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Data Analytics on Graphs Part II: Signals on Graphs

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ABSTRACT

The area of Data Analytics on graphs deals with information processing of data acquired on irregular but structured graph domains. The focus of Part I of this monograph has been on both the fundamental and higher-order graph properties, graph topologies, and spectral representations of graphs. Part I also establishes rigorous frameworks for vertex clustering and graph segmentation, and illustrates the power of graphs in various data association tasks. Part II embarks on these concepts to address the algorithmic and practical issues related to data/signal processing on graphs, with the focus on the analysis and estimation of both deterministic and random data on graphs. The fundamental ideas related to graph signals are introduced through a simple and intuitive, yet general enough case study of multisensor temperature field estimation. The concept of systems on graph

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is defined using graph signal shift operators, which generalize the corresponding principles from traditional learning systems. At the core of the spectral domain representation of graph signals and systems is the Graph Fourier Transform (GFT), defined based on the eigendecomposition of both the adjacency matrix and the graph Laplacian. Spectral domain representations are then used as the basis to introduce graph signal filtering concepts and address their design, including Chebyshev series polynomial approximation. Ideas related to the sampling of graph signals, and in particular the challenging topic of data dimensionality reduction through graph subsampling, are presented and further linked with compressive sensing. The principles of time-varying signals on graphs and basic definitions related to random graph signals are next reviewed. Localized graph signal analysis in the joint vertex-spectral domain is referred to as the vertex-frequency analysis, since it can be considered as an extension of classical time-frequency analysis to the graph serving as signal domain. Important aspects of the local graph Fourier transform (LGFT) are covered, together with its various forms including the graph spectral and vertex domain windows and the inversion conditions and relations. A link between the LGFT with a varying spectral window and the spectral graph wavelet transform (SGWT) is also established. Realizations of the LGFT and SGWT using polynomial (Chebyshev) approximations of the spectral functions are further considered and supported by examples. Finally, energy versions of the vertex-frequency representations are introduced, along with their relations with classical time-frequency analysis, including a vertex-frequency distribution that can satisfy the marginal properties. The material is supported by illustrative examples.

Keywords: graph theory; random data on graphs; big data on graphs; signal processing on graphs; machine learning on graphs; graph

topology learning; systems on graphs; vertex-frequency estimation; graph neural networks; graphs and tensors.

1

Introduction

Graphs are structures, often irregular, constructed in a way to represent the observed data and to account, in a natural way, the specific interrelationships between the data sources. However, traditional approaches have been established outside Machine Learning and Signal Processing, with which largely focus on analyzing the underlying graphs rather than dealing with signals on graphs. Moreover, given the rapidly increasing availability of multisensor and multinode measurements, likely recorded on irregular or ad-hoc grids, it would be extremely advantageous to analyze such structured data as "signals on graphs" and thus benefit from the ability of graphs to account for spatial sensing awareness, physical intuition and sensor importance, together with the inherent "local versus global" sensor association. The aim of Part II of this monograph is therefore to establish a common language between graph signals which are observed on irregular signal domains, and some of the fundamental paradigms in Learning Systems, Signal Processing and Data Analytics, such as spectral analysis, system transfer function, digital filter design, parameter estimation, and optimal denoising.

In classical Data Analytics and Signal Processing, the signal domain is determined by equidistant time instants or by a set of spatial sensing points on a uniform grid. However, increasingly the actual data sensing domain may not even be related to the physical dimensions of time and/or space, and it typically does exhibit various forms of irregularity, as, for example, in social or web-related networks, where the sensing points and their connectivity pertain to specific objects/nodes and ad-hoc topology of their links. It should be noted that even for the data acquired on well defined time and space domains, the introduction of new relations between the signal samples, through graphs, may yield new insights into the analysis and provide enhanced data processing (for example, based on local similarity, through neighborhoods). We therefore set out to demonstrate that the advantage of graphs over classical data domains is that graphs account naturally and comprehensively for irregular data relations in the problem definition, together with the corresponding data connectivity in the analysis (Chen *et al.*, 2014; Ekambaram, 2014; Gavili and Zhang, 2017; Hamon *et al.*, 2016; Moura, 2018; Sandryhaila and Moura, 2013; Shuman *et al.*, 2013; Vetterli *et al.*, 2014).

To build up the intuition behind the fundamental ideas of signals/data on graphs, a simple yet general example of multisensor temperature estimation is first considered in Section 2. Basic concepts regarding the signals and systems on graphs are presented in Section 3, including basic definitions, operations and transforms, which generalize the foundations of traditional signal processing. Systems on graphs are interpreted starting from a comprehensive account of the existing and the introduction of a novel, isometric, graph signal shift operator. Further, graph Fourier transform is defined based on both the adjacency matrix and the graph Laplacian and it serves as the basis to introduce graph signal filtering concepts. Various ideas related to the sampling of graph signals, and particularly, the challenging topic of their subsampling, are reviewed in Section 4. Sections 6 and 7 present the concepts of time-varying signals on graphs and introduce basic definitions related to random graph signals. Localized graph signal behavior can be simultaneously characterized in the vertex-frequency domain, which is discussed in Section 8. This section also covers the important topics of local graph Fourier transform, various forms of its inversion, relations with the frames and links with the graph wavelet transform. Energy versions of the vertex-frequency representations are also considered, along with their relations with classical time-frequency analysis.

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