An Introduction into Robust Mechanism Design

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Outside North America:
now Publishers Inc.
PO Box 179
2600 AD Delft
The Netherlands
Tel. +31-6-51115274

The preferred citation for this publication is D. Bergemann and S. Morris, An Introduction into Robust Mechanism Design, Foundation and Trends® in Microeconomics, vol 8, no 3, pp 169–230, 2012

ISBN: 978-1-60198-644-3

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Foundations and Trends® in Microeconomics, 2012, Volume 8, 4 issues. ISSN paper version 1547-9846. ISSN online version 1547-9854. Also available as a combined paper and online subscription.

Foundations and Trends® in
Microeconomics
Vol. 8, No. 3 (2012) 169–230

DOI: 10.1561/0700000057

An Introduction to Robust Mechanism Design*

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Abstract

This essay provides an introduction to our recent work on robust mechanism design. The objective is to provide an overview of the research agenda and its results. We present the main results and illustrate many of them in terms of a common and canonical example, the single unit auction with interdependent values. In addition, we provide an extended discussion about the role of alternative assumptions about type spaces in our work, and the literature at large, in order to explain

* We would like to thank Eric Maskin for inviting us to publish the work covered in this survey in a collection of the World Scientific Series in Economic Theory edited by Eric. An early version of this essay appeared as an introduction in Bergemann and Morris (2012b). We would like to thank our co-authors Hanming Fang, Moritz Meyer-ter-Vehn, Karl Schlag, Satoru Takahashi and Olivier Tercieux in this research agenda and Nemanja Antic, Andreas Blume, Tilman Borgers, Jacques Cremer, Moritz Meyer-ter-Vehn, Phil Reny and Olivier Tercieux for comments on this essay. We had the opportunity to deliver the present material at a number of invited lectures, notably at Boston University, Northwestern University and the European and North American Econometric Society Meetings and a set of slides which cover and accompany this essay can be found at http://dirkbergemann.commons.yale.edu/files/2010/12/robustmechanismdesign1.pdf.

the common logic of the informational robustness approach that unifies the work.

Keywords: Mechanism design; robust mechanism design; common knowledge; universal type space; interim equilibrium; ex post equilibrium; dominant strategies; rationalizability; partial implementation; full implementation; robust implementation.

JEL Codes: C79, D82

Contents

1

Introduction

This essay brings together and presents a number of results on the theme of robust mechanism design and robust implementation that we have been working on in the past decade. This work examines the implications of relaxing the strong informational assumptions that drive much of the mechanism design literature. It discusses joint work of the two of us with each other and with co-authors Hanming Fang, Moritz Meyer-ter-Vehn, Karl Schlag, Satoru Takahashi, and Olivier Tercieux.

The objective of this essay is to provide the reader with an overview of the research agenda pursued in these papers. We present the main results of these papers and illustrate many of them in terms of a common and canonical example, the single unit auction with interdependent values. It is our hope that the use of this example facilitates the presentation of the results and that it brings the main insights within the context of an important economic mechanism, the generalized second price auction. In addition, we include an extended discussion about the role of alternative assumptions about type spaces in our work and the literature, in order to explain the common logic of the informational robustness approach that unifies the work surveyed in this essay.

The mechanism design literature of the last thirty years has been a huge success on a number of different levels. There is a beautiful theoretical literature that has shown how a wide range of institutional design questions can be formally posed as mechanism design problems with a common structure. Elegant characterizations of optimal mechanisms have been obtained. Market design has become more important in many economic arenas, both because of new insights from theory and developments in information and computing technologies, which enable the implementation of large scale trading mechanisms. A very successful econometric literature has tested auction theory in practise.

However, there has been an unfortunate disconnect between the general theory and the applications/empirical work: mechanisms that work in theory or are optimal in some class of mechanisms often turn out to be too complicated to be used in practise. Practitioners have then often been led to argue in favor of using simpler but apparently sub-optimal mechanisms. It has been argued that the optimal mechanisms are not "robust" — i.e., they are too sensitive to fine details of the specified environment that will not be available to the designer in practise. These concerns were present at the creation of the theory and continue to be widespread today.[1] In response to the concerns, researchers have developed many attractive and influential results by imposing (in a somewhat ad hoc way) stronger solution concepts and/or simpler mechanisms motivated by robustness considerations. Our starting point is the influential concern of Wilson (1987) regarding the robustness of the game theoretic analysis to the common knowledge assumptions:

> "Game theory has a great advantage in explicitly analyzing the consequences of trading rules that presumably are really common knowledge; it is deficient to the extent it assumes other features to be common

[1] Hurwicz (1972) discussed the need for "non-parametric" mechanisms wich are independent of the distributional assumptions regarding the willingness-to-pay of the agents. Wilson (1985) states that trading rules should be "belief-free" by requiring that they "should not rely on features of the agents' common knowledge, such as their probability assessments." Dasgupta and Maskin (2000) seek "detail-free" auction rules "that are independent of the details — such as functional forms or distribution of signals - of any particular application and that work will in a broad range of circumstances."

> knowledge, such as one agent's probability assessment about another's preferences or information. I foresee the progress of game theory as depending on successive reductions in the base of common knowledge required to conduct useful analyses of practical problems. Only by repeated weakening of common knowledge assumptions will the theory approximate reality."

Wilson emphasized that as analysts we are tempted to assume that too much information is common knowledge among the agents, and suggested that more robust conclusions would arise if researchers were able to relax those common knowledge assumptions. Harsanyi (1967–68) had the original insight that relaxing common knowledge assumptions is equivalent to working with a type space which is larger if there is less common knowledge. A natural theoretical question then is to ask whether it is possible to explicitly model the robustness considerations in such a way that stronger solution concepts and/or simpler mechanisms emerge endogenously. In other words, if the optimal solution to the planner's problem is too complicated or too sensitive to be used in practice, it is presumably because the original description of the planner's problem was itself flawed. We would like to investigate if improved modelling of the planner's problem endogenously generates the "robust" features of mechanisms that researchers have been tempted to assume. Our research agenda in robust mechanism design is therefore to *first* make explicit the implicit common knowledge assumptions and then *second* to weaken them.

Thus, formally, our approach suggests asking what happens to the conventional insights in the theory of mechanism design when confronted with larger and richer type spaces with weaker requirements regarding the common knowledge of between the designer and the agents. In this respect, a very important contribution is due to Neeman (2004) who showed that the small type space assumption is of special importance for the full surplus extraction results, as in Myerson (1981) and Cremer and McLean (1988). In particular, he showed that the full surplus extraction results fail to hold if agents' private information doesn't display a one-to-one relationship between each agent's beliefs about the other agents and his preferences (valuation). The extended dimensionality relative to the standard model essentially allows for a richer set of higher-order beliefs.

Similarly, in an analysis of the first price auction, Fang and Morris (2006) look at the role of richer type spaces by considering private values but allowing for multidimensional types. There, each bidder observes his own private valuation and a noisy signal of his opponent's private valuation. This model of private information stands in stark contrast to the standard analysis of auctions with private values, where each agent's belief about his competitor is simply assumed to coincide with the common prior. In the presence of multidimensional private signals, Fang and Morris (2006) find that the celebrated revenue equivalence result between the first and the second price auction fails to hold. With the richer type space, it is not even possible to rank the auction format with respect to their expected revenue.

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