# Interference in Large Wireless Networks

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## Abstract

Since interference is the main performance-limiting factor in most wireless networks, it is crucial to characterize the interference statistics. The two main determinants of the interference are the network geometry (spatial distribution of concurrently transmitting nodes) and the path loss law (signal attenuation with distance). For certain classes of node distributions, most notably Poisson point processes, and attenuation laws, closed-form results are available, for both the interference itself as well as the signal-to-interference ratios, which determine the network performance.

This monograph presents an overview of these results and gives an introduction to the analytical techniques used in their derivation. The node distribution models range from lattices to homogeneous and clustered Poisson models to general motion-invariant ones. The analysis of the more general models requires the use of Palm theory, in particular conditional probability generating functionals, which are briefly introduced in the appendix.

# Contents

# 1

# Introduction

Due to the scarcity of the wireless spectrum, it is not possible in large wireless networks to separate concurrent transmissions completely in frequency. Some transmissions will necessarily occur at the same time in the same frequency band, separated only in space, and the signals from many undesired or *interfering* transmitters are added to the desired transmitter's signal at a receiver. This interference can be mitigated quite efficiently in systems with centralized control, where a base station or access point can coordinate the channelization and the power levels of the individual terminals, or where sophisticated multi-user detection or interference cancellation schemes can be implemented. However, many emerging classes of wireless systems, such as ad hoc and sensor networks, mesh networks, cognitive networks, and cellular networks with multihop coverage extensions, do not permit the same level of centralized control but require a more distributed resource allocation. For example, channel access schemes are typically based on carrier sensing, and power control is performed on a pairwise rather than a network-wide basis, if at all. In these networks, interference is not tightly controllable and subject to considerable uncertainty. Consequently, interference is the main performance-limiting factor in most emerging wireless networks, and the statistical characterization of the interference power becomes critical.

In this monograph, we derive results for the interference statistics in large wireless networks that are subject to one or several sources of randomness, including the node distribution, the channel access scheme, and the channel or fading states. There are two main factors that shape the interference: First, since interfering signals are only separated in space, the *spatial distribution of the concurrently transmitting nodes*; second, since the amount of interference caused depends on the signal attenuation with distance, the *path loss law*. The first factor consists of two parts, the node distribution on the one hand and the channel access scheme (MAC) on the other. It is their combination that determines the distribution of transmitting nodes. For example, even if the nodes are very randomly distributed, a good MAC scheme will ensure a certain spacing between concurrent transmitters or, better, between receivers and interferers; hence the distribution of the transmitters at any given moment may be fairly regular. Since the performance of a network is determined by the signal-to-interference-and-noise ratios (SINRs) or, in the pure interference-limited case, by the signal-to-interference ratios (SIRs), the SIR distributions are also derived, usually in the form of outage probabilities $\mathbb{P}(\mathsf{SINR} < \theta)$, which correspond to the cumulative distributions.

The exact characterization of the interference or SIRs for general node distributions and MAC schemes is a very challenging problem. Since our focus in this monograph is on analytical results and on the underlying mathematical techniques, the network models are partly chosen for their tractability, not necessarily because they are the most realistic ones. The analytical methods are best illustrated when applied to simple models, and the results derived will provide bounds for more elaborate ones, in particular when the models considered are in some sense extreme, such as lattice networks on one end and "completely spatially irregular" networks (Poisson networks) on the other. Also, general design principles and guidelines can be inferred more easily from analytical results, and it is our hope the analytical techniques are described in enough detail to enable the reader to apply them to other types of networks.

We restrict ourselves to the statistics of the (aggregate) interference *power* when the sources of randomness include the node distribution, the fading states of the channels, and the channel access scheme. We will not be discussing the amplitude statistics of the interference, which depend strongly on the type of signaling employed and may, conditioned on the power, be well approximated by a Gaussian or not [22]. With Gaussian codebooks, the interference amplitude is certainly conditionally Gaussian, and if it is treated as noise at the receiver, its variance or power is the relevant statistic for the achievable link performance. While not optimum in general, treating interference as noise is, in fact, optimum in the Gaussian weak interference or *noisy interference* regime [42]. In this regime, sophisticated multi-user detectors do not perform better than simple single-user detectors, and the expected value of $\log_2(1 + \mathsf{SINR})$ is the actual (bandwidth-normalized) capacity.

This monograph is organized as follows:

Section 2 derives the interference for networks with deterministic node placement, in particular lattices. Section 3 is devoted to Poisson networks, where the nodes are distributed as a Poisson point process (PPP). The PPP model is by far the most popular, thanks to its analytical tractability. It lends itself for extended analyses, including the impact of power control and spread-spectrum and interference cancellation techniques, and the derivation of interference correlation coefficients. The following two sections provide generalizations to the Poisson model. In Section 4, the interference properties in clustered Poisson networks are studied, while Section 5 is devoted to general motion-invariant node distributions.

Sections 2 and 3 only require a basic knowledge in probability, while the results in Sections 4 and 5 were obtained using Palm theory, in particular conditional probability generating functionals. The appendix provides a brief introduction of the mathematical techniques used in this monograph.

The results and analytical techniques derived in this monograph will hopefully serve as guidelines for the design of large wireless systems with random user locations. They provide answers to such questions as how the interference statistics and outage probabilities are affected by the user density and distribution, the path loss law, the fading statistics, and power control. In turn, given system constraints such as outage or rate requirements, they permit the tuning of the network parameters for optimum performance.

## 1.1 Interference Characterization

The main quantity of interest is the (cumulated) interference. Measured at a point $y \in \mathbb{R}^d$ it is given by

$$I(y) = \sum_{\mathsf{x} \in \mathcal{T}} P_{\mathsf{x}} h_{\mathsf{x}} \ell(\|y - \mathsf{x}\|), \tag{1.1}$$

where $\mathcal{T} \subset \mathbb{R}^d$ denotes the set of all transmitting nodes, $P_{\mathsf{x}}$ the transmit power of node $\mathsf{x}$, $h_{\mathsf{x}}$ the (power) fading coefficient, and $\ell$ the path loss function, assumed to depend only on the distance $\|y - \mathsf{x}\|$ from node $\mathsf{x}$ to the point $y$.

In a large wireless system, the unknowns are $\mathcal{T}$, $h_{\mathsf{x}}$, and perhaps $P_{\mathsf{x}}$. The locations of the interfering nodes, together with the path loss law, determine the interference to first order. The impact of fading is smaller but certainly non-negligible, as we shall see. So, in essence, it is the *network geometry* or, more precisely, the *interference geometry*, that determines the distribution of the interference. The geometry consists of the underlying node distribution that, together with the channel access scheme, determines the locations of the interfering nodes, and the path loss law, which determines the strength of the interfering power given the distance.

The nodes may be arranged deterministically, for example in a lattice, or in a random fashion, in which case the uncertainty in the nodes' locations is usually represented by a stochastic point process $\Phi$ on $\mathbb{R}^2$ or $\mathbb{R}^3$ or a subset thereof. Assuming that the point process is simple, i.e., there are no two nodes at the same position, we can write the point process as a random set, $\Phi = \{\mathsf{x}_1, \mathsf{x}_2, \dots, \mathsf{x}_N\}$, where the (possibly random) total number of nodes $N$ may be finite or infinite. At any moment in time, the MAC scheme selects a subset of nodes as transmitters. This makes $\mathcal{T}$ in (1.1) and, in turn, the interference, time dependent. In some cases, the interference is stationary, both in time and space, so neither a time index nor a spatial location needs to be specified, and we can simply talk about the distribution of the interference $I$.

Throughout this monograph, unless otherwise specified, we will assume unit transmit powers at all nodes and the fading to be iid with $\mathbb{E}(h) = 1$.

## 1.2 Signal-to-Interference-Plus-Noise Ratio and Outage

### 1.2.1 Definitions

The performance of a wireless network critically depends on the signal-to-interference-plus-noise (SINR) levels at the receivers.

---

**Definition 1.1 (Signal-to-interference-plus noise ratio (SINR)).** The SINR for a receiver placed at the origin $o$ in the two- or three-dimensional Euclidean space is

$$\mathsf{SINR} = \frac{S}{W + I}, \tag{1.2}$$

where $S$ is the desired signal power, $W$ is the noise power, and $I$ the interference power given by (1.1).

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For a fixed modulation and coding scheme and with interference treated as noise, e.g., by using a simple linear receiver, a well accepted model for packetized transmissions is that they succeed if the SINR exceeds a certain threshold $\theta$. So we define the success probability as follows:

---

**Definition 1.2 (Transmission success probability).**

$$p_s(\theta) = \mathbb{P}(\mathsf{SINR} > \theta). \tag{1.3}$$

---

Its complement $1 - p_s$ is the outage probability, which is the same as the cumulative distribution function (CDF) of the SINR, and we may express the achievable rate (with interference treated as noise) of a link as

$$\mathbb{E}\log_2(1 + \mathsf{SINR}) = -\int \log_2(1 + x)\mathrm{d}p_s(x),$$

assuming that the interference amplitude is Gaussian. In the weak-interference regime, this expression is the actual bandwidth-normalized capacity [42].

### 1.2.2 Outage in Rayleigh Fading

In the case of Rayleigh fading, the desired signal power $S$ is exponentially distributed. Assuming $\mathbb{E}S = 1$,

$$p_s(\theta) = \mathbb{P}(S > \theta(W + I)) = \underbrace{\exp(-\theta W)}_{p_s^W} \cdot \underbrace{\exp(-\theta I)}_{p_s^I},$$

which shows that the success probability is the product of two factors, a noise term $p_s^W \triangleq \exp(-\theta W)$ that does not depend on the interference, and an interference term $p_s^I \triangleq \exp(-\theta I)$ that does not depend on the noise. This allows a significant simplification of outage analyses since the joint impact of noise and interference is captured by the product of the success probabilities in the noiseless and the interference-free cases. Moreover, since $\exp(-\theta I)$ is the Laplace transform of the interference evaluated at $\theta$, i.e.,

$$p_s^I(\theta) = \mathcal{L}_I(s)\big|_{s=\theta}, \tag{1.4}$$

the interference component of the success probability can be calculated by determining the Laplace transform of $I$, as was noted in [3, 31, 54]. It turns out that this is easier in many cases than determining the distribution. In other words, the SIR distribution when $S$ is Rayleigh fading is known for more types of networks than the distribution of just the interference itself.

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