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**Stochastic Geometry  
and Wireless Networks:  
Volume II Applications**

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# Stochastic Geometry and Wireless Networks: Volume II Applications

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## Stochastic Geometry and Wireless Networks: Volume II Applications

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### Abstract

This volume bears on wireless network modeling and performance analysis. The aim is to show how stochastic geometry can be used in a more or less systematic way to analyze the phenomena that arise in this context. It first focuses on medium access control mechanisms used in ad hoc networks and in cellular networks. It then discusses the use of stochastic geometry for the quantitative analysis of routing algorithms in mobile ad hoc networks. The appendix also contains a concise summary of wireless communication principles and of the network architectures considered in the two volumes.

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## Preface

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A wireless communication network can be viewed as a collection of nodes, located in some domain, which can in turn be transmitters or receivers (depending on the network considered, nodes may be mobile users, base stations in a cellular network, access points of a WiFi mesh, etc.). At a given time, several nodes transmit simultaneously, each toward its own receiver. Each transmitter–receiver pair requires its own wireless link. The signal received from the link transmitter may be jammed by the signals received from the other transmitters. Even in the simplest model where the signal power radiated from a point decays in an isotropic way with Euclidean distance, the geometry of the locations of the nodes plays a key role since it determines the *signal to interference and noise ratio* (SINR) at each receiver and hence the possibility of establishing simultaneously this collection of links at a given bit rate. The interference seen by a receiver is the sum of the signal powers received from all transmitters, except its own transmitter.

Stochastic geometry provides a natural way of defining and computing macroscopic properties of such networks, by averaging over all potential geometrical patterns for the nodes, in the same way as queuing theory provides response times or congestion, averaged over all potential arrival patterns within a given parametric class.

## 2 Preface

Modeling wireless communication networks in terms of stochastic geometry seems particularly relevant for large scale networks. In the simplest case, it consists in treating such a network as a snapshot of a stationary random model in the whole Euclidean plane or space and analyzing it in a probabilistic way. In particular the locations of the network elements are seen as the realizations of some point processes. When the underlying random model is ergodic, the probabilistic analysis also provides a way of estimating *spatial averages* which often capture the key dependencies of the network performance characteristics (connectivity, stability, capacity, etc.) as functions of a relatively small number of parameters. Typically, these are the densities of the underlying point processes and the parameters of the protocols involved. By spatial average, we mean an empirical average made over a large collection of ‘locations’ in the domain considered; depending on the cases, these locations will simply be certain points of the domain, or nodes located in the domain, or even nodes on a certain route defined on this domain. These various kinds of spatial averages are defined in precise terms in the monograph. This is a very natural approach e.g., for ad hoc networks, or more generally to describe user positions, when these are best described by random processes. But it can also be applied to represent both irregular and regular network architectures as observed in cellular wireless networks. In all these cases, such a space average is performed on a large collection of nodes of the network executing some common protocol and considered at some common time when one takes a snapshot of the network. Simple examples of such averages are the fraction of nodes which transmit, the fraction of space which is covered or connected, the fraction of nodes which transmit their packet successfully, and the average geographic progress obtained by a node forwarding a packet towards some destination. This is rather new to classical performance evaluation, compared to time averages.

Stochastic geometry, which we use as a tool for the evaluation of such spatial averages, is a rich branch of applied probability particularly adapted to the study of random phenomena on the plane or in higher dimension. It is intrinsically related to the theory of point processes. Initially its development was stimulated by applications to biology, astronomy and material sciences. Nowadays, it is also used in

image analysis and in the context of communication networks. In this latter case, its role is similar to that played by the theory of point processes on the real line in classical queuing theory.

The use of stochastic geometry for modeling communication networks is relatively new. The first papers appeared in the engineering literature shortly before 2000. One can consider Gilbert's paper of 1961 [34] both as the first paper on continuum and Boolean percolation and as the first paper on the analysis of the connectivity of large wireless networks by means of stochastic geometry. Similar observations can be made on [35] concerning Poisson–Voronoi tessellations. The number of papers using some form of stochastic geometry is increasing fast. One of the most important observed trends is to take better account in these models of specific mechanisms of wireless communications.

Time averages have been classical objects of performance evaluation since the work of Erlang (1917). Typical examples include the random delay to transmit a packet from a given node, the number of time steps required for a packet to be transported from source to destination on some multihop route, the frequency with which a transmission is not granted access due to some capacity limitations, etc. A classical reference on the matter is [58]. These time averages will be studied here either on their own or in conjunction with space averages. The combination of the two types of averages unveils interesting new phenomena and leads to challenging mathematical questions. As we shall see, the order in which the time and the space averages are performed matters and each order has a different physical meaning.

This monograph surveys recent results of this approach and is structured in two volumes.

Volume I focuses on the theory of spatial averages and contains three parts. Part I in Volume I provides a compact survey on *classical* stochastic geometry models. Part II in Volume I focuses on *SINR* stochastic geometry. Part III in Volume I is an appendix which contains mathematical tools used throughout the monograph. Volume II bears on more practical wireless network modeling and performance analysis. It is in this volume that the interplay between wireless communications and stochastic geometry is deepest and that the time–space framework alluded to above is the most important. The aim is to show

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how stochastic geometry can be used in a more or less systematic way to analyze the phenomena that arise in this context. Part IV in Volume II is focused on medium access control (MAC). We study MAC protocols used in ad hoc networks and in cellular networks. Part V in Volume II discusses the use of stochastic geometry for the quantitative analysis of routing algorithms in MANETs. Part VI in Volume II gives a concise summary of wireless communication principles and of the network architectures considered in the monograph. This part is self-contained and readers not familiar with wireless networking might either read it before reading the monograph itself, or refer to it when needed.

Here are some comments on what the reader will obtain from studying the material contained in this monograph and on possible ways of reading it.

For readers with a background in applied probability, this monograph provides direct access to an emerging and fast growing branch of spatial stochastic modeling (see, e.g., the proceedings of conferences such as IEEE Infocom, ACM Sigmetrics, ACM Mobicom, etc. or the special issue [38]). By mastering the basic principles of wireless links and of the organization of communications in a wireless network, as summarized in Volume II and already alluded to in Volume I, these readers will be granted access to a rich field of new questions with high practical interest. SINR stochastic geometry opens new and interesting mathematical questions. The two categories of objects studied in Volume II, namely medium access and routing protocols, have a large number of variants and of implications. Each of these could give birth to a new stochastic model to be understood and analyzed. Even for classical models of stochastic geometry, the new questions stemming from wireless networking often provide an original viewpoint. A typical example is that of route averages associated with a Poisson point process as discussed in Part V in Volume II. Reader already knowledgeable in basic stochastic geometry might skip Part I in Volume I and follow the path:

Part II in Volume I  $\Rightarrow$  Part IV in Volume II  $\Rightarrow$  Part V in  
Volume II,

using Part VI in Volume II for understanding the physical meaning of the examples pertaining to wireless networks.

For readers whose main interest in wireless network design, the monograph aims to offer a new and comprehensive methodology for the performance evaluation of large scale wireless networks. This methodology consists in the computation of both time and space averages within a unified setting. This inherently addresses the scalability issue in that it poses the problems in an infinite domain/population case from the very beginning. We show that this methodology has the potential to provide both qualitative and quantitative results as below:

- Some of the most important qualitative results pertaining to these infinite population models are in terms of *phase transitions*. A typical example bears on the conditions under which the network is spatially connected. Another type of phase transition bears on the conditions under which the network delivers packets in a finite mean time for a given medium access and a given routing protocol. As we shall see, these phase transitions allow one to understand how to tune the protocol parameters to ensure that the network is in the desirable “phase” (i.e. well connected and with small mean delays). Other qualitative results are in terms of scaling laws: for instance, how do the overhead or the end-to-end delay on a route scale with the distance between the source and the destination, or with the density of nodes?
- Quantitative results are often in terms of closed form expressions for both time and space averages, and this for each variant of the involved protocols. The reader will hence be in a position to discuss and compare various protocols and more generally various wireless network organizations. Here are typical questions addressed and answered in Volume II: is it better to improve on Aloha by using a collision avoidance scheme of the CSMA type or by using a channel-aware extension of Aloha? Is Rayleigh fading beneficial or detrimental when using a given MAC scheme? How does geographic routing compare to shortest path routing in a mobile ad hoc

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network? Is it better to separate the medium access and the routing decisions or to perform some cross layer joint optimization?

The reader with a wireless communication background could either read the monograph from beginning to end, or start with Volume II i.e. follow the path

Part IV in Volume II  $\Rightarrow$  Part V in Volume II  $\Rightarrow$  Part II in Volume I

and use Volume I when needed to find the mathematical results which are needed to progress through Volume II.

We conclude with some comments on what the reader will *not* find in this monograph:

- We do not discuss statistical questions and give no measurement based validation of certain stochastic assumptions used in the monograph: e.g., when are Poisson-based models justified? When should one rather use point processes with some repulsion or attraction? When is the stationarity/ergodicity assumption valid? Our only aim is to show what can be done with stochastic geometry when assumptions of this kind can be made.
- We will not go beyond SINR models either. It is well known that considering interference as noise is not the only possible option in a wireless network. Other options (collaborative schemes, successive cancellation techniques) can offer better rates, though at the expense of more algorithmic overhead and the exchange of more information between nodes. We believe that the methodology discussed in this monograph has the potential of analyzing such techniques but we decided not to do this here.

Here are some final technical remarks. Some sections, marked with a \* sign, can be skipped at the first reading as their results are not used in what follows; the index, which is common to the two volumes, is designed to be the main tool to navigate within and between the two volumes.



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## Preface to Volume II

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The two first parts of volume II (Part IV and Part V) are structured in terms of the key ingredients of wireless communications, namely medium access and routing. The general aim of this volume is to show how stochastic geometry can be used in a more or less systematic way to analyze the key phenomena that arise in this context. We limit ourselves to simple (yet not simplistic) models and basic protocols. This volume is nevertheless expected to convince the reader that much more can be done for improving the realism of the models, for continuing the analysis and for extending the scope of the methodology.

Part IV is focused on medium access control (MAC). We study MAC protocols used both in mobile ad hoc networks (MANETs) and in cellular networks. We analyze spatial Aloha schemes in terms of Poisson shot-noise processes in Chapters 16 and 17 and carrier sense multiple access (CSMA) schemes in terms of Matérn point processes in Chapter 18. The analytical results are then used to perform various optimizations on these schemes. For instance, we determine the tuning of the protocol parameters which maximizes the number of successful transmissions or the throughput per unit of space. We also determine the

protocol parameters for which end-to-end delays have a finite mean, etc. Chapter 19 is focused on the Code Division Multiple Access (CDMA) schemes with *power control* which are used in cellular networks. The terminal nodes associated with a given concentration node (base station, access point) are those located in its Voronoi cell w.r.t. the point process of concentration nodes. For analyzing these systems, we use both shot noise processes and tessellations. When the terminal nodes require a fixed bit rate, and power is controlled so as to maximize the number of terminal nodes that can be served by such a cellular network, powers become functionals of the underlying point processes. We study admission control and capacity within this context.

Part V discusses the use of stochastic geometry for the qualitative and quantitative analysis of routing algorithms in a MANET where the nodes are some realization of a Poisson point process (p.p.) of the plane. In the point-to-point routing case, the main object of interest is the path from some source to some destination node. In the point-to-multipoint case, this is the tree rooted in the source node and spanning a set of destination nodes. The motivations are multihop diffusion in MANETs. We also analyze the multipoint-to-point case, which is used for instance for concentration in wireless sensor communication networks where information has to be gathered at some central node. These random geometric objects are made of a set of wireless links, which have to be either simultaneously or successively feasible. Chapter 20 is focused on optimal routing, like, e.g., shortest path and minimal weight routing. The main tool is subadditive ergodic theory. In Chapter 21, we analyze various types of suboptimal (greedy) geographic routing schemes. We show how to use stochastic geometry to analyze local functionals of the random paths/tree such as the distribution of the length of its edges or the mean degree of its nodes. Chapter 22 bears on time-space routing. This class of routing algorithms leverages the interaction between MAC and routing and belongs to the so called *cross-layer* framework. More precisely, these algorithms take advantage of the time and space diversity of fading variables and MAC decisions to route packets from source to destination. Typical qualitative results bear on the ‘convergence’ of these routing algorithms or on the fact that the velocity of a packet on a route is positive or zero. Typical

quantitative results are in terms of the comparison of the mean time it takes to transport a packet from some source node to some destination node.

Part VI is an appendix which contains a concise summary of wireless communication principles and of the network architectures considered in the monograph. Chapter 23 is focused on propagation issues and on statistical channel models for fading such as Rayleigh or Rician fading. Chapter 24 bears on detection with a special focus on the fundamental limitations of wireless channels. As for architecture, we describe both MANETs and cellular networks in Chapter 25. MANETs are “flat” networks, with a single type of nodes which are at the same time transmitters, receivers and relays. Examples of MAC protocols used within this framework are described as well as multihop routing principles. Cellular networks have two types of network elements: base stations and users. Within this context, we discuss power control and its feasibility as well as admission control. We also consider other classes of heterogeneous networks like WiFi mesh networks, sensor networks or combinations of WiFi and cellular networks.

Let us conclude with a few general comments on the wireless channels and the networks to be considered throughout the volume.<sup>1</sup>

Two basic communication models are considered:

- A *digital communication model*, where the throughput on a link (measured in bits per seconds) is determined by the SINR at the receiver through a Shannon-like formula.
- A *packet model*, where the SINR at the receiver determines the probability of reception (also called probability of capture) of the packet and where the throughput on a link is measured in packets per time slot.

In most models, time is slotted and the time slot is assumed to be such that fading is constant over a time slot (see Chapter 23 for more on the

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<sup>1</sup>For those not familiar with wireless networks, a full understanding of these comments might require a preliminary study of Part VI

physical meaning of this assumption). There are hence at least three time scales:

- The time scale of *symbol transmissions*. In this volume, this time scale is considered small compared to the time slot, so that many symbols are sent during one slot. At this time scale, the additive noise is typically assumed to be a Gaussian white noise and spreading techniques can be invoked to justify the representation of the interference on each channel as a Gaussian additive white noise (see Section 24.3.3). Shannon's formula can then in turn be invoked to determine the bit-rate of each channel over a given time slot in terms of the ratio of the mean signal power to the mean interference-and-noise power seen on the channel; the latter mean is the sum of the variance of noise and of the variance of the Gaussian representation of interference; the bit rate is an ergodic average over the many symbols sent in one slot.
- The time scale of *slots*. At this time scale, only the mean interference and noise powers for each channel and each time slot are retained from the symbol transmission time scale. These quantities change from a time slot to the next due to the fact that MAC decisions and fading may change. For example, with Aloha, the MAC decisions are resampled at each time slot; as for fading, we consider a *fast fading* scenario,<sup>2</sup> where the fading between a transmitter and a receiver changes (e.g., is resampled) from a time slot to the next (for instance do the motion of reflectors – see Chapter 23) and a *slow fading* scenario, where it remains unchanged over time slots. At this time scale, the interference powers are hence again random processes, fully determined by the fading scenario and the MAC. As we shall see, their laws (which are not Gaussian anymore) can be determined using the Shot-Noise theory of Section 2 in Volume I.

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<sup>2</sup>Notice that this definition of fast fading differs from the definition used in many papers of literature, where fast fading often means that the channel conditions fluctuate much over a given time slot.

- The time scale of *mobility*. In this monograph, this time scale is considered large compared to time slots. In particular in the part on routing, we primarily focus on scenarios where all nodes are static and where routes are established on this static network. The rationale is that the time scale of packet transmission on a route is smaller than that of node mobility. Stated differently, we do not consider here the class of delay tolerant networks which leverage node mobility for the transport of packets.

## References

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- [1] D. Aldous and J. M. Steele, “Asymptotics for euclidean minimal spanning trees on random points,” *Probability Theory and Related Fields*, vol. 92, pp. 247–258, 1992.
- [2] S. Asmussen, P. Fiorini, L. Lipsky, T. Rolski, and R. Sheahan, “Asymptotic behavior of total times for jobs that must start over if a failure occurs,” *Mathematics of Operations Research*, vol. 33, no. 4, pp. 932–944, 2008.
- [3] Audrey M. Viterbi and A. J. Viterbi, “Erlang capacity of a power controlled CDMA system,” *IEEE Journal on Selected Areas in Communications*, vol. 11, no. 6, pp. 892–899, 8, 1993.
- [4] F. Baccelli and B. Błaszczyszyn, “A new phase transitions for local delays in MANETs,” in *Proceedings of IEEE Infocom*, San Diego, CA, USA, March 2010.
- [5] F. Baccelli, B. Błaszczyszyn, and M. Karray, “Up and downlink admission/congestion control and maximal load in large homogeneous CDMA networks,” *MONET*, vol. 9, no. 6, pp. 605–617, December 2004.
- [6] F. Baccelli, B. Błaszczyszyn, and M. Karray, “Blocking rates in large CDMA networks via a spatial Erlang formula,” in *Proceedings of IEEE Infocom*, Miami, FL, USA, March 2005.
- [7] F. Baccelli, B. Błaszczyszyn, and M. Karray, “A spatial markov queuing process and its applications to wireless loss systems,” INRIA, Technical Report 00159330, 2007.
- [8] F. Baccelli, B. Błaszczyszyn, and O. Mirsadeghi, “Optimal paths on the space-time SINR random graphs,” *submitted for publication*, <http://arxiv.org/abs/0911.3721>, 2009.
- [9] F. Baccelli, B. Błaszczyszyn, and P. Mühlethaler, “An Aloha protocol for multi-hop mobile wireless networks,” in *Proceedings of the Allerton Conference*,

- pp. 99, 161, Urbana Champaign, Illinois. and *IEEE Transactions on Information Theory*, vol. 52, no. 2, pp. 421–436, 2006, November 2003.
- [10] F. Baccelli, B. Błaszczyszyn, and P. Mühlethaler, “A stochastic model for spatial and opportunistic aloha,” *JSAC*, pp. 21, 99, Special Issue on Stochastic Geometry and Random Graphs for Wireless Networks, to appear, 2009.
  - [11] F. Baccelli, B. Błaszczyszyn, and P. Mühlethaler, “Time-space opportunistic routing in wireless ad hoc networks: Algorithms and performance optimization by stochastic geometry,” *The Computer Journal*, p. 161, June 2009.
  - [12] F. Baccelli, B. Błaszczyszyn, and F. Tournois, “Downlink capacity and admission/congestion control in CDMA networks,” in *Proceedings of IEEE INFOCOM*, p. 100, San Francisco, 2003.
  - [13] F. Baccelli and C. Bordenave, “The radial spanning tree of a poisson point process,” *Annals of Applied Probab.*, vol. 17, no. 1, pp. 305–359, 127, 128, 129, 161, 2007.
  - [14] P. Billingsley, *Probability and Measure*. Wiley, 3rd edition, 1995.
  - [15] S. Biswas and R. Morris, “Exor: opportunistic multi-hop routing for wireless networks,” in *Proceedings of SIGCOMM '05*, pp. 133–144, Philadelphia, Pennsylvania, USA, 2005.
  - [16] B. Błaszczyszyn and M. Karray, “An efficient analytical method for dimensioning of cdma cellular networks serving streaming calls,” in *Proceedings of ACM/ICST VALUETOOLS*, Athens, Greece, 2008.
  - [17] B. Błaszczyszyn and M. Karray, “Dimensioning of the downlink in OFDMA cellular networks via an erlang’s loss model,” in *Proceedings of European Wireless Conference*, Aalborg, 2009.
  - [18] B. Błaszczyszyn and M. K. Karray, “Performance evaluation of scalable congestion control schemes for elastic traffic in cellular networks with power control,” in *Proceedings of IEEE INFOCOM*, May 2007.
  - [19] B. Błaszczyszyn and M. K. Karray, “An efficient analytical method for dimensioning of CDMA cellular networks serving streaming calls,” in *Proceedings of ValueTools*, 2008.
  - [20] B. Błaszczyszyn and P. Mühlethaler, “Stochastic analysis of non-slotted Aloha in wireless ad-hoc networks,” in *Proceedings of IEEE Infocom*, San Diego, CA, USA, March 2010.
  - [21] B. Błaszczyszyn, P. Mühlethaler, and Y. Toor, “Maximizing throughput of Linear Vehicular Ad-hoc NETWORKS (VANETs) — A stochastic approach,” in *Proceedings of European Wireless Conference*, Aalborg, 2009.
  - [22] B. Błaszczyszyn and B. Radunović, “M/D/1/1 loss system with interference and applications to transmit-only sensor networks,” in *Proceedings of IEEE Spaswin 2007*, 2007.
  - [23] B. Błaszczyszyn and B. Radunović, “Using transmit-only sensors to reduce deployment cost of wireless sensor networks,” in *Proceedings of IEEE INFOCOM*, Phoenix, AZ, 2008.
  - [24] B. Blum, T. He, S. Son, and J. Stankovic, “Igf: A state-free robust communication protocol for wireless sensor networks,” Technical Report CS-2003-1, University of Virginia CS Department, 2003.
  - [25] C. Bordenave, “Navigation on a Poisson point process,” Technical Report RR-5790, INRIA, 2006.



- [26] P. Brémaud, *Mathematical Principles of Signal Processing*. New York: Springer, 2002.
- [27] E. W. Dijkstra, "A note on two problems in connexion with graphs," *Numerische Mathematik*, vol. 1, no. 3, pp. 269–271, 1959.
- [28] M. Durvy, O. Dousse, and P. Thiran, "Self-organization properties of csma/ca systems and their consequences on fairness," *IEEE Transactions on Information Theory*, vol. 55, March 2009.
- [29] N. Ehsan and R. L. Cruz, "On the optimal sinr in random access networks with spatial reuse," in *Proceedings of the CISS 2006 Conference*, Princeton University, NJ, USA, 2006.
- [30] J. Evans and D. Everitt, "On the teletraffic capacity of CDMA cellular networks," *IEEE Transactions on Vehicular Technology*, vol. 48, January 1999.
- [31] W. Feller, *An Introduction to Probability Theory and its Applications, volume II*. Wiley, 1971.
- [32] R. K. Ganti and M. Haenggi, "Bounds on Information Propagation Delay in Interference-Limited ALOHA Networks," in *Proc. of Workshop on Spatial Stochastic Models for Wireless Networks*, 2009. <http://www.nd.edu/~mhaenggi/pubs/spaswin09.pdf>.
- [33] R. K. Ganti and H. M., "Spatial and temporal correlation of the interference in aloha ad hoc networks," *IEEE Communications Letters*, To appear, 2009.
- [34] E. N. Gilbert, "Random plane networks," *SIAM Journal*, vol. 9, pp. 533–543, iv, 1961.
- [35] E. N. Gilbert, "Random subdivisions of space into crystals," *The Annals of Mathematical Statistics*, vol. 33, no. 3, pp. 958–972, 1962.
- [36] M. Haenggi, "On routing in random rayleigh fading networks," *IEEE Transactions on Wireless Communications*, vol. 4, pp. 1553–1562, July 2005.
- [37] M. Haenggi, "Outage, local throughput, and capacity of random wireless networks," *IEEE Transactions on Wireless Communications*, To appear, 2009.
- [38] M. Haenggi, J. Andrews, F. Baccelli, O. Dousse, and M. Franceschetti, eds., *Stochastic Geometry and Random Graphs for Wireless Networks*. Special Issue of IEEE JSAC, 2009.
- [39] S. Hanly, "Congestion measures in DS-CDMA networks," *IEEE Transactions on Communications*, vol. 47, no. 3, 1999.
- [40] C. D. Howard and C. M. Newman, "Euclidean models of first-passage percolation," *Probability Theory and Related Fields*, vol. 108, pp. 153–170, 1997.
- [41] A. Hunter, J. Andrews, and S. Weber, "Capacity scaling of ad hoc networks with spatial diversity," *submitted for publication*, 2008.
- [42] P. Jacquet, "Realistic wireless network model with explicit capacity evaluation," *submitted for publication*, Rapport de recherche INRIA RR-6407, 2008.
- [43] P. Jacquet, B. Mans, P. Mühlethaler, and G. Rodolakis, "Opportunistic Routing in Wireless Ad Hoc Networks: Upper Bounds for the Packet Propagation Speed," *IEEE JSAC, special issue on Stochastic Geometry and Random Graphs for Wireless Networks*, vol. 27, pp. 1192–1202, 2009.
- [44] V. Jarník, "About a certain minimal problem," *Práce Moravské Přírodovědecké Společnosti*, vol. 6, pp. 57–63, in Czech, 1930.

310 *References*

- [45] P. Jelenković and J. Tan, “Can retransmissions of superexponential documents cause subexponential delays?,” in *Proc. of INFOCOM*, (Anchorage, AL, USA), IEEE, 2007.
- [46] P. Jelenković and J. Tan, “Is ALOHA Causing Power Law Delays?,” in *Managing Traffic Performance in Converged Networks*, (L. Mason, T. Drwiega, and J. Yan, eds.), Berlin: Springer, 2007.
- [47] N. Jindal, J. Andrews, and S. Weber, “Fractional power control for decentralized wireless networks,” *IEEE Transactions on Wireless Communications*, vol. 7, pp. 5482–5492, December 2008.
- [48] N. Jindal, S. Weber, and J. Andrews, “Bandwidth partitioning in decentralized wireless networks,” *IEEE Transactions on Wireless Communications*, vol. 7, pp. 5408–5419, December 2008.
- [49] B. Karp, “Geographic routing for wireless networks,” PhD thesis, Harvard University, 2000.
- [50] B. Karp and H. T. Kung, “Gpsr: Greedy perimeter stateless routing for wireless networks,” in *6th Annual International Conference on Mobile Computing and Networking, MobiCom 2000, August 6.-11., 2000, Boston, Massachusetts, USA*, pp. 243–254, ACM/IEEE, August 2000.
- [51] M. K. Karray, “Analytic evaluation of wireless cellular networks performance by a spatial Markov process accounting for their geometry, dynamics and control schemes,” PhD thesis, ENST, 2007.
- [52] B. Kauffmann, F. Baccelli, F. Chaintreau, V. Mhatre, K. Papagiannaki, and C. Diot, “Measurement-based self organization of interfering 802.11 wireless access networks,” in *Proceedings of IEEE INFOCOM’07*, pp. 1451–1459, 2007.
- [53] J. M. Keil and C. Gutwin, “Classes of graphs which approximate the complete Euclidean graph,” *Discrete Computing Geometry*, vol. 7, pp. 13–28, 1992.
- [54] F. P. Kelly, “Loss networks; special invited paper,” *Annals of Applied Probability*, vol. 1, pp. 319–378, 1991.
- [55] S. Keshav, *An Engineering Approach to Computer Networking*. Addison-Wesley, 1997.
- [56] J. Kingman, “Subadditive ergodic theory,” *Annals of Probability*, vol. 1, pp. 883–899, 1973.
- [57] B. Kitchens, *Symbolic Dynamics*. Springer Berlin, 1998.
- [58] L. Kleinrock, *Queueing Systems*. John Wiley & Sons, iv, 1975.
- [59] Z. Liu and M. E. Zarki, “Sir-based call admission control for DS-CDMA cellular systems,” *Selected Areas in Communications, IEEE Journal on*, vol. 12, pp. 638–644, May 1994.
- [60] S. Meyn and R. Tweedie, *Markov Chains and Stochastic Stability*. Springer-Verlag, 1993.
- [61] J. Møller, *Lectures on Random Voronoi Tessellations*. Vol. 87 of *Lecture Notes in Statistics*. New York: Springer-Verlag, 1994.
- [62] H. Nguyen, F. Baccelli, and D. Kofman, “A stochastic geometry analysis of dense IEEE 802.11 networks,” in *Proceedings of IEEE Infocom 2007*, pp. 1199–1207, 2007.

- [63] L. Pimentel, “The time constant and critical probabilities in percolation models,” *Electronic Communications in Probability*, vol. 11, pp. 160–168, 2006.
- [64] R. C. Prim, “Shortest connection networks and some generalizations,” *Bell System Technical Journal*, vol. 36, pp. 1389–1401, 1957.
- [65] L. Russo, “A note on percolation,” *Z. Wahr. verw. Geb.*, vol. 43, pp. 39–48, 1987.
- [66] R. Serfozo, *Introduction to stochastic networks*. New York: Springer, 1999.
- [67] M. Sidi and D. Starobinski, “New call blocking versus handoff blocking in cellular networks,” *Wireless Networks*, vol. 3, 1997.
- [68] I. Stojmenovic and X. Lin, “Loop-free hybrid single-path/flooding routing algorithms with guaranteed delivery for wireless networks,” *IEEE Transaction on Parallel and Distributed Systems*, vol. 12, pp. 1023–1032, 2001.
- [69] I. Stojmenovic and X. Lin, “Power-aware localized routing in wireless networks,” *IEEE Transaction on Parallel and Distributed Systems*, vol. 12, no. 11, pp. 1122–1133, 2001.
- [70] G. L. Stuber, *Principles of Mobile Communication*. Boston: Kluwer AP, 2001.
- [71] H. Takagi and L. Kleinrock, “Optimal transmission ranges for randomly distributed packet radio networks,” *IEEE Transactions on Communication*, vol. 32, no. 3, pp. 246–257, 1984.
- [72] D. Tse and P. Viswanath, *Foundamentals of Wireless Communication*. Cambridge University Press, 2005.
- [73] M. Q. Vahidi-Asl and J. C. Wierman, “First-passage percolation on the Voronoi tessellation and Delaunay triangulation,” in *Random Graphs’87; Based on Proceedings of the 3rd International Seminar on Random Graphs and Probabilistic Methods in Combinatorics, June 27–July 3*, (M. Karoński, J. Jaworski, and A. Ruciński, eds.), pp. 341–359, Chichester: John Wiley & sons, 1990.
- [74] J. Venkataraman and M. Haenggi, “Optimizing the throughput in random wireless ad hoc networks,” in *Proceedings of the 42st Annual Allerton Conference on Communication, Control, and Computing*, Monticello, IL, USA, October 2004.
- [75] S. Weber, J. Andrews, and N. Jindal, “The effect of fading, channel inversion, and threshold scheduling on ad hoc networks,” *IEEE Trans. Information Theory*, vol. 53, pp. 4127–4149, November 2007.
- [76] S. Weber, N. Jindal, R. K. Ganti, and M. Haenggi, “Longest-edge routing on the spatial aloha graph,” in *Proceedings of IEEE Global Communications Conference (GLOBECOM’08)*, New Orleans, LA, USA, December 2008.