

# Fundamentals of Diffusion-Based Molecular Communication in Nanonetworks

---

**Massimiliano Pierobon**

Department of Computer Science & Engineering,  
University of Nebraska-Lincoln  
pierobon@cse.unl.edu

**Ian F. Akyildiz**

School of Electrical and Computer Engineering,  
Georgia Institute of Technology  
ian@ece.gatech.edu

**now**

the essence of knowledge

Boston — Delft

## Foundations and Trends<sup>®</sup> in Networking

*Published, sold and distributed by:*

now Publishers Inc.  
PO Box 1024  
Hanover, MA 02339  
United States  
Tel. +1-781-985-4510  
[www.nowpublishers.com](http://www.nowpublishers.com)  
[sales@nowpublishers.com](mailto:sales@nowpublishers.com)

*Outside North America:*

now Publishers Inc.  
PO Box 179  
2600 AD Delft  
The Netherlands  
Tel. +31-6-51115274

The preferred citation for this publication is

M. Pierobon and I. F. Akyildiz. *Fundamentals of Diffusion-Based Molecular Communication in Nanonetworks*. Foundations and Trends<sup>®</sup> in Networking, vol. 8, no. 1-2, pp. 1–147, 2013.

*This Foundations and Trends<sup>®</sup> issue was typeset in L<sup>A</sup>T<sub>E</sub>X using a class file designed by Neal Parikh. Printed on acid-free paper.*

ISBN: 978-1-60198-817-1

© 2014 M. Pierobon and I. F. Akyildiz

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, mechanical, photocopying, recording or otherwise, without prior written permission of the publishers.

Photocopying. In the USA: This journal is registered at the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923. Authorization to photocopy items for internal or personal use, or the internal or personal use of specific clients, is granted by now Publishers Inc for users registered with the Copyright Clearance Center (CCC). The 'services' for users can be found on the internet at: [www.copyright.com](http://www.copyright.com)

For those organizations that have been granted a photocopy license, a separate system of payment has been arranged. Authorization does not extend to other kinds of copying, such as that for general distribution, for advertising or promotional purposes, for creating new collective works, or for resale. In the rest of the world: Permission to photocopy must be obtained from the copyright owner. Please apply to now Publishers Inc., PO Box 1024, Hanover, MA 02339, USA; Tel. +1 781 871 0245; [www.nowpublishers.com](http://www.nowpublishers.com); [sales@nowpublishers.com](mailto:sales@nowpublishers.com)

now Publishers Inc. has an exclusive license to publish this material worldwide. Permission to use this content must be obtained from the copyright license holder. Please apply to now Publishers, PO Box 179, 2600 AD Delft, The Netherlands, [www.nowpublishers.com](http://www.nowpublishers.com); e-mail: [sales@nowpublishers.com](mailto:sales@nowpublishers.com)

**Foundations and Trends<sup>®</sup> in Networking**  
Volume 8, Issue 1-2, 2013  
**Editorial Board**

**Editor-in-Chief**

**Anthony Ephremides**  
University of Maryland  
United States

**Editors**

François Baccelli  
*ENS Paris*

Victor Bahl  
*Microsoft Research*

Helmut Bölcskei  
*ETH Zurich*

J.J. Garcia-Luna Aceves  
*UC Santa Cruz*

Andrea Goldsmith  
*Stanford University*

Roch Guerin  
*University of Pennsylvania*

Bruce Hajek  
*UIUC*

Jean-Pierre Hubaux  
*EPFL*

Frank Kelly  
*University of Cambridge*

P.R. Kumar  
*Texas A&M University*

Steven Low  
*Caltech*

Eytan Modiano  
*MIT*

Keith Ross  
*Polytechnic Institute of NYU*

Henning Schulzrinne  
*Columbia University*

Mani Srivastava  
*UCLA*

Leandros Tassioulas  
*University of Thessaly*

Lang Tong  
*Cornell University*

Ozan Tonguz  
*Carnegie Mellon University*

Don Towsley  
*University of Massachusetts, Amherst*

Nitin Vaidya  
*UIUC*

Pravin Varaiya  
*UC Berkeley*

Roy Yates  
*Rutgers University*

Raymond Yeung  
*Chinese University of Hong Kong*

## Editorial Scope

### Topics

Foundations and Trends<sup>®</sup> in Networking publishes survey and tutorial articles in the following topics:

- Modeling and analysis of:
  - Ad hoc wireless networks
  - Sensor networks
  - Optical networks
  - Local area networks
  - Satellite and hybrid networks
  - Cellular networks
  - Internet and web services
- Protocols and cross-layer design
- Network coding
- Energy-efficiency incentives/pricing/utility-based
- Games (co-operative or not)
- Security
- Scalability
- Topology
- Control/Graph-theoretic models
- Dynamics and asymptotic behavior of networks

### Information for Librarians

Foundations and Trends<sup>®</sup> in Networking, 2013, Volume 8, 4 issues. ISSN paper version 1554-057X. ISSN online version 1554-0588. Also available as a combined paper and online subscription.

Foundations and Trends® in Networking  
Vol. 8, No. 1-2 (2013) 1–147  
© 2014 M. Pierobon and I. F. Akyildiz  
DOI: 10.1561/13000000033



## **Fundamentals of Diffusion-Based Molecular Communication in Nanonetworks**

Massimiliano Pierobon  
Department of Computer Science & Engineering,  
University of Nebraska-Lincoln  
pierobon@cse.unl.edu

Ian F. Akyildiz  
School of Electrical and Computer Engineering,  
Georgia Institute of Technology  
ian@ece.gatech.edu

## Contents

---

<b>1</b>	<b>Introduction</b>	<b>2</b>
1.1	Biological Nanomachines and Nanonetworks . . . . .	3
1.2	Potential Applications of Nanonetworks Enabled by MC . . . . .	5
1.3	Research Objectives and Solutions . . . . .	6
1.4	Article Outline . . . . .	7
<b>2</b>	<b>Previous Work on Molecular Communication</b>	<b>8</b>
2.1	Molecular Communication Types . . . . .	8
2.2	Diffusion-based Molecular Communication in Nanonetworks . . . . .	10
<b>3</b>	<b>Deterministic Model of a Molecular Communication Link</b>	<b>13</b>
3.1	Motivation and Related Work . . . . .	13
3.2	A Basic Design of a Diffusion-based MC Link . . . . .	14
3.3	The Emission Process . . . . .	15
3.4	The Diffusion Process . . . . .	20
3.5	The Reception Process . . . . .	24
3.6	End-to-end Normalized Gain and Delay . . . . .	29
3.7	Numerical Results . . . . .	29
3.8	Conclusions . . . . .	34
<b>4</b>	<b>Stochastic Models of the Noise Sources in a MC Link</b>	<b>35</b>
4.1	Motivation and Related Work . . . . .	35

4.2	Definition of Noise Sources in a Diffusion-based MC Link . . . . .	37
4.3	The Particle-sampling Noise . . . . .	38
4.4	The Particle-counting Noise . . . . .	43
4.5	The Ligand-receptor-kinetics Noise . . . . .	49
4.6	Conclusions . . . . .	55
<b>5</b>	<b>Capacity Analysis of a Molecular Communication Link</b>	<b>57</b>
5.1	Motivation and Related Work . . . . .	57
5.2	Capacity Analysis through Thermodynamics . . . . .	61
5.3	Capacity Analysis with Channel Memory & Molecular Noise	70
5.4	Conclusions . . . . .	95
<b>6</b>	<b>Interference of Multiple MC Links in a Nanonetwork</b>	<b>97</b>
6.1	Motivation and Related Work . . . . .	97
6.2	Intersymbol Interference and Co-Channel Interference . . . . .	100
6.3	Statistical-physical Model of Interference . . . . .	113
6.4	Conclusions . . . . .	134
<b>7</b>	<b>Conclusions and Future Work</b>	<b>136</b>
	<b>References</b>	<b>140</b>

## Abstract

Molecular communication (MC) is a promising bio-inspired paradigm for the interconnection of autonomous nanotechnology-enabled devices, or nanomachines, into nanonetworks. MC realizes the exchange of information through the transmission, propagation, and reception of molecules, and it is proposed as a feasible solution for nanonetworks. This idea is motivated by the observation of nature, where MC is successfully adopted by cells for intracellular and intercellular communication. MC-based nanonetworks have the potential to be the enabling technology for a wide range of applications, mostly in the biomedical, but also in the industrial and surveillance fields. The focus of this article is on the most fundamental type of MC, i.e., diffusion-based MC, where the propagation of information-bearing molecules between a transmitter and a receiver is realized through free diffusion in a fluid. The objectives of the research presented in this article are to analyze an MC link from the point of view of communication engineering and information theory, and to provide solutions to the modeling and design of MC-based nanonetworks. First, a deterministic model is realized to study each component, as well as the overall diffusion-based-MC link, in terms of gain and delay. Second, the noise sources affecting a diffusion-based-MC link are identified and statistically modeled. Third, upper/lower bounds to the capacity are derived to evaluate the information-theoretic performance of diffusion-based MC. Fourth, an analysis of the interference produced by multiple diffusion-based MC links in a nanonetwork is provided. This research provides fundamental results that establish a basis for the modeling, design, and realization of future MC-based nanonetworks, as novel technologies and tools are being developed.

---

M. Pierobon and I. F. Akyildiz. *Fundamentals of Diffusion-Based Molecular Communication in Nanonetworks*. Foundations and Trends<sup>®</sup> in Networking, vol. 8, no. 1-2, pp. 1–147, 2013.

DOI: 10.1561/13000000033.

# 1

---

## Introduction

---

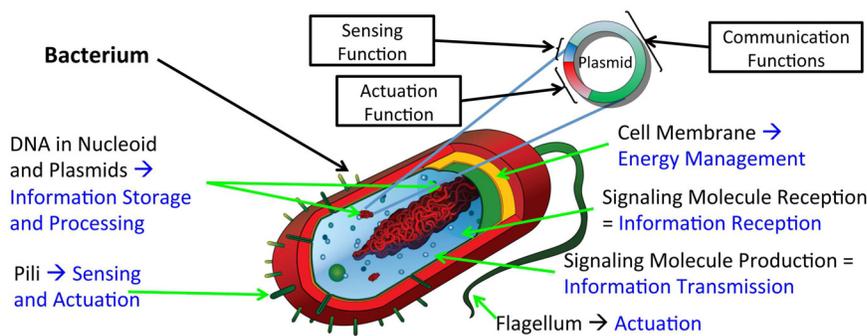
Molecular communication (MC) [2] is a bio-inspired paradigm where the exchange of information is realized through the transmission, propagation, and reception of molecules. This paradigm was first studied in biology, since it is successfully adopted in nature by cells for intracellular and intercellular communication [73]. MC is considered a promising option for communications in nanonetworks [5], which are defined as the interconnections of intelligent autonomous nanometer-scale devices, or nanomachines. Thanks to the feasibility of MC in biological environments, MC-based nanonetworks have the potential to be the enabling technology for a wide range of applications [5], mostly in the biomedical, but also in the industrial and surveillance fields. The objectives of the research presented in this article are to analyze the MC paradigm from the point of view of communication engineering and information theory, and to provide solutions to the modeling and design of MC-based nanonetworks.

## 1.1 Biological Nanomachines and Nanonetworks

Among the more promising research fields of today, nanotechnology is enabling the manipulation of matter at an atomic and molecular scale, from one to a hundred nanometers. One of the goals of nanotechnology is to engineer functional systems based on the unique phenomena and properties of matter at the nanoscale [32]. Currently, a great research effort is spent in the attempt to realize nanoscale machines, also called molecular machines or nanomachines, defined by E. Drexler as “mechanical devices that perform useful functions using components of nanometer-scale and defined molecular structure” [33]. More specifically, nanomachines [5, 3, 4] are expected to have the ability to sense, compute, actuate, manage their energy, and interconnect into networks, termed nanonetworks, to overcome their individual limitations and benefit from collaborative efforts.

Two main types of nanomachines can be identified within the aforementioned definition, namely, synthetic and biological. On the one hand, the synthetic nanomachines are realized either by downscaling from the current micro-scale technologies, such as microelectronics or micro-electro mechanics, or through the use of chemically synthesized nanomaterials [3]. On the other hand, the biological nanomachines are realized either by reusing biological components (e.g., DNA-based memories [52], flagellum-based actuators [18]), or by programming the behavior of biological cells from nature, such as through the genetic engineering of bacteria [41], as illustrated in 1.1.

While the engineering of fully synthetic nanomachines is still in its infancy, the research on the genetic engineering of biological cells is currently in rapid progress, thanks to the advancements made by biotechnology [16]. Several key techniques developed under the umbrella of synthetic biology have made possible today the realization of simple biological nanomachines [87]. As illustrated in Figure 1.1, through the insertion of engineered genetic code in the form of a circular DNA strand (i.e., plasmid) in a bacterium, it will be soon possible to program complete functions, including sensing, actuation, and communication, and have access to the main functionalities of the cell, such as the storage and the processing of information through DNA code, the



**Figure 1.1:** The expected functions of a biological nanomachine realized through the genetic engineering of a bacterium.

sensing and actuation through the use of the *pili* (hairlike appendages), the management of the cell energy through the cell membrane, and the transmission and reception of information through the production and the reception of signaling molecules.

The exchange of information between nanomachines, and their interconnection into nanonetworks, is key to overcome their individual limitations in size, energy and computational capabilities, and benefit from collaborative efforts. In nanonetworks, the applicability of classical communication technologies is limited by several constraints. In particular, the very restricted size of the nanomachines and the peculiarities of the environments in which they are envisioned to operate (e.g., biological scenarios) demand for novel solutions from the perspective of both the choice of the communication medium and the study of suitable communication techniques. While a possible solution to the problem of communication between synthetic nanomachines is suggested by recent studies [3] on nano-structures and on the properties of carbon nano-electronics, the imminent availability of biological nanomachines encourages to study and adopt the communication techniques naturally adopted by biological cells. In this direction, the **Molecular Communication (MC) paradigm**, inspired by the natural cell communication in biology, where message-carrying molecules are synthesized, emitted, collected, and converted to cellular responses through biochemical processes, is expected to be especially attractive

because of its inherent feasibility in a biocompatible environment [2, 5].

## 1.2 Potential Applications of Nanonetworks Enabled by Molecular Communication

Given the tight integration of MC within the biological environment and its feasibility at the cellular scale (nm -  $\mu\text{m}$ ), MC is studied not only as a candidate for nanonetwork communication, but also as a possible tool for the future nanonetworks to interact with the living organisms and their biological processes. As a consequence, the number of potential applications of MC-enabled nanonetworks is very large. Amongst others, the following three main areas deserve a special attention.

**Biomedical applications**, such as disease control and infectious agent detection [93], smart drug delivery systems [43], and intelligent intrabody systems for monitoring glucose, sodium, and cholesterol [34, 60]. These applications are expected to greatly benefit from the use of nanomachines deployed over the body (e.g., through tattoo-like patches) or inside the body (e.g., through pills or intramuscular injection). Since MC is naturally adopted by cells, nanonetworks enabled by this paradigm are envisioned to better integrate with the intra-body biological processes and to show higher biocompatibility when compared to other possible solutions.

**Industrial applications**, such as the monitoring and control of microbial formations. As an example, applications based on bacterial biofilms [27], which are used to clean residual waters coming from different manufacturing processes or to treat organic waste [56], could be greatly enhanced by MC-enabled nanonetworks, since microbial organisms naturally produce and respond to molecular stimuli.

**Surveillance applications** will make use of biological and chemical nanosensors that have an unprecedented sensing accuracy [85, 95]. Nanonetworks composed by several MC-enabled nanosensors could serve for surveillance against biological and chemical attacks [95] by detecting toxic or infectious agents diffusing in the environment.

### 1.3 Research Objectives and Solutions

The focus of this article is on diffusion-based MC, where the propagation of information-bearing molecules between a transmitter and a receiver is realized through free diffusion in a fluid. This choice is motivated by a preliminary analysis, detailed in Chapter 2, which identifies the diffusion-based as the most fundamental type of MC among different options suggested in the literature. As a consequence of the differences between the diffusion-based MC paradigm and classical electromagnetic communication paradigms, the classical communication engineering models and techniques are not directly applicable for the study and the design of diffusion-based MC-enabled nanonetworks. These differences include, but are not limited to, the following:

- The process of diffusion-based molecule propagation is based on radically different phenomena with respect to the electromagnetic wave propagation in classical communication systems. While electromagnetic waves operate the propagation of energy at the speed of light, the molecule diffusion process is caused by the random walk of the molecule Brownian motion in a fluid [76, 29]. As a consequence, while an electromagnetic wave propagates in a defined direction, and with negligible delay for most of the terrestrial communication systems, molecules subject to Brownian motion propagate with a random direction and with a high delay for almost all the transmission ranges of interest.
- The biologically-inspired physical processes that can be adopted to transmit and receive information in a diffusion-based MC-enabled nanonetworks are based on different mechanisms with respect to the modulation and reception of electromagnetic radiations in classical communication systems. While in classical systems antennas transmit and receive electromagnetic radiations through moving charges in metallic conductors, in biological cell bio-signaling [73] information is transmitted through the chemical synthesis of signaling molecules, and received through chemical reactions between incoming signaling molecules and chemical receptors.

As a consequence, there is a need of to build a complete understanding of the diffusion-based MC paradigm from the ground up. The research objectives addressed in this article, and the proposed solutions, have been identified to specifically target this need, and they are summarized as follows. The first research objective is to develop of a deterministic model of diffusion-based MC link, which provides a mathematical characterization of the main physical processes involved in the transmission, propagation, and reception of molecules for the exchange of information between a transmitter and a receiver. The second research objective is to identify and stochastically model the noise sources that affect a diffusion-based MC link. The third research objective is to provide an estimate of the achievable performance of a diffusion-based MC link in terms of information capacity. The fourth research objective is to analyze the interference produced by multiple diffusion-based MC links when present at the same time in a nanonetwork.

#### **1.4 Article Outline**

The rest of this article is organized as follows. A preliminary analysis of different MC options from the literature is contained in Chapter 2, which also includes a survey of the results from previous works pertinent to the study of diffusion-based MC. The results obtained through the design and end-to-end modeling of a basic diffusion-based-MC link are presented in Chapter 3, where the contributions of each component of the system are analyzed in terms of gain and delay. In Chapter 4, the most relevant noise sources affecting a diffusion-based MC link are studied through the mathematical expression of their underlying physical processes, and modeled through the use of statistical parameters. Analytical expressions of upper and lower bounds to the information capacity of a diffusion-based MC link are derived in Chapter 5, first by using tools from thermodynamics, and then through a pure information-theoretic approach. In Chapter 6, an analysis of the interference produced by multiple diffusion-based MC links in a nanonetwork is detailed. Finally, a conclusion with the possible future avenues for this research field is provided in Chapter 7.

## References

---

- [1] M. Abramowitz and I. A. Stegun. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*. New York: Dover Publications, 1972.
- [2] I. F. Akyildiz, F. Brunetti, and C. Blazquez. Nanonetworks: a new communication paradigm at molecular level. *Computer Networks (Elsevier) Journal*, 52(12):2260–2279, August 2008.
- [3] I. F. Akyildiz and J. M. Jornet. Electromagnetic wireless nanosensor networks. *Nano Communication Networks (Elsevier) Journal*, 1(1):3–19, June 2010.
- [4] I. F. Akyildiz and J. M. Jornet. The internet of nano-things. *IEEE Wireless Communication Magazine*, 17(6):58–63, December 2010.
- [5] I. F. Akyildiz, J. M. Jornet, and M. Pierobon. Nanonetworks: a new frontier in communications. *Communications of the ACMs*, 54(11):84–89, November 2011.
- [6] R. Aleixo and E. Capelas de Oliveira. Green’s function for the lossy wave equation. *Revista Brasileira de Ensino de Fisica*, 30(1):1302, 2008.
- [7] G. Alfano and D. Miorandi. On information transmission among nanomachines. In *Proc. of First Intl. Conf. on Nano-Networks and Workshops*, pages 1–5, September 2006.
- [8] Y.M. Ali and L.C. Zhang. Relativistic heat conduction. *International Journal of Heat and Mass Transfer*, 48:2397–2406, March 2005.
- [9] G Arfken. Green’s functions—two and three dimensions. *Mathematical Methods for Physicists*, pages 480–491, 1985.

- [10] D. Arifler. Capacity analysis of a diffusion-based short-range molecular nano-communication channel. *Computer Networks Journal (Elsevier)*, 55(6):1426–1434, April 2011.
- [11] B. Atakan and O. B. Akan. An information theoretical approach for molecular communication. In *Proc. of Second Intl. Conf. on Bio-Inspired Models of Network, Information and Computing Systems (Bionetics)*, pages 33–40, December 2007.
- [12] B. Atakan, S. Galmes, and O. B. Akan. Nanoscale communication with molecular arrays in nanonetworks. *IEEE Transactions on NanoBio-science*, 11(2):164–168, June 2012.
- [13] Baris Atakan and Ozgur B. Akan. On molecular multiple-access, broadcast, and relay channels in nanonetworks. In *Proc. of Third Intl. Conf. on Bio-Inspired Models of Network, Information and Computing Systems (Bionetics)*, pages 16:1–16:8, November 2008.
- [14] Baris Atakan and Ozgur B. Akan. On channel capacity and error compensation in molecular communication. *Springer Transaction on Computational System Biology*, 10:59–80, December 2009.
- [15] Baris Atakan and Ozgur B. Akan. Deterministic capacity of information flow in molecular nanonetworks. *Nano Communication Networks Journal (Elsevier)*, 1(1):31–42, March 2010.
- [16] David Baker, George Church, Jim Collins, Drew Endy, Joseph Jacobson, Jay Keasling, Paul Modrich, Christina Smolke, and Ron Weiss. Engineering life: building a fab for biology. *Scientific American*, 294(6):44–51, June 2006.
- [17] M. S. Bartlett. *An introduction to stochastic processes, with special reference to methods and applications*. Press Syndicate of the University of Cambridge, 1978.
- [18] Bahareh Behkam and Metin Sitti. Bacterial flagella-based propulsion and on/off motion control of microscale objects. *Appl. Phys. Lett.*, 90(2):023902, January 2007.
- [19] A. Ben-Naim. *A Farewell to Entropy: Statistical Thermodynamics Based on Information*. World Scientific Publishing Company, 2008.
- [20] H.C. Berg and E.M. Purcell. Physics of chemoreception. *Biophysical Journal*, 20(2):193–219, November 1977.
- [21] J. M. Bernardo. Psi (digamma) function. *Journal of the Royal Statistical Society. Series C (Applied Statistics)*, 25(3):315–317, 1976.
- [22] Michael J. Berridge. The AM and FM of calcium signalling. *Nature*, 386(6627):759–780, April 1997.

- [23] William Bialek and Sima Setayeshgar. Physical limits to biochemical signaling. *Proceedings of the National Academy of Sciences (PNAS) of the USA*, 102(29):10040–10045, July 2005.
- [24] William H. Bossert and Edward O. Wilson. The analysis of olfactory communication among animals. *Journal of Theoretical Biology*, 5(3):443–469, November 1963.
- [25] C. Bustamante, D. Keller, and G. Oster. The physics of molecular motors. *Accounts of Chemical Research*, 34(6):412–420, 2001.
- [26] L.J Clancy. *Aerodynamics*. Pitman Publishing Limited, 1975.
- [27] J. W. Costerton, P. S. Stewart, and E. P. Greenberg. Bacterial biofilms: a common cause of persistent infections. *Science*, 67(5418):1318–1322, May 1999.
- [28] Thomas M. Cover and Joy A. Thomas. *Elements of Information Theory, 2nd Edition*. Wiley, 2006.
- [29] E. L. Cussler. *Diffusion. Mass Transfer in Fluid Systems*. 2nd edition, Cambridge University Press, 1997.
- [30] Brian Davies. *Integral transforms and their applications*. Springer, New York, 2002.
- [31] B. S. Donahue and R. F. Abercrombie. Free diffusion coefficient of ionic calcium in cytoplasm. *Cell Calcium*, 8(6):437–48, December 1987.
- [32] E. Drexler. Molecular engineering: Assemblers and future space hardware. American Astronautical Society: AAS-86-415, 1986.
- [33] E. Drexler. *Nanosystems: Molecular Machinery, Manufacturing, and Computation*. John Wiley and Sons, Inc, 1992.
- [34] J. Matthew Dubach, Daniel I. Harjes, and Heather A. Clark. Fluorescent ion-selective nanosensors for intracellular analysis with improved lifetime and size. *Nano Letters*, 7(6):1827–2831, 2007.
- [35] E.R.G. Eckert and R.M. Drake. *Analysis of Heat and Mass Transfer*. McGraw-Hill, 1972.
- [36] Arash Einolghozati, Mohsen Sardari, Ahmad Beirami, and Faramarz Fekri. Capacity of discrete molecular diffusion channels. In *Proc. of 2011 IEEE International Symposium on Information Theory (ISIT)*, pages 603–607, July 2011.
- [37] Arash Einolghozati, Mohsen Sardari, and Faramarz Fekri. Capacity of diffusion-based molecular communication with ligand receptors. In *Proc. of 2011 IEEE Information Theory Workshop (ITW)*, pages 85–89, October 2011.

- [38] Arash Einolghozati, Mohsen Sardari, and Faramarz Fekri. Collective sensing-capacity of bacteria populations. In *Proc. of 2011 IEEE International Symposium on Information Theory (ISIT)*, pages 2959–2963, July 2012.
- [39] Arash Einolghozati, Mohsen Sardari, and Faramarz Fekri. Molecular communication between two populations of bacteria. In *Proc. of 2012 IEEE Information Theory Workshop (ITW)*, pages 437–441, September 2012.
- [40] Albert Einstein. On the electrodynamics of moving bodies. *Annalen der Physik*, 17:891–921, 1905.
- [41] Drew Endy. Foundations for engineering biology. *Nature*, 438(24):449–453, November 2005.
- [42] E. Fermi. *Thermodynamics*. Prentice-Hall Company, 1936.
- [43] R. A. Freitas. *Nanomedicine, Volume I: Basic Capabilities*. Landes Bioscience, Georgetown, TX, 1999.
- [44] Robert A. Freitas. Pharmacytes: An ideal vehicle for targeted drug delivery. *Journal of Nanoscience and Nanotechnology*, 6:2769–2775, 2006.
- [45] A. Gelman, J. B. Carlin, H. S. Stern, and D. B. Rubin. *Bayesian Data Analysis, Second Edition*. Chapman and Hall/CRC, 2003.
- [46] J. W. Gibbs, H. A. Bumstead, and R. G. Van Name. *Scientific Papers of J. Willard Gibbs, Vol. 1: Thermodynamics*. Longmans, Green and Company, 1961.
- [47] Daniel T. Gillespie. The chemical langevin equation. *Journal of Chemical Physics*, 113(1):297–306, July 2000.
- [48] Daniel T. Gillespie. Stochastic simulation of chemical kinetics. *Annual Review of Physical Chemistry*, 58:35–55, May 2007.
- [49] Maria Gregori and Ian F. Akyildiz. A new nanonetwork architecture using flagellated bacteria and catalytic nanomotors. *IEEE Journal on Selected Areas in Communications (JSAC)*, 28(4):602–611, May 2010.
- [50] Maria Gregori, Ignacio Llatser, Albert Cabellos-Aparicio, and Eduard Alarcón. Physical channel characterization for medium-range nanonetworks using flagellated bacteria. *Computer Networks*, 55(3):779–791, 2011.
- [51] Kapil Gulati, Brian L. Evans, Jeffrey G. Andrews, and Keith R. Tinsley. Statistics of co-channel interference in a field of poisson and poisson-poisson clustered interferers. *IEEE Transactions on Signal Processing*, 58(12):6207–6222, December 2010.

- [52] Yu-Chueh Hung, Wei-Ting Hsu, Ting-Yu Lin, and Ljiljana Fruk. Photoinduced write-once read-many-times memory device based on dna biopolymer nanocomposite. *Appl. Phys. Lett.*, 99:253301, October 2011.
- [53] J. Ilow and D. Hatzinakos. Analytic alpha-stable noise modeling in a poisson field of interferers or scatterers. *IEEE Transactions On Signal Processing*, 46(6):1601–1611, June 1998.
- [54] E. T. Jaynes. Information theory and statistical mechanics. *The Physical Review*, 106(4):620–630, May 1957.
- [55] S. Kadloor and R. Adve. A framework to study the molecular communication system. In *Proc. of 18th Intl. Conf. on Computer Communications and Networks*, pages 1–6, August 2009.
- [56] C.J. Kerr, K.S. Osborn, G.D. Robson, and P.S. Handley. The relationship between pipe material and biofilm formation in a laboratory model system. *Journal of Applied Microbiology*, 85:295–385, 1999.
- [57] M. S. Kuran, H. B. Yilmaz, T. Tugcu, and I. F. Akyildiz. Interference effects on modulation techniques in diffusion based nanonetworks. *Nano Communication Networks (Elsevier) Journal*, 3(1):65–73, March 2012.
- [58] M. S. Kuran, H. B. Yilmaz, T. Tugcu, and I. F. Akyildiz. Interference effects on modulation techniques in diffusion based nanonetworks. *Nano Communication Networks (Elsevier) Journal*, 3:65–73, March 2012.
- [59] M. P. Langevin. Paul langevin's 1908 paper on the theory of brownian motion. *American Journal of Physics*, 65(11):1079–1081, November 1997.
- [60] Jianping Li, Tuzhi Peng, and Yuqiang Peng. A cholesterol biosensor based on entrapment of cholesterol oxidase in a silicic sol-gel matrix at a prussian blue modified electrode. *Electroanalysis*, 15(12):1031–1037, 2003.
- [61] Jian-Qin Liu and Tadashi Nakano. An information theoretic model of molecular communication based on cellular signaling. In *Proc. of Second Intl. Conf. on Bio-Inspired Models of Network, Information and Computing Systems (Bionetics)*, pages 316–321, December 2007.
- [62] M.U. Mahfuz, D. Makrakis, and H.T. Mouftah. Characterization of intersymbol interference in concentration-encoded unicast molecular communication. In *Proc. of 24th Canadian Conference on Electrical and Computer Engineering (CCECE)*, pages 139–146, May 2011.
- [63] Andreas Mandelis. Diffusion waves and their uses. *Physics Today*, 53(29):29–34, August 2000.

- [64] Andreas Mandelis. *Diffusion-wave fields: mathematical methods and Green functions*. Springer-Verlag, 2001.
- [65] M. L. McGlashan. *Physico-Chemical Quantities and Units*. London: Royal Institute of Chemistry, 1968.
- [66] Donald A. McQuarrie. Kinetics of small systems. I. *The Journal of Chemical Physics*, 38(2):433–436, January 1963.
- [67] Milan Merkle. Convexity in the theory of the gamma function. *International Journal of Applied Mathematics & Statistics*, 11(V07):103–117, November 2007.
- [68] M A Model and G M Omann. Ligand-receptor interaction rates in the presence of convective mass transport. *Biophysical Journal*, 69(5):1712–1720, 1995.
- [69] M. Moore, A. Enomoto, T. Nakano, R. Egashira, T. Suda, A. Kayasuga, H. Kojima, H. Sakakibara, and K. Oiwa. A design of a molecular communication system for nanomachines using molecular motors. In *Proc. of Fourth Annual IEEE Intl. Conf. on Pervasive Computing and Communications Workshops*, pages 6–12, March 2006.
- [70] M.-J. Moore, T. Suda, and K. Oiwa. Molecular communication: modeling noise effects on information rate. *IEEE Transactions on NanoBioscience*, 85:295–385, June 2009.
- [71] T. Nakano, T. Suda, T. Koujin, T. Haraguchi, and Y. Hiraoka. Molecular communication through gap junction channels: system design, experiments and modeling. In *Proc. of 2nd Intl. Conf. on Bio-Inspired Models of Network, Information and Computing Systems (Bionetics)*, pages 139–146, December 2007.
- [72] T. Nakano, T. Suda, M. Moore, R. Egashira, A. Enomoto, and K. Arima. Molecular communication for nanomachines using intercellular calcium signaling. In *Proc. of Fifth IEEE Intl. Conf. on Nanotechnology*, volume 2, pages 478–481, July 2005.
- [73] D. L. Nelson and M. M. Cox. *Lehninger Principles of Biochemistry*, chapter 12.2, pages 425–429. W. H. Freeman, 2005.
- [74] A. Papoulis and S. U. Pillai. *Probability, Random Variables and Stochastic Processes, 4th ed.* McGraw-Hill, 2002.
- [75] Lluís Parcerisa and Ian F. Akyildiz. Molecular communication options for long range nanonetworks. *Computer Networks (Elsevier) Journal*, 53(16):2753–2766, August 2009.
- [76] Jean Philibert. One and a half century of diffusion: Fick, Einstein, before and beyond. *Diffusion Fundamentals*, 4:6.1–6.19, 2006.

- [77] M Pierobon and I. F. Akyildiz. A physical end-to-end model for molecular communication in nanonetworks. *IEEE Journal on Selected Areas in Communications (JSAC)*, 28(4):602–611, May 2010.
- [78] M Pierobon and I. F. Akyildiz. Diffusion-based noise analysis for molecular communication in nanonetworks. *IEEE Transactions on Signal Processing*, 59(6):2532–2547, June 2011.
- [79] M. Pierobon and I. F. Akyildiz. Information capacity of diffusion-based molecular communication in nanonetworks. In *in Proc. of IEEE Intl. Conf. on Computer Communication, INFOCOM, Miniconference*, April 2011.
- [80] M Pierobon and I. F. Akyildiz. Noise analysis in ligand-binding reception for molecular communication in nanonetworks. *IEEE Transactions on Signal Processing*, 59(9):4168–4182, September 2011.
- [81] M. Pierobon and I. F. Akyildiz. Intersymbol and co-channel interference in diffusion-based molecular communication. In *in Proc. of the 2nd IEEE Intl. Workshop on Molecular and Nano Scale Communication (MoNaCom), ICC*, June 2012.
- [82] M Pierobon and I. F. Akyildiz. Capacity of a diffusion-based molecular communication system with channel memory and molecular noise. *IEEE Transactions on Information Theory*, 59(2):942–954, June 2013.
- [83] Klaus Prank, Fabrizio Gabbiani, and Georg Brabant. Coding efficiency and information rates in transmembrane signaling. *Biosystems*, 55(1-3):15–22, February 2000.
- [84] Michael Reed and Ron Rohrer. *Applied introductory circuit analysis for electrical and computer engineers*. Prentice-Hall, Inc. Upper Saddle River, NJ, USA, 1999.
- [85] Jordi Riu, Alicia Maroto, and F. Xavier Rius. Nanosensors in environmental analysis. *Talanta*, 69(2):288–301, 2006.
- [86] Jean-Pierre Rospars, Vlastimil Křivan, and Ptr Lánský. Perireceptor and receptor events in olfaction. Comparison of concentration and flux detectors: a modeling study. *Chemical Senses*, 25(3):293–311, June 2000.
- [87] Howard Salis, Alvin Tamsir, and Christopher Voigt. Engineering bacterial signals and sensors. *Contrib. Microbiol.*, 16:194–225, June 2009.
- [88] E. Schrödinger. *Statistical thermodynamics*. Dover Publications New York, 1989.

- [89] A. Sezginer and Weng Cho Chew. Closed form expression of the green's function for the time-domain wave equation for a lossy two-dimensional medium. *IEEE Transactions on Antennas and Propagation*, 32(5):527–528, 1984.
- [90] C. E. Shannon. A mathematical theory of communication. *The Bell System Technical Journal*, 27:379–423, July 1948.
- [91] Jeffrey S. Simonoff. *Smoothing Methods in Statistics*. Springer Series in Statistics. Springer-Verlag New York, 1998.
- [92] D. J. Spencer, S. K. Hampton, P. Park, J. P. Zurkus, and P. J. Thomas. The diffusion-limited biochemical signal-relay channel. *Advances in Neural Information Processing Systems*, 16, 2004.
- [93] P. Tallury, A. Malhotra, L. M. Byrne, and S. Santra. Nanobioimaging and sensing of infectious diseases. *Advanced Drug Delivery Reviews*, 62(4-5):424–437, March 2010.
- [94] Gabriel Vasilescu. *Electronic Noise and Interfering Signals: Principles and Applications*. Springer Series in Signals and Communication Technology. Springer, 2005.
- [95] C. R. Yonzon, D. A. Stuart, X. Zhang, A. D. McFarland, C. L. Haynes, and R. P. V. Duyne. Towards advanced chemical and biological nanosensors - an overview. *Talanta*, 67(3):438–448, 2005.
- [96] Steven S. Zumdahl. *Thermochemistry*. Chemistry. Cengage Learning., 2008.