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Machine Learning for Automated Theorem Proving: Learning to Solve SAT and QSAT

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ABSTRACT

The decision problem for Boolean satisfiability, generally referred to as SAT, is the archetypal NP-complete problem, and encodings of many problems of practical interest exist allowing them to be treated as SAT problems. Its generalization to quantified SAT (QSAT) is PSPACE-complete, and is useful for the same reason. Despite the computational complexity of SAT and QSAT, methods have been developed allowing large instances to be solved within reasonable resource constraints. These techniques have largely exploited algorithmic developments; however machine learning also exerts a significant influence in the development of state-of-the-art solvers. Here, the application of machine learning is delicate, as in many cases, even if a relevant learning problem can be solved, it may be that incorporating the result into a SAT or QSAT solver is counterproductive, because the run-time of such solvers can be sensitive to small implementation changes. The application of better machine learning methods in this area is thus an ongoing challenge, with characteristics unique to the field. This work provides a comprehensive review of the research to date on incorporating machine learning into SAT and QSAT solvers, as a resource for those interested in further advancing the field.

1

Introduction

Automated theorem proving represents a significant and long-standing area of research in computer science, with numerous applications. A large proportion of the methods developed to date for the implementation of *automated theorem provers (ATPs)* have been algorithmic, sharing a great deal in common with the wider study of heuristic search algorithms (Harrison, 2009). However in recent years researchers have begun to incorporate *machine learning (ML)* methods (Murphy, 2012) into ATPs in an effort to extract better performance.

ATPs represent a compelling area in which to explore the application of ML. It is well-known that theorem-proving problems are computationally intractable, with the exception of specific, limited cases. Even in the apparently simple case of *propositional logic* the task is NP-hard, and adding *quantifiers* makes the task PSPACE-complete (Garey and Johnson, 1979). Taking a small step further we arrive at *first-order logic (FOL)*, which is undecidable (Boolos *et al.*, 2007). In addition to the general computational complexity of theorem-proving problems, they have a common property that makes them challenging as a target for ML: even the most trivial change to the statement of a problem can have a huge impact on the complexity of any subsequent proof

attempt (Fuchs and Fuchs, 1998; Hutter *et al.*, 2007; Hutter *et al.*, 2009; Biere and Fröhlich, 2015; Biere and Fröhlich, 2019).

The aim of this work is to review the research that has appeared to date on incorporating ML methods into solvers for propositional *satisfiability (SAT)* problems, and also solvers for its immediate variants such as *quantified SAT (QSAT)*.

In a sense, these are some of the simplest possible ATP problems. (Any instance of a SAT problem can be represented as a Boolean formula in conjunctive normal form, and it is undeniably hard to propose anything much simpler.) But the combination of the computational challenges such problems present, and the enormous range of significant, practical applications that can be addressed this way, makes general solvers for SAT and its friends a compelling target for research. Marques-Silva (2008) reviews applications of SAT solvers circa 2008, and the interested reader might consult work applying them to bounded model checking (Biere *et al.*, 1999; Clarke *et al.*, 2001), planning (Kautz and Selman, 1992; Kautz, 2006), bioinformatics (Lynce and Marques-Silva, 2006; Graça *et al.*, 2010), allocation of radio spectrum (Fréchet *et al.*, 2016), and software verification (Babić and Hu, 2007). A further notable application has been the solution of the *Boolean Pythagorean triples* problem by Heule *et al.* (2016), resulting in what is currently considered the longest mathematical proof in history.

1.1 Coverage

Work on applying ML in this context appears to have started with Ertel *et al.* (1989) and Johnson (1989). At that time the limited availability of computing power and the limitations of existing solvers made the studies necessarily small by current standards, in terms of the size of the problems addressed, and also of the ML methods applied. This review is the result of a systematic search for literature appearing from then until late 2020.

SAT/QSAT solving and machine learning are both large and long-standing areas of research, and each has a correspondingly large literature. As these are two apparently rather unrelated fields, it is therefore inevitable that any reader versed in one might feel less confident with

the other. (It has certainly been my experience in talking to researchers from both domains that this is often the case.) It would not be feasible to explain either, let alone both, areas in full detail here; and in any case, this is not intended to be a textbook on either subject. I have provided an introduction to each, but experts in either area might find one presentation overly elementary and the other too brief. The aim has been to provide sufficient information to make this work self-contained for both sides while maintaining a manageable length; however I expect that for many there will be areas where further reading will be necessary.

I wrote this work guided by two central aims for what the reader should gain from it. First, they should know *what has been tried*. In presenting the material, I concentrate on the learning methods used and the way in which they have been incorporated into solvers. As the literature rarely if ever presents methods not leading to performance improvements of some kind, less consideration is given to the details of the level of improvement achieved, because I believe such details are secondary to my second aim, which is: that the reader should understand the often *complex interaction between ATP and ML that is needed for success* in these undeniably challenging applications.

In order to achieve these aims it was necessary to be quite selective in the level of detail used to present various methods. Some research is presented in very great detail, relating to the learning method and its relationship with a solver, the description of the data used, or the experimental method employed. Other research is presented in less detail, although I hope at a level sufficient to allow the reader to understand what was done, and why. With the exception of the Chapters on ATP and ML, each Chapter presents a discussion summarizing what I believe are the central lessons to be taken from it. Where methods have been presented in greater detail, it is generally in the service of these arguments.

1.2 Outline of the review

Chapter 2 presents an introduction to the SAT problem, and to contemporary methods for its solution. Much of this section is devoted to summarizing the operation of *Conflict-Driven Clause Learning (CDCL)*

solvers;¹ first, as these form the core of many of the most successful SAT solvers available; and second, because there are many distinct areas of their operation that have provided a point at which to introduce ML, and this therefore provides a road map for a large portion of the review. This section also briefly describes *portfolio solvers* and *local search* solvers, which have also been targets for ML, and which will be described further in later Chapters.

Chapter 3 provides a complementary introduction to some of the ML methods most commonly applied to SAT and QSAT solvers; this work spans supervised and unsupervised learning in addition to n -armed bandits, reinforcement learning, neural networks and evolutionary computing. In addition we describe some of the main sources of problems available for testing SAT and QSAT solvers; as these are often annotated such that we know which problems are satisfiable, and which are not, they provide a valuable resource for training ML methods.

Many applications of ML in this domain have required a phase of *feature engineering*, whereby a problem, typically expressed in *conjunctive normal form (CNF)*, is converted into a vector of real numbers suitable for use by an ML method. Chapter 4 reviews common sets of features that have been used, and that continue to form the basis for many ongoing studies. More recent work has made significant use of *graph neural networks* to (partially) automate the feature engineering process, and we introduce these here also.

There are, broadly-speaking, four ways in which ML has been applied to SAT solvers: by treating SAT directly as a classification problem; by building *portfolios* of existing SAT solvers; by modifying CDCL solvers; and by treating the problem as a form of *local search*.

In Chapter 5 we describe work aiming to identify satisfiability directly, without necessarily also obtaining a satisfying assignment of variables if one exists. Here, the SAT problem is treated as a classification

¹There is an important distinction to be made here for the avoidance of confusion. The term ‘learning’ in the context of a CDCL solver is, at least at first glance, unconnected to the idea of machine learning. It is used to describe the addition of one or more new clauses to a problem after analysing a conflict during the search for a satisfying assignment; this is explained in more detail in Section 2.4.4. The use of the term ‘learning’ in both contexts is ubiquitous however, and we expand on the distinction a little further in Section 3.1.5.

problem: given a formula f , we aim to return the answer ‘yes’ or ‘no’, indicating whether or not the problem is satisfiable. In some cases it may be possible to extract a satisfying assignment as a side-effect.

Portfolio solvers are addressed in Chapter 6. Here, a collection of different SAT solvers is used in some combination to attack a problem. Chapter 7 then reviews the application of ML to CDCL solvers, addressing in turn the way in which ML has been applied to the individual elements described in Chapter 2. Chapter 8 describes the application of ML to local search SAT solvers.

In Chapter 9 we address attempts to introduce ML into solvers for QSAT. This area has received comparatively little attention, but work has appeared addressing ML for both portfolio solvers, and individual solvers.

While this review mainly addresses solvers for SAT and QSAT—these problems having received considerable attention as they have clear and significant applications—in Chapter 10 we briefly address machine learning applied to *intuitionistic propositional logic (IPL)* (Dalen, 2001). While this logic is of more foundational interest, having few applications beyond the philosophy of mathematics, it is related sufficiently closely to propositional logic that I feel attempts to apply machine learning to the search for proofs in IPL are relevant.

Chapter 11 concludes.

1.3 Limits to Coverage

A body of research exists addressing methods for automatically configuring algorithms that expose parameters—a process sometimes referred to as the *algorithm configuration problem*. Effective methods such as ParamILS (Hutter *et al.*, 2009) and, perhaps the best-known system of this kind, *Sequential Model-based Algorithm Configuration (SMAC)* (Hutter *et al.*, 2011), are now common. Algorithms in this class can clearly be applied to SAT/QSAT and related solvers, which invariably have parameters governing aspects of their operation. In compiling this review, I have aimed to focus on material that has a *specific emphasis* on SAT, QSAT and (closely) related problems. As a result, I decided not to describe in detail work such as that of Kadioglu

et al. (2010) and Malitsky *et al.* (2013), that develops a general method for algorithm configuration and uses SAT as a test case, or Hutter *et al.* (2007) and Mangla *et al.* (2020), that is predominantly an application of an existing algorithm configuration method to SAT. For the same reasons, I have not included work that mainly relies on the application of general methods for selecting an algorithm from a collection of candidates; see Kotthoff (2016) for a review of such methods.

1.4 What Should the Reader Gain?

It is my hope that ML researchers might gain from this work, an understanding of state-of-the-art SAT and QSAT solvers that is sufficient to make new opportunities for applying their own ML research to this domain clearly visible. It is equally my hope that ATP researchers will gain a complementary understanding, giving them a clear appreciation of how state-of-the-art machine learning might help them to design better solvers. For both constituencies, I aim to show what has already been achieved at the time of writing, at a level of detail sufficient to provide a basis for new work.

Acknowledgements

In 2016 Josef Urban invited me to speak at the *1st Conference on Artificial Intelligence and Theorem Proving (AITP)*. I offered to give a survey talk on applications of machine learning to automated theorem provers. Having given the talk it seemed like a good idea to write it up in full.

I thought this would be a straightforward process, but it did not take long to discover that the full extent of the literature on the subject is genuinely impressive. In any case, here is the result.

Thanks for the invitation Josef.

I've done my share of reviewing, and I'm aware that reviewing a work of this length is a major undertaking. I therefore offer great thanks to the anonymous reviewer for their careful reading and numerous useful suggestions.

In reading the literature underlying this work there were inevitably occasions where I felt the need to contact the original authors for clarification. All responded quickly and helpfully. Thanks to all of them.

Appendices

A

Abbreviations

Abbreviation	Meaning
ATP	Automated theorem-prover
CAL	Clauses active list
CDCL	Conflict-Driven Clause Learning
CHB	Conflict history-based
CIG	Clause incidence graph
CNF	Conjunctive normal form
CNN	Convolutional neural network
CSP	Constraint satisfaction problem
CVIG	Clause-variable incidence graph
DAG	Directed acyclic graph
DPLL	Davis, Putnam, Logemann, Loveland
DRAT	Deletion Resolution Asymmetric Tautology
EHM	Empirical hardness model
EP	Evolutionary programming
ES	Evolutionary strategy

Continued overleaf...

Abbreviation	Meaning
ERWA	Exponential recency weighted average
EVSIDS	Exponential VSIDS
GA	Genetic algorithm
GLR	Global learning rate
GNN	Graph neural network
GP	Genetic program
IPL	Intuitionistic propositional logic
LBD	Literals blocks distance
LRB	Learning rate branching
LSTM	Long short-term memory
MAB	Multi-armed bandit
ML	Machine learning
MLB	Machine learning-based restart
MLP	Multi-layer perceptron
MPNN	Message-passing neural network
NN	Neural network
QSAT	Quantified satisfiability
RL	Reinforcement learning
SAT	Satisfiability
SVM	Support vector machine
SGDB	Stochastic Gradient Descent Branching
UC	Unsatisfiable core
UCB	Upper confidence bound
UIP	Unique implication point
VIG	Variable incidence graph
VSIDS	Variable State Independent Decaying Sum

B

Symbols

General

I	Identity matrix
\mathbb{R}^i	Set of i -dimensional vectors with real elements
v_i	Element i of a vector \mathbf{v}
$\mathbb{R}^{i \times j}$	Set of i by j matrices with real elements
$M_{i,j}$	Element at row i , column j of a matrix \mathbf{M}
\mathbb{I}	Indicator function: $\mathbb{I}[P]$ is 1 if P is true and 0 otherwise
$\mathbf{1}_{ij}$	i by j matrix with all elements equal to 1.
$N(\mathbf{x}; \boldsymbol{\mu}, \boldsymbol{\Sigma})$	Multivariate normal density with mean $\boldsymbol{\mu}$ and covariance $\boldsymbol{\Sigma}$
\otimes	Element-by-element multiplication of vectors
$[n]$	The set $\{1, \dots, n\}$

Continued overleaf...

 The SAT Problem

V	Set of variables
C	Set of clauses
v	A variable. or $ V $, according to context
c	A clause, or $ C $, according to context
l	A literal
f, ϕ, ψ	Propositional formulas
A	Assignment
$a(v)$	Activity of a variable
$a(c)$	Activity of a clause

 Machine Learning

n	Dimension of feature space for a classifier
m	Size of training set
\mathbf{s}	Sequence containing m training examples
F	Function mapping instances of a problem to feature vectors
\mathbb{A}	Learning algorithm
\mathcal{H}	Hypothesis space
Z	Constant normalizing a probability distribution
C	Random variable denoting a class
\mathbf{x}	Instance vector
k	Dimension of the extended space
p	Number of basis functions
ϕ_i	Basis functions
λ	Regularization parameter
$\phi(\mathbf{x})$	Mapping from instance \mathbf{x} to the extended space
Φ	Matrix of $\phi(\mathbf{x})$ for \mathbf{x} in a training sequence
$\sigma(x)$	Step or sigmoid function
$\boldsymbol{\theta}, \mathbf{w}$	Vectors of parameters
K	Number of clusters

Continued overleaf...

Machine Learning

r_i	Reward sequence
$r_{i,t}$	Reward from arm i of a multi-armed bandit at time t
α	EWMA discounting factor
γ	Bandit or reinforcement learning discount factor
\hat{r}_T	Estimated bandit reward at time T
\mathcal{S}	RL state set
\mathcal{A}	RL action set
p	RL policy
R	RL discounted reward
μ	Step size for gradient descent
c	Number of classes in a problem
\mathbf{K}	CNN kernel
t	Step in a sequence
T	Final step in a sequence
O	Objective function

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