Continuous-Time
Linear Models

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Abstract

I translate familiar concepts of discrete-time time series to contnuous-time equivalent. I cover lag operators, ARMA models, the relation between levels and differences, integration and cointegration, and the Hansen–Sargent prediction formulas.

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Introduction

Discrete-time linear ARMA processes and lag operator notation are convenient for lots of calculations. Continuous-time representations often simplify economic models, and can handle interesting nonlinearities as well. But standard treatments of continuous-time processes typically don't mention how to adapt the discrete-time linear model concepts and lag operator methods to continuous time. Here I attempt that translation.

The point of this monograph is to exposit the techniques, understand the intuition, and to make the translation from familiar discrete-time ideas. I do not pretend to offer anything new. I also don't discuss the technicalities. Hansen and Sargent (1991) is a good reference. Heaton (1993) describes many of these methods and provides a useful application. I assume basic knowledge of discrete-time time-series representation methods and continuous-time representations. Cochrane (2005a,b) cover the necessary background, but any standard reference covers the same material.

The concluding section collects the important formulas in one place.

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