Property Testing: A Learning Theory Perspective

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www.nowpublishers.com
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Outside North America:
now Publishers Inc.
PO Box 179
2600 AD Delft
The Netherlands
Tel. +31-6-51115274

The preferred citation for this publication is Dana Ron, Property Testing: A Learning Theory Perspective, Foundation and Trends® in Machine Learning, vol 1, no 3, pp 307–402, 2008

ISBN: 978-1-60198-182-0

Foundations and Trends® in Machine Learning

Volume 1 Issue 3, 2008

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Foundations and Trends® in Machine Learning, 2008, Volume 1, 4 issues. ISSN paper version 1935-8237. ISSN online version 1935-8245. Also available as a combined paper and online subscription.

Foundations and Trends® in
Machine Learning
Vol. 1, No. 3 (2008) 307–402

DOI: 10.1561/2200000004

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Abstract

Property testing deals with tasks where the goal is to distinguish between the case that an object (e.g., function or graph) has a prespecified property (e.g., the function is linear or the graph is bipartite) and the case that it differs significantly from any such object. The task should be performed by observing only a very small part of the object, in particular by querying the object, and the algorithm is allowed a small failure probability.

One view of property testing is as a relaxation of learning the object (obtaining an approximate representation of the object). Thus property testing algorithms can serve as a preliminary step to learning. That is, they can be applied in order to select, very efficiently, what hypothesis class to use for learning. This survey takes the learning-theory point of view and focuses on results for testing properties of functions that are of interest to the learning theory community. In particular, we cover results for testing algebraic properties of functions such as linearity, testing properties defined by concise representations, such as having a small DNF representation, and more.

Contents

1

Introduction

Property testing [82, 128] is the study of the following class of problems.

> *Given the ability to perform (local) queries concerning a particular object the problem is to determine whether the object has a predetermined (global) property or differs significantly from any object that has the property. In the latter case we say it is* far *from (having) the property. The algorithm is allowed a small probability of failure, and typically it inspects only a small part of the whole object.*

For example, the object may be a graph and the property is that it is bipartite, or the object may be a function and the property is that it is linear. It is usually assumed that the property testing algorithm is given query access to the object. When the object is a function f the queries are of the form: "what is $f(x)$?" while if the object is a graph then queries may be: "is there an edge between vertices u and v" or: "what vertex is the i^{th} neighbor of v?". In order to determine what it means to be far from the property, we need a distance measure between objects. In the case of functions it is usually the weight according to the uniform distribution of the symmetric difference between the functions, while in the case of graphs it is usually the number of edge modifications divided by some upper bound on the number of edges. When dealing with other objects (e.g., the object may be a set of points and the property may be that the set of points can be clustered in a certain way) one has to define both the types of queries allowed and the distance measure.

1.1 Property Testing as Relaxed Decision

Property testing problems are often viewed as a relaxation of decision problems. Namely, instead of requiring that the algorithm decide whether the object has the property or does not have the property, the algorithm is required to decide whether the object has the property or is far from having the property. Given this view there are several scenarios in which property testing may be useful.

- If the object is very large, then it is infeasible to examine all of it and we must design algorithms that examine only a small part of the object and make an approximate decision based on what they view.
- Another scenario is when the object is not too large to fully examine, but the exact decision problem is $\mathcal{NP}$-hard. In such a case some form of approximate decision is necessary if one seeks an efficient algorithm and property testing suggest one such form. We note that in some cases the approximation essentially coincides with *standard* notions of approximation problems (e.g., Max-Cut [82]) while in others it is quite different (e.g., k-Colorability [82]).
- It may be the case that the object is not very large and there is an efficient (polynomial-time) algorithm for solving the problem exactly. However, we may be interested in a *very* efficient (sublinear-time) algorithm, and are willing to tolerate the approximation/error it introduces.
- Finally, there are cases in which typical no-instances of the problem (that is, objects that do not have the property) are actually relatively far from having the property. In such cases we may first run the testing algorithm. If it rejects the object

then we reject it and otherwise we run the exact decision procedure. Thus, we save time on the typical no-instances. This is in particular useful if the testing algorithm has one-sided error so that it never rejects yes-instances (that have the property).

In all the aforementioned scenarios we are interested in testing algorithms that are much more efficient than the corresponding decision algorithms, and in particular have complexity that is sublinear in the size of the object.

1.2 Property Testing and Learning (Estimation)

Here when we say *learning* we mean outputting a good estimate of the target object.[1] Thus, another view of property testing is as a relaxation of learning (with queries and under the uniform distribution).[2] Namely, instead of asking that the algorithm output a good estimate of the function (object), which is assumed to belong to a particular class of functions $\mathcal{F}$, we only require that the algorithm decide whether the function belongs to $\mathcal{F}$ or is far from any function in $\mathcal{F}$. Given this view, a natural motivation for property testing is to serve as a preliminary step before learning (and in particular, agnostic learning (e.g., [107]) where no assumption is made about the target function but the hypothesis should belong to a particular class of functions): we can first run the testing algorithm to decide whether to use a particular class of functions as our hypothesis class.

Here too we are interested in testing algorithms that are more efficient than the corresponding learning algorithms. As observed in [82], property testing is no harder than *proper* learning (where the learning algorithm is required to output a hypothesis from the same class of functions as the target function). Namely, if we have a proper learning

[1] One may argue that property testing is also a certain form of learning as we learn information about the object (i.e., whether it has a certain property or is far from having the property). However, we have chosen to adopt the notion of learning usually used in the computational learning theory community.

[2] Testing under non-uniform distributions (e.g., [1, 92]) and testing with random examples (e.g., [105]) have been considered (and are discussed in this survey), but most of the work in property testing deals with testing under the uniform distributions and with queries.

algorithm for a class of functions F then we can use it as a subroutine to test the property: "does the function belong to F" (see Section 2.2 for a formal statement and proof).

Choosing between the two viewpoints. The choice of which of the aforementioned views to take is typically determined by the type of objects and properties in question. Much of property testing deals with combinatorial objects and in particular graphs. For such objects it is usually more natural to view property testing as a relaxation of exact decision. Indeed, there are many combinatorial properties for which there are testing algorithms that are much more efficient than the corresponding (exact) decision algorithms. On the other hand, when the objects are functions, then it is usually natural to look at property testing from a learning theory perspective. In some cases, both viewpoints are appropriate. This survey focuses on the latter perspective.

1.3 Property Testing and Hypothesis Testing

The notion of property testing is related to that of *hypothesis testing* (see e.g., [108, Chap. 8]) and indeed the distinction between estimation and testing is well known in the mathematical statistics literature. In this context, having the tested property (belonging to the corresponding class of objects) is called the *null* hypothesis, while being ϵ-far from the property (where ϵ is the distance parameter that the algorithm is given as input) is the *alternative* hypothesis. There are two major mathematical approaches to the study of testing in statistics (see, e.g., [136] and [113]). In the first, the alternative is taken to approach the null hypothesis at a certain rate as a function of the number of data points; when the correct rate is chosen the error probabilities stabilize at values strictly greater than zero and strictly less than one. In the second approach, the alternative is held fixed as the number of data points grows; in this case error probabilities go to zero and large deviation methods are used to assess the rate at which error probabilities go to zero. Aspects of both of these approaches can be found in the property testing literature.

While in many cases the particular problems studied in the property testing literature are somewhat different from those typically studied in the mathematical statistics literature, the work on testing properties of distributions (which is discussed shortly in Section 6.3) deals with problems that are similar (or even the same) as those studied in mathematical statistics.

We also note that there are several works with a mathematical statistics flavor that are related to property testing and appeared in the computational learning literature (e.g., [33, 112, 137]).

1.4 Topics and Organization

We start with some preliminaries, which include basic definitions and notations. The preliminaries also include a precise statement and proof of the simple but important observation that testing is no harder than learning.

In Section 3, we consider the first type of properties that were studied in the context of property testing: algebraic properties. These include testing whether a function is (multi-)linear and more generally whether it is a polynomial of bounded degree. This work has implications to coding theory, and some of the results played an important role in the design of Probabilistically Check Proof (PCP) systems.

In Section 4, we turn to the study of function class that have a concise (propositional logic) representation such as singletons, monomials, and small DNF formula. This section includes a general result that applies to many classes of functions, where the underlying idea is that testing is performed by *implicit* learning.

The results in Sections 3 and 4 are in the *standard* model of testing. That is, the underlying distribution is uniform and the algorithm may perform queries to the function. In Section 5, we discuss distribution-free testing, and testing from random examples alone.

Finally, in Section 6, we give a more brief survey of other results in property testing. These include testing monotonicity, testing of clustering, testing properties of distributions, and more.

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