Monotonic Optimization in Communication and Networking Systems

Monotonic Optimization in Communication and Networking Systems

Ying Jun (Angela) Zhang

The Chinese University of Hong Kong
Hong Kong
yjzhang@ie.cuhk.edu.hk

Liping Qian

Zhejiang University of Technology
China
qianjoe@gmail.com

Jianwei Huang

The Chinese University of Hong Kong
Hong Kong
jwhuang@ie.cuhk.edu.hk

now
the essence of knowledge
Boston – Delft

Foundations and Trends® in Networking

Published, sold and distributed by:
now Publishers Inc.
PO Box 1024
Hanover, MA 02339
USA
Tel. +1-781-985-4510
www.nowpublishers.com
sales@nowpublishers.com

Outside North America:
now Publishers Inc.
PO Box 179
2600 AD Delft
The Netherlands
Tel. +31-6-51115274

The preferred citation for this publication is Y. J. (Angela) Zhang, L. Qian and J. Huang, Monotonic Optimization in Communication and Networking Systems, Foundations and Trends® in Networking, vol 7, no 1, pp 1–75, 2012

ISBN: 978-1-60198-708-2

Foundations and Trends® in Networking
Volume 7 Issue 1, 2012

Editorial Scope

Foundations and Trends® in Networking publishes survey and tutorial articles in the following topics:

- Modeling and Analysis of:
  - Ad Hoc Wireless Networks
  - Sensor Networks
  - Optical Networks
  - Local Area Networks
  - Satellite and Hybrid Networks
  - Cellular Networks
  - Internet and Web Services
- Protocols and Cross-Layer Design
- Network Coding
- Energy-Efficiency Incentives/Pricing/Utility-based
- Games (co-operative or not)
- Security
- Scalability
- Topology
- Control/Graph-theoretic models
- Dynamics and Asymptotic Behavior of Networks

Information for Librarians

Foundations and Trends® in Networking, 2012, Volume 7, 4 issues. ISSN paper version 1554-057X. ISSN online version 1554-0588. Also available as a combined paper and online subscription.

Foundations and Trends® in
Networking
Vol. 7, No. 1 (2012) 1–75

DOI: 10.1561/1300000038

Monotonic Optimization in Communication and Networking Systems

Ying Jun (Angela) Zhang[1], Liping Qian[2] and Jianwei Huang[3]

[1] *Department of Information Engineering, The Chinese University of Hong Kong, Hong Kong, yjzhang@ie.cuhk.edu.hk*
[2] *College of Information Engineering, Zhejiang University of Technology, China, qianjoe@gmail.com*
[3] *Department of Information Engineering, The Chinese University of Hong Kong, Hong Kong, jwhuang@ie.cuhk.edu.hk*

Abstract

Optimization has been widely used in recent design of communication and networking systems. One major hurdle in this endeavor lies in the nonconvexity of many optimization problems that arise from practical systems. To address this issue, we observe that most nonconvex problems encountered in communication and networking systems exhibit monotonicity or hidden monotonicity structures. A systematic use of the monotonicity properties would substantially alleviate the difficulty in obtaining the global optimal solutions of the problems. This monograph provides a succinct and accessible introduction to *monotonic optimization*, including the formulation skills and solution algorithms. Through several application examples, we will illustrate modeling techniques and algorithm details of monotonic optimization in various scenarios. With this promising technique, many previously difficult problems can now be solved with great efficiency. With this monograph, we wish to spur new research activities in broadening the scope of application of monotonic optimization in communication and networking systems.

Contents

1

Introduction

1.1 Monotonic Optimization Theory and Applications

The global data traffic has reached 885 petabytes per month in 2012, which is more than ten times the global Internet traffic in the entire year of 2000. The rapid demand growth drives the research community to develop evolutionary and revolutionary approaches that push the communication and networking system performance toward new limits. To this end, optimization techniques have been proved extremely useful in approaching the utmost capacity of the limited available radio resources. Indeed, optimization methods have been successfully employed to obtain the optimal strategies for, for example, radio resource allocation, routing and scheduling, power control and interference avoidance, MIMO transceiver design, TCP flow control, and localization, just to name a few.

Most recent advances of optimization techniques rely crucially on the convexity of the problem formulation. Nonetheless, many problems encountered in practical engineering systems are nonconvex *by their very nature*. These problems are not only nonconvex in their original forms, but also cannot be equivalently transformed to convex ones by any existing means.[1] One such example is power control for throughput maximization in wireless networks. Another example is general utility maximization in random access networks.

An encouraging observation, however, is that a majority of nonconvex problems encountered in communication and networking systems exhibit monotonicity or hidden monotonicity structures. For example, the capacity and reliability of a wireless link monotonically increase with the bandwidth and SINR (signal to interference and noise ratio) of the link, and the quality of service provided to a user is a nondecreasing function of the amount of radio resources dedicated to the user. A systematic use of monotonicity properties may substantially alleviate the difficulty in obtaining the global optimal solution(s) of the problems, and this is indeed the key idea behind the *monotonic optimization* theory.

The theory of monotonic optimization has been established relatively recently by a series of papers by Tuy [22, 31, 32, 33, 34, 35, 37] and others [17, 27]. To intuitively understand the potential advantages offered by a monotonicity structure, recall that the search for a global optimal solution of a nonconvex optimization problem can involve examining every feasible point in the entire feasible region. If the objective function $f(\mathbf{z}) : \mathcal{R}^n \to \mathcal{R}$ to be maximized is increasing, however, then once a feasible point $\mathbf{z}$ is known, one can ignore the whole cone $\mathbf{z} - \mathcal{R}^n_+$,[2] because no better feasible solution can be found in this cone. On the other hand, if the function $g(\mathbf{z}) : \mathcal{R}^n \to \mathcal{R}$ in a constraint like $g(\mathbf{z}) \leq 0$ is increasing, then once a point $\mathbf{z}$ is known to be infeasible, the whole cone $\mathbf{z} + \mathcal{R}^n_+$ can be ignored, because no feasible solution can be found in this cone. As such, the monotonic nature of the objective function and constraints allows us to limit the global search to a much smaller region of the feasible set, thus drastically simplifying the problem.

Only very recently was monotonic optimization introduced to the communication and networking research community. The first attempt

[1] Note that there are also problems that are seemingly nonconvex, but can be equivalently transformed to convex problems by existing known methods, for example, change of variables. Such problems are NOT considered as nonconvex in our context.

[2] $\mathbf{z} - \mathcal{R}^n_+$ and $\mathbf{z} + \mathcal{R}^n_+$ correspond to the sets $\{\mathbf{z}'|\mathbf{z}' \leq \mathbf{z}\}$ and $\{\mathbf{z}'|\mathbf{z}' \geq \mathbf{z}\}$, respectively.

was made by Qian et al. [24], where the global optimal power control solution of ad hoc networks was found by exploiting the hidden monotonicity of the nonconvex power control problem. This work was subsequently followed by a number of researchers [5, 9, 12, 13, 15, 16, 19, 23, 25, 38, 39, 41, 43], where monotonicity or hidden monotonicity structures were exploited to solve a variety of nonconvex problems arising from areas including capacity maximization, scheduling, MIMO precoding and detection, distributed antenna coordination, and optimal relaying, etc. By and large, the application of monotonic optimization in communication and networking systems is still at its infancy stage, mainly because the technique is relatively new and unfamiliar to the communication and networking community. This is contrasted by the fact that most nonconvex problems considered in the communication and networking community are indeed monotonic.

The purpose of this monograph is to provide a succinct and accessible introduction to the theory and algorithms of monotonic optimization. Through several application examples, we will illustrate modeling techniques and algorithm details of monotonic optimization in various engineering scenarios. This is a humble attempt to spur new research activities in substantially broadening the scope of application of this promising technique in communication and networking systems.

1.2 Outline

There are two main parts in this monograph. Part I focuses on the theory and Part II on the application.

Part I consists of Sections 2 and 3, and is mainly based on the work of Tuy et al. [15, 22, 31, 32, 33, 34, 35, 37]. In particular, Section 2 discusses the formulation techniques, including the canonical formulation of monotonic optimization problems and problems that can be transformed into the canonical form. Section 3 introduces the polyblock outer approximation algorithm and its various enhancements that expedite the algorithm. The discussion is then extended to problems with discrete variables.

Part II consists of Sections 4–7. In particular, Section 4 discusses nonconvex power control in wireless interference channels, where the problem formulations belong to a special class of monotonic optimization problems, namely, general linear fractional programming. Section 5 discusses power controlled scheduling problems, where we show how to reduce the variable size by exploiting the convexity of some set. Section 6 extends the discussion to multi-antenna systems where the objective is to optimize the transmitter beamforming. In this section we illustrate how to deal with vector variables in the polyblock outer approximation algorithm. Finally, Section 7 concerns network utility maximization in random access networks, where the problem is to maximize an increasing function of polynomials. Through this problem, we illustrate the use of auxiliary variables to convert a "difference of monotonic" optimization problem to a canonical monotonic optimization problem.

1.3 Notations

Throughout this monograph, vectors are denoted by boldface lower case letters and matrices are denoted by boldface upper case letters. The i^{th} entry of a vector $\mathbf{x}$ is denoted by x_i. We use $\mathcal{R}$, $\mathcal{R}_+$, and $\mathcal{R}_{++}$ to denote the set of real numbers, nonnegative real numbers, and positive real numbers, respectively. The set of n-dimensional real, nonnegative real, and positive real vectors are denoted by $\mathcal{R}^n$, $\mathcal{R}^n_+$, $\mathcal{R}^n_{++}$, respectively. $\mathbf{e}^i \in \mathcal{R}^n$ denotes the i^{th} unit vector of $\mathcal{R}^n$, i.e., the vector such that $e^i_i = 1$ and $e^i_j = 0$ for all $j \neq i$. $\mathbf{e} \in \mathcal{R}^n$ is an all-one vector.

For any two vectors $\mathbf{x}, \mathbf{y} \in \mathcal{R}$, we say $\mathbf{x} \leq \mathbf{y}$ (or $\mathbf{x} < \mathbf{y}$) if $x_i \leq y_i$ (or $x_i < y_i$) for all $i = 1, \cdots, n$. When $\mathbf{x} \leq \mathbf{y}$, we also say $\mathbf{y}$ dominates $\mathbf{x}$ or $\mathbf{x}$ is dominated by $\mathbf{y}$. Moreover, $\mathbf{x} - \mathcal{R}^n_+$ and $\mathbf{x} + \mathcal{R}^n_+$ correspond to the cones $\{\mathbf{x}'|\mathbf{x}' \leq \mathbf{x}\}$ and $\{\mathbf{x}'|\mathbf{x}' \geq \mathbf{x}\}$, respectively. $\cup$, $\cap$, and$\setminus$ represent set union, set intersection, and set difference operators, respectively.

References

[1] G. Bianchi, "Performance analysis of the IEEE 802.11 distributed coordination function," *IEEE Journal on Selected Areas in Communications*, vol. 18, no. 3, pp. 535–547, 2000.

[2] S. Boyd, S. Kim, L. Vandenberghe, and A. Hassibi, "A tutorial on geometric programming," *Optimization and Engineering*, vol. 8, no. 1, pp. 67–127, 2007.

[3] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge university press, 2004.

[4] M. Chiang, C. Tan, D. Palomar, D. O'Neill, and D. Julian, "Power control by geometric programming," *IEEE Transactions on Wireless Communications*, vol. 6, no. 7, pp. 2640–2651, 2007.

[5] G. Fodor, M. Johansson, and P. Soldati, "Near optimum power control under fairness constraints in CoMP systems," in *IEEE GLOBECOM*, 2009.

[6] Y. Gao, D.-M. Chiu, and J. Lui, "Determining the end-to-end throughput capacity in multi-hop networks: Methodology and applications," in *ACM SIGMETRICS Performance Evaluation Review,* vol. 34, no. 1, pp. 39–50, 2006.

[7] M. Garetto, T. Salonidis, and E. W. Knightly, "Modeling per-flow throughput and capturing starvation in CSMA multi-hop wireless networks," *IEEE/ACM Transactions on Networking*, vol. 16, no. 4, pp. 864–877, 2008.

[8] S. He, Z.-Q. Luo, J. Nie, and S. Zhang, "Semidefinite relaxation bounds for indefinite homogeneous quadratic optimization," *SIAM Journal on Optimization*, vol. 19, no. 2, pp. 503–523, 2008.

[9] C. Hellings, W. Utschick, and M. Joham, "Power minimization in parallel vector broadcast channels with separate linear precoding," in *Proceedings of the European Signal Processing Conference, Barcelona, Spain*, 2011.

[10] R. Horst, N. Thoai, and H. Benson, "Concave minimization via conical partitions and polyhedral outer approximation," *Mathematical Programming*, vol. 50, no. 1, pp. 259–274, 1991.

[11] J. Huang, R. Berry, and M. Honig, "Distributed interference compensation for wireless networks," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 5, pp. 1074–1084, 2006.

[12] E. A. Jorswieck and E. G. Larsson, "Monotonic optimization framework for the two-user MISO interference channel," *IEEE Transactions on Communications*, vol. 58, no. 7, pp. 2159–2168, 2010.

[13] S. Joshi, P. Weeraddana, M. Codreanu, and M. Latva-Aho, "Weighted sum-rate maximization for MISO downlink cellular networks via branch and bound," *IEEE Transactions on Signal Processing*, vol. 60, no. 4, pp. 2090–2095, 2012.

[14] S. C. Liew, C. H. Kai, H. C. Leung, and P. Wong, "Back-of-the-envelope computation of throughput distributions in CSMA wireless networks," *IEEE Transactions on Mobile Computing*, vol. 9, no. 9, pp. 1319–1331, 2010.

[15] L. Liu, R. Zhang, and K. Chua, "A new approach to weighted sum-rate maximization for the K-user gaussian interference channel," in *IEEE International Conference on Wireless Communications and Signal Processing (WCSP)*, Nanjing, China, 2011.

[16] L. Liu, R. Zhang, and K. Chua, "Achieving global optimality for weighted sum-rate maximization in the K-user gaussian interference channel with multiple antennas," *IEEE Transactions on Wireless Communications*, vol. 11, no. 5, pp. 1933–1945, May 2012.

[17] L. T. Luc, "Reverse polyblock approximation for optimization over the weakly efficient set and efficient set," *Acta Mathematica Vietnamica*, vol. 26, no. 1, pp. 65–80, 2001.

[18] D. G. Luenberger, "Quasi-convex programming," *SIAM Journal on Applied Mathematics*, vol. 16, no. 5, pp. 1090–1095, 1968.

[19] J. Mikael and S. Pablo et al., "Near optimum power control and precoding under fairness constraints in network MIMO systems," *International Journal of Digital Multimedia Broadcasting*, 2010.

[20] A. Mohsenian-Rad, J. Huang, M. Chiang, and V. Wong, "Utility-optimal random access: Reduced complexity, fast convergence, and robust performance," *IEEE Transactions on Wireless Communications,*, vol. 8, no. 2, pp. 898–911, 2009.

[21] P. C. Ng and S. C. Liew, "Throughput analysis of IEEE 802.11 multi-hop ad hoc networks," *IEEE/ACM Transactions on Networking (TON)*, vol. 15, no. 2, pp. 309–322, 2007.

[22] N. T. H. Phuong and H. Tuy, "A unified monotonic approach to generalized linear fractional programming," *Journal of Global Optimization*, vol. 26, no. 3, pp. 229–259, 2003.

[23] L. Qian and Y. Zhang, "S-MAPEL: Monotonic optimization for non-convex joint power control and scheduling problems," *IEEE Transactions on Wireless Communications*, vol. 9, no. 5, pp. 1708–1719, 2010.

[24] L. Qian, Y. J. Zhang, and J. Huang, "MAPEL: Achieving global optimality for a non-convex wireless power control problem," *IEEE Transactions on Wireless Communications*, vol. 8, no. 3, pp. 1553–1563, 2009.

[25] M. Riemensberger, I. Abdelghani, J. Lenz, and W. Utschick, "Weighted sum rate maximization on two user Gaussian interference channels with rate splitting," in *Asilomar Conference on Signals, Systems and Computers*, Asilomar, CA. USA, 2009.

[26] J. W. Robinson and T. S. Randhawa, "Saturation throughput analysis of IEEE 802.11 e enhanced distributed coordination function," *IEEE Journal on Selected Areas in Communications*, vol. 22, no. 5, pp. 917–928, 2004.

[27] A. Rubinov, H. Tuy, and H. Mays, "An alogrithm for monotonic global optimization problems," *Optimization*, vol. 49, no. 3, pp. 205–221, 2001.

[28] H. D. Sherali, "Global optimization of nonconvex polynomial programming problems having rational exponents," *Journal of Global Optimization*, vol. 12, no. 3, pp. 267–283, 1998.

[29] H. D. Sherali and C. H. Tuncbilek, "A global optimization algorithm for polynomial programming problems using a reformulation-linearization technique," *Journal of Global Optimization*, vol. 2, no. 1, pp. 101–112, 1992.

[30] P. D. Tao et al., "The DC (difference of convex functions) programming and DCA revisited with dc models of real world nonconvex optimization problems," *Annals of Operations Research*, vol. 133, no. 1–4, pp. 23–46, 2005.

[31] H. Tuy, "Monotonic optimization: Problems and solution approaches," *SIAM Journal on Optimization*, vol. 11, pp. 464–494, 2000.

[32] H. Tuy, "Monotonicity in the framework of generalized convexity," *Generalized Convexity, Generalized Monotonicity and Applications*, pp. 61–85, 2005.

[33] H. Tuy and F. Al-Khayyal, "Monotonic optimization revisited," Working paper, http://www. math. ac. vn/library/download/e-print/03/pdf/htuy23. pdf. Accessed in Feburary, Technical Report, 2004.

[34] H. Tuy, F. Al-Khayyal, and P. Thach, "Monotonic optimization: branch and cut methods," *Essays and Surveys in Global Optimization*, pp. 39–78, 2005.

[35] H. Tuy and N. Hoai-Phuong, "Optimization under composite monotonic constraints and constrained optimization over the efficient set," *Global Optimization*, pp. 3–31, 2006.

[36] H. Tuy, M. Minoux, and N. Hoai-Phuong, "Discrete monotonic optimization with application to a discrete location problem," *SIAM Journal on Optimization*, vol. 17, no. 1, pp. 78–97, 2006.

[37] H. Tuy, P. Thach, and H. Konno, "Optimization of polynomial fractional functions," *Journal of Global Optimization*, vol. 29, no. 1, pp. 19–44, 2004.

[38] W. Utschick and A. Brack, "Computing upper bounds for coordinated multipoint transmission in small networks," in *IEEE International ITG Workshop on Smart Antennas (WSA)*, Aachen, Germany, 2011.

[39] W. Utschick and J. Brehmer, "Monotonic optimization framework for coordinated beamforming in multi-cell networks," *IEEE Transactions on Signal Processing*, vol. 60, no. 4, pp. 1899–1909, 2012.

[40] J. P. Vielma, S. Ahmed, and G. Nemhauser, "Mixed-integer models for nonseparable piecewise-linear optimization: Unifying framework and extensions," *Operations research*, vol. 58, no. 2, pp. 303–315, 2010.

[41] Y. Wang, W. Feng, L. Xiao, Y. Zhao, and S. Zhou, "Coordinated multi-cell transmission for distributed antenna systems with partial CSIT," *IEEE Communications Letters*, vol. 16, no. 7, pp. 1044–1047, 2012.

[42] Y. Xiao, "Performance analysis of priority schemes for IEEE 802.11 and IEEE 802.11e wireless lans," *IEEE Transactions on Wireless Communications*, vol. 4, no. 4, pp. 1506–1515, 2005.

[43] J. Zhang, F. Roemer, M. Haardt, A. Khabbazibasmenj, and S. A. Vorobyov, "Sum rate maximization for multi-pair two-way relaying with single-antenna amplify and forward relays," in *IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, Kyoto, Japan, 2012.