Learning-Aided Asynchronous ADMM for Optimal Power Flow”. *IEEE Transactions on Power Systems*. 37(3): 1671–1681. DOI: [10.1109/TPWRS.2021.3120260](https://doi.org/10.1109/TPWRS.2021.3120260).
- Mohammadi, J., G. Hug, and S. Kar. (2018). “Agent-Based Distributed Security Constrained Optimal Power Flow”. *IEEE Transactions on Smart Grid*. 9(2): 1118–1130. DOI: [10.1109/TSG.2016.2577684](https://doi.org/10.1109/TSG.2016.2577684).
- Molzahn, D., F. Dörfler, H. Sandberg, S. Low, S. Chakrabarti, R. Baldick, and J. Lavaei. (2017). “A Survey of Distributed Optimization and Control Algorithms for Electric Power Systems”. *IEEE Transactions on Smart Grid*. PP(July): 1–1. DOI: [10.1109/TSG.2017.2720471](https://doi.org/10.1109/TSG.2017.2720471).
- Molzahn, D. and I. Hiskens. (2019). “A Survey of Relaxations and Approximations of the Power Flow Equations”. *Foundations and Trends® in Electric Energy Systems*. 4(1-2): 1–221. DOI: [10.1561/3100000012](https://doi.org/10.1561/3100000012).
- Morrison, D. R., S. H. Jacobson, J. J. Sauppe, and E. C. Sewell. (2016). “Branch-and-bound algorithms: A survey of recent advances in searching, branching, and pruning”. *Discrete Optimization*. 19: 79–102. URL: <https://doi.org/10.1016/j.disopt.2016.01.005>.
- Nazir, F. U., B. C. Pal, and R. A. Jabr. (2020). “Distributed Solution of Stochastic Volt/VAR Control in Radial Networks”. *IEEE Transactions on Smart Grid*. 11(6): 5314–5324. DOI: [10.1109/TSG.2020.3002100](https://doi.org/10.1109/TSG.2020.3002100).
- Nedic, A., A. Ozdaglar, and P. A. Parrilo. (2010). “Constrained Consensus and Optimization in Multi-Agent Networks”. *IEEE Transactions on Automatic Control*. 55(4): 922–938. DOI: [10.1109/TAC.2010.2041686](https://doi.org/10.1109/TAC.2010.2041686).
- Nejad, R. R. and W. Sun. (2022). “Enhancing Active Distribution Systems Resilience by Fully Distributed Self-Healing Strategy”. *IEEE Transactions on Smart Grid*. 13(2): 1023–1034. DOI: [10.1109/TSG.2021.3127518](https://doi.org/10.1109/TSG.2021.3127518).

- Nocedal, J. and S. J. Wright. (2006). *Numerical Optimization*. 2e. New York, NY, USA: Springer.
- Ortmann, L., A. Prostejovsky, K. Heussen, and S. Bolognani. (2020). “Fully distributed peer-to-peer optimal voltage control with minimal model requirements”. *Electric Power Systems Research*. 189: 106717. URL: <https://www.sciencedirect.com/science/article/pii/S0378779620305204>.
- Patari, N., A. K. Srivastava, G. Qu, and N. Li. (2021). “Distributed Voltage Control for Three-Phase Unbalanced Distribution Systems With DERs and Practical Constraints”. *IEEE Transactions on Industry Applications*. 57(6): 6622–6633. DOI: [10.1109/TIA.2021.3114388](https://doi.org/10.1109/TIA.2021.3114388).
- Patari, N., V. Venkataramanan, A. Srivastava, D. K. Molzahn, N. Li, and A. Annaswamy. (2022). “Distributed Optimization in Distribution Systems: Use Cases, Limitations, and Research Needs”. *IEEE Transactions on Power Systems*. 37(5): 3469–3481. DOI: [10.1109/TPWRS.2021.3132348](https://doi.org/10.1109/TPWRS.2021.3132348).
- Peng, Q. and S. H. Low. (2018). “Distributed Optimal Power Flow Algorithm for Radial Networks, I: Balanced Single Phase Case”. *IEEE Transactions on Smart Grid*. 9(1): 111–121. DOI: [10.1109/TSG.2016.2546305](https://doi.org/10.1109/TSG.2016.2546305).
- Potra, F. and S. Wright. (2000). “Interior-point methods”. *Journal of Computational and Applied Mathematics*. 124(Dec.): 281–302. DOI: [10.1016/S0377-0427\(00\)00433-7](https://doi.org/10.1016/S0377-0427(00)00433-7).
- Putratama, M. A., R. Rigo-Mariani, V. Debusschere, and Y. Besanger. (2021). “Parameter Tuning for LV Centralized and Distributed Voltage Control with High PV Production”. In: *2021 IEEE Madrid PowerTech*. 1–6. DOI: [10.1109/PowerTech46648.2021.9494802](https://doi.org/10.1109/PowerTech46648.2021.9494802).
- Raju, L., S. Sankar, and R. Milton. (2015). “Distributed Optimization of Solar Micro-grid Using Multi Agent Reinforcement Learning”. *Procedia Computer Science*. 46: 231–239. URL: <https://www.sciencedirect.com/science/article/pii/S1877050915000800>.
- Roofegari Nejad, R. and W. Sun. (2019). “Distributed Load Restoration in Unbalanced Active Distribution Systems”. *IEEE Transactions on Smart Grid*. 10(5): 5759–5769. DOI: [10.1109/TSG.2019.2891419](https://doi.org/10.1109/TSG.2019.2891419).

- Roofegari Nejad, R., W. Sun, and A. Golshani. (2019). “Distributed Restoration for Integrated Transmission and Distribution Systems With DERs”. *IEEE Transactions on Power Systems*. 34(6): 4964–4973. DOI: [10.1109/TPWRS.2019.2920123](https://doi.org/10.1109/TPWRS.2019.2920123).
- Rui, X., M. Liu, M. Sahraei-Ardakani, and T. R. Nudell. (2022). “ADMM-Based Distributed DC Optimal Power Flow with Power Flow Control”. In: *2022 North American Power Symposium (NAPS)*. 1–6. DOI: [10.1109/NAPS56150.2022.10012190](https://doi.org/10.1109/NAPS56150.2022.10012190).
- Ryu, M. and K. Kim. (2022). “A Privacy-Preserving Distributed Control of Optimal Power Flow”. *IEEE Transactions on Power Systems*. 37(3): 2042–2051. DOI: [10.1109/TPWRS.2021.3120056](https://doi.org/10.1109/TPWRS.2021.3120056).
- Sadnan, R., T. Asaki, and A. Dubey. (2021). “Online Distributed Optimization in Radial Power Distribution Systems: Closed-Form Expressions”. In: *2021 IEEE International Conference on Communications, Control, and Computing Technologies for Smart Grids (SmartGridComm)*. 51–56. DOI: [10.1109/SmartGridComm51999.2021.9632288](https://doi.org/10.1109/SmartGridComm51999.2021.9632288).
- Al-Saffar, M. and P. Musilek. (2021). “Distributed Optimization for Distribution Grids With Stochastic DER Using Multi-Agent Deep Reinforcement Learning”. *IEEE Access*. 9: 63059–63072. DOI: [10.1109/ACCESS.2021.3075247](https://doi.org/10.1109/ACCESS.2021.3075247).
- Sarker, P., S. Majumder, M. F. Rafy, and A. Srivastava. (2022a). “Impact Analysis of Cyber-Events on Distributed Voltage Control with Active Power Curtailment”. In: DOI: [10.1109/PEDES56012.2022.10080183](https://doi.org/10.1109/PEDES56012.2022.10080183).
- Sarker, P. S., N. Patari, B. Ha, S. Majumder, and A. K. Srivastava. (2022b). “Cyber-Power Testbed for Analyzing Distributed Control Performance during Cyber-Events”. In: *2022 10th Workshop on Modelling and Simulation of Cyber-Physical Energy Systems (MSCPES)*. 1–6. DOI: [10.1109/MSCPES55116.2022.9770160](https://doi.org/10.1109/MSCPES55116.2022.9770160).
- Shen, F., J. C. López, Q. Wu, M. J. Rider, T. Lu, and N. D. Hatziairgiou. (2020a). “Distributed Self-Healing Scheme for Unbalanced Electrical Distribution Systems Based on Alternating Direction Method of Multipliers”. *IEEE Transactions on Power Systems*. 35(3): 2190–2199. DOI: [10.1109/TPWRS.2019.2958090](https://doi.org/10.1109/TPWRS.2019.2958090).
- Shen, F., Q. Wu, and Y. Xue. (2020b). “Review of Service Restoration for Distribution Networks”. *Journal of Modern Power Systems and Clean Energy*. 8(1): 1–14. DOI: [10.35833/MPCE.2018.000782](https://doi.org/10.35833/MPCE.2018.000782).

- Shi, G., K. H. Johansson, and Y. Hong. (2013). “Reaching an Optimal Consensus: Dynamical Systems That Compute Intersections of Convex Sets”. *IEEE Transactions on Automatic Control*. 58(3): 610–622. DOI: [10.1109/TAC.2012.2215261](https://doi.org/10.1109/TAC.2012.2215261).
- Shi, W., Q. Ling, G. Wu, and W. Yin. (2015). “EXTRA: An Exact First-Order Algorithm for Decentralized Consensus Optimization”. *SIAM Journal on Optimization*. 25(2): 944–966. DOI: [10.1137/14096668X](https://doi.org/10.1137/14096668X).
- Shi, W., Q. Ling, K. Yuan, G. Wu, and W. Yin. (2014). “On the Linear Convergence of the ADMM in Decentralized Consensus Optimization”. *IEEE Transactions on Signal Processing*. 62(7): 1750–1761. DOI: [10.1109/TSP.2014.2304432](https://doi.org/10.1109/TSP.2014.2304432).
- Subhonmesh, B., S. H. Low, and K. M. Chandy. (2012). “Equivalence of branch flow and bus injection models”. In: *2012 50th Annual Allerton Conference on Communication, Control, and Computing (Allerton)*. 1893–1899. DOI: [10.1109/Allerton.2012.6483453](https://doi.org/10.1109/Allerton.2012.6483453).
- Suchithra, J., D. Robinson, and A. Rajabi. (2023). “Hosting Capacity Assessment Strategies and Reinforcement Learning Methods for Coordinated Voltage Control in Electricity Distribution Networks: A Review”. *Energies*. 16(5): 2371.
- Sun, K. and X. A. Sun. (2021). “A Two-Level ADMM Algorithm for AC OPF With Global Convergence Guarantees”. *IEEE Transactions on Power Systems*. 36(6): 5271–5281. DOI: [10.1109/TPWRS.2021.3073116](https://doi.org/10.1109/TPWRS.2021.3073116).
- Tian, Z., W. Wu, B. Zhang, and A. Bose. (2016). “Mixed-integer second-order cone programming model for VAR optimisation and network reconfiguration in active distribution networks”. *IET Generation, Transmission & Distribution*. 10(8): 1938–1946. URL: <https://doi.org/10.1049/iet-gtd.2015.1228>.
- Vosughi, A., A. Tamimi, A. B. King, S. Majumder, and A. K. Srivastava. (2022). “Cyber–physical vulnerability and resiliency analysis for DER integration: A review, challenges and research needs”. *Renewable and Sustainable Energy Reviews*. 168: 112794. URL: <https://www.sciencedirect.com/science/article/pii/S1364032122006785>.

- Wang, L., A. Dubey, A. H. Gebremedhin, A. K. Srivastava, and N. Schulz. (2022). “MPC-Based Decentralized Voltage Control in Power Distribution Systems With EV and PV Coordination”. *IEEE Transactions on Smart Grid*. 13(4): 2908–2919. DOI: [10.1109/TSG.2022.3156115](https://doi.org/10.1109/TSG.2022.3156115).
- Wang, Y., L. Wu, and S. Wang. (2017). “A Fully-Decentralized Consensus-Based ADMM Approach for DC-OPF With Demand Response”. *IEEE Transactions on Smart Grid*. 8(6): 2637–2647. DOI: [10.1109/TSG.2016.2532467](https://doi.org/10.1109/TSG.2016.2532467).
- Wu, T., C. Zhao, and Y.-J. A. Zhang. (2021). “Privacy-Preserving Distributed Optimal Power Flow With Partially Homomorphic Encryption”. *IEEE Transactions on Smart Grid*. 12(5): 4506–4521. DOI: [10.1109/TSG.2021.3084934](https://doi.org/10.1109/TSG.2021.3084934).
- Yang, Q., Y. Liu, T. Chen, and Y. Tong. (2019). “Federated machine learning: Concept and applications”. *ACM Transactions on Intelligent Systems and Technology (TIST)*. 10(2): 1–19.
- Yang, Y., X. Guan, Q.-S. Jia, L. Yu, B. Xu, and C. J. Spanos. (2022). “A Survey of ADMM Variants for Distributed Optimization: Problems, Algorithms and Features”. DOI: [10.48550/arXiv.2208.03700](https://doi.org/10.48550/arXiv.2208.03700).
- Zeng, S., A. Kody, Y. Kim, K. Kim, and D. K. Molzahn. (2022). “A reinforcement learning approach to parameter selection for distributed optimal power flow”. *Electric Power Systems Research*. 212: 108546. URL: <https://www.sciencedirect.com/science/article/pii/S0378779622006319>.
- Zhang, H., G. T. Heydt, V. Vittal, and J. Quintero. (2013). “An Improved Network Model for Transmission Expansion Planning Considering Reactive Power and Network Losses”. *IEEE Transactions on Power Systems*. 28(3): 3471–3479. DOI: [10.1109/TPWRS.2013.2250318](https://doi.org/10.1109/TPWRS.2013.2250318).
- Zhang, Y., X. Wang, J. Wang, and Y. Zhang. (2021). “Deep Reinforcement Learning Based Volt-VAR Optimization in Smart Distribution Systems”. *IEEE Transactions on Smart Grid*. 12(1): 361–371. DOI: [10.1109/TSG.2020.3010130](https://doi.org/10.1109/TSG.2020.3010130).

- Zhao, C., J. He, P. Cheng, and J. Chen. (2017). “Analysis of Consensus-Based Distributed Economic Dispatch Under Stealthy Attacks”. *IEEE Transactions on Industrial Electronics*. 64(6): 5107–5117. DOI: [10.1109/TIE.2016.2638400](https://doi.org/10.1109/TIE.2016.2638400).
- Zhao, J., Q. Zhang, Z. LIU, and X. WU. (2021). “A Distributed Black-Start Optimization Method for Global Transmission and Distribution Network”. *IEEE Transactions on Power Systems*. 36(5): 4471–4481. DOI: [10.1109/TPWRS.2021.3056096](https://doi.org/10.1109/TPWRS.2021.3056096).
- Zheng, W., W. Wu, B. Zhang, H. Sun, and Y. Liu. (2016). “A Fully Distributed Reactive Power Optimization and Control Method for Active Distribution Networks”. *IEEE Transactions on Smart Grid*. 7(2): 1021–1033. DOI: [10.1109/TSG.2015.2396493](https://doi.org/10.1109/TSG.2015.2396493).
- Zhou, X., E. Dall’Anese, and L. Chen. (2020). “Online Stochastic Optimization of Networked Distributed Energy Resources”. *IEEE Transactions on Automatic Control*. 65(6): 2387–2401. DOI: [10.1109/TAC.2019.2927925](https://doi.org/10.1109/TAC.2019.2927925).