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Multi-hop Wireless Networks: A Unified Approach to Relaying and Interference Management

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Abstract

Multi-hop communication paradigms are expected to play a central role in future wireless networks by enabling a higher spatial reuse of the spectrum. A major challenge in multi-hop multi-user (or multi-flow) wireless networks is that "interference management" and "relaying" are coupled with each other. In other words, wireless relay nodes must play a dual role: they serve as intermediate steps for multi-hop communication and as part of the mechanism that allows interference management schemes. Nonetheless, in the communications, networking and information theory literature, these two tasks have traditionally been addressed separately, and the fundamental principles of the "wireless networks of the future" are currently not well understood. In this monograph, we take a unified approach to relaying and interference management, and seek to develop tools to study the fundamentals of multi-hop multi-flow wireless networks.

We first consider multi-hop two-flow – or *two-unicast* – wireless networks. In order to handle networks with an arbitrary number of hops and arbitrary interference patterns, we introduce the idea of *network condensation*, by which a network with an arbitrary number of layers is effectively reduced to a network with at most four layers. This is done by identifying *key layers* and letting the nodes in all other layers apply random linear coding to relay the messages. Only the nodes in the remaining key layers need to be "smart" and perform coupled relaying and interference management operations. In addition, we introduce the new notion of *paths with manageable interference*, which represents a first attempt at finding flow-like structures in multi-user wireless networks, and develop novel outer bounds that capture the interference structure of a given topology. These techniques yield a complete characterization of the degrees of freedom of two-unicast layered networks as a function of the network graph.

Extending these results for general K-unicast networks is quite challenging. To make progress on this front, we focus on the $K \times K \times K$ wireless network, a two-hop network consisting of K sources, K relays, and K destinations. This network represents a canonical example of a multi-hop multi-flow wireless network for which previously there was a

large gap between known inner and outer bounds, even from a degrees-of-freedom perspective. We introduce a coding scheme called *Aligned Network Diagonalization* (AND) that couples relaying and interference management in a way that all interference experienced by the destinations is simultaneously neutralized. This proves that $K \times K \times K$ wireless networks have K sum degrees of freedom and demonstrates the significant gains that can be obtained with a unified approach to relaying and interference management. Moreover, this automotically yields the optimal scheme and degrees-of-freedom characterization for layered K unicast networks with fully connected hops.

We then describe ideas and preliminary results for K-unicast networks with general topologies. Besides discussing how the tools developed for two-unicast networks and for $K \times K \times K$ networks can be extended to this general setting, we present a novel outer-bounding technique, which improves over the cut-set bound and can capture limitations imposed by the interference between different users. The new bound can be understood as computing the flow across multiple "nested cuts", as opposed to a single cut, as is the case in the classical cut-set bound. This technique allows us to establish a graph-theoretic notion of manageable interference in $K \times K \times K$ wireless networks with arbitrary connectivity.

Throughout the monograph, many extensions and future directions are addressed. At the end of each chapter, related work is also described and several open problems are presented. Important research directions such as accounting for the lack of global channel state information in large networks and reducing the complexity of relaying operations are discussed, and recent results along these lines are described.

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Introduction

Relaying and interference management are two key aspects of communication in wireless networks. While the relaying strategy is what allows information to flow in the network, in the wireless setting, flows originated at different sources interfere with each other, and interference management schemes are crucial for reliable communication. Hence, in *multi-hop* wireless networks; i.e., networks where messages must traverse multiple nodes before reaching their destination, wireless relay nodes must play a dual role: they serve as intermediate steps for relaying schemes and as part of the mechanism that allows interference management schemes.

Nevertheless, from a communications, networking and information theory perspective, these two components have traditionally been addressed separately. The relaying problem is usually studied in the context of multi-hop *single-flow* wireless networks (or, simply, relay networks), where inter-user interference is not an issue. Starting with simple strategies such as decode-and-forward and amplify-and-forward, many relaying strategies have been developed and had their performances studied, initially in the context of the simple relay channel [14] and later in more general relay networks. Recently, in [4], a new relaying strategy called *quantize-map-and-forward* was introduced and shown to achieve to within a constant number of bits of the capacity of general single-flow relay networks. Later on, other strategies such as lattice quantization followed by map-and-forward [43] and compress-and-forward [37] were also shown to achieve the capacity of relay networks to within a constant number of bits.

In the realm of interference management, starting with simple ideas such as *treating interference as noise*, several techniques for coping with undesired interference have also been proposed. For instance, the idea of decoding the interference and subtracting it from the received signal, or *Successive Interference Cancellation*, is particularly useful in strong interference scenarios, and when side information about the interference is available at the transmitter, *Dirty Paper Coding* [13] ideas can be employed. In general, most of the information-theoretic research on interference management tends to focus in *multi-flow single-hop* wireless networks, or interference channels. The Han-Kobayashi scheme [25], where users split their messages into a private part and a public part, is a simple yet powerful way of dealing with interference in these networks. Particularly for the two-user interference channel, it has been used to establish the capacity of the interference channel to within a single bit [17]. When we increase the number of users, however, the problem becomes significantly more difficult, and the interference management techniques required become far more sophisticated. The main example is the idea of *Interference Alignment*, which consists of trying to "align" the interference from all unintended transmitters at each receiver, so that they are perceived as a single interference signal. This technique has led to an approximation to the capacity of K-user interference channels in terms of sum degrees of freedom [10, 40].

Once we consider multi-hop multi-flow networks, the challenge is to combine the insights obtained in the study of single-hop multi-flow networks and multi-hop single-flow networks in order to design schemes that handle relaying and interference management simultaneously. In this monograph, we discuss the gains that can be obtained by taking such a *unified* approach to relaying and interference management. We base ourselves on the recent developments presented in [29, 46, 48,

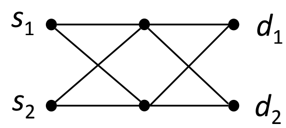

Figure 1.1: A fully-connected $2 \times 2 \times 2$ wireless network.

50] to study fundamental principles of these unified constructions and discuss interesting directions for future research and open problems.

Our initial guiding example will be the $2 \times 2 \times 2$ wireless network, shown in Fig. 1.1. The unified framework we will propose in this case for handling relaying and interference management simultaneously will have an interesting interpretation: interference management techniques will be responsible for modifying the effective network topology to facilitate the information relaying to the destinations. This suggests that a coupled approach to relaying and interference management may provide new coding opportunities that are absent otherwise, and that relays have the potential of being *game-changers* in interference management schemes.

As exact capacity characterizations for multi-hop multi-flow networks are still a distant goal, our performance metric of choice will be the *sum degrees of freedom*. The sum degrees of freedom of a multi-flow network can be understood as the pre-log factor of the sum capacity. More precisely, if each node in the network receives its signals at a given signal-to-noise ratio (SNR), and the sum capacity is represented by $C_\Sigma(\mathrm{SNR})$, then the sum degrees of freedom are defined as

$$D_\Sigma = \lim_{\mathrm{SNR}\to\infty} \frac{C_\Sigma(\mathrm{SNR})}{\log \mathrm{SNR}}.$$

Therefore, the sum degrees of freedom can be thought of as providing a first-order approximation of the sum capacity given by $C_\Sigma(\mathrm{SNR}) = D_\Sigma \log \mathrm{SNR} + o(\log \mathrm{SNR})$. As it turns out, designing communication schemes from a degrees-of-freedom perspective; i.e., trying to achieve the optimal degrees of freedom, reveals many fundamental insights about multi-hop multi-flow networks, and gives rise to sophisticated novel schemes that significantly outperform previously known schemes that took a non-unified approach to interference management and relaying.

This monograph will be organized as follows. After formalizing the problem and providing necessary definitions in Chapter 2, we consider the $2 \times 2 \times 2$ wireless network, which consists of two source-destination pairs communicating with each other via two relays, as a motivating example. First, we will consider restricting ourselves to *linear schemes*; i.e., schemes where the relays are restricted to performing linear operations on their received signals. Interestingly, we show that with such schemes we can achieve $4/3$ degrees of freedom by effectively creating distinct network topologies at different times and coding across them. This illustrates a key role that relays can play for interference management in multi-hop multi-flow networks; namely *changing the end-to-end interference topology and creating the potential for coding across them*, and suggests that a coupled approach to relaying and interference management can indeed offer new opportunities of coding that go beyond what decoupled schemes can do. In fact, as shown in [22], by using more sophisticated coupled schemes than the linear schemes required to achieve $4/3$ degrees of freedom, 2 degrees of freedom can be achieved in $2 \times 2 \times 2$ wireless networks. Motivated by this example, we then study more general network topologies and the fundamental limits of a unified approach to relaying and interference management in those networks.

A natural direction of extension of the $2 \times 2 \times 2$ wireless network consists of increasing the number of hops and allowing general connectivity patterns for each hop. This gives raise to a large class of layered networks where the characterization of the degrees of freedom, even in the case of two source-destination pairs (otherwise known as *two-unicast* networks) is challenging, and several questions can be asked. Which network topologies behave like $2 \times 2 \times 2$ wireless networks and provide enough opportunities for the effective interference at the destinations to be fully neutralized? When this full neutralization is not possible, what can coupled schemes accomplish? When do more sophisticated schemes such as the one from [22] are necessary and what is the potential of linear schemes such as the one from [29]?

These questions motivate Chapter 3, where we study two-unicast wireless networks. Our main result is a general characterization of the degrees of freedom of two-unicast layered networks, based on the network graph. In order to understand which networks are like the $2\times2\times2$, in the sense that full interference neutralization is possible and 2 degrees of freedom are achievable, we introduce the new notion of *paths with manageable interference*. This represents a first attempt at finding flow-like structures in multi-user wireless networks. In addition, we introduce the idea of network condensation, which allows us to transform any two-unicast layered network into a network with at most four layers but the same sum degrees of freedom, and develop novel outer bounds that capture the interference structure of a given topology in order to obtain an outer bound that is tighter than the classical cut-set bound. These techniques allow us to completely characterize the sum degrees of freedom (and later the full degrees-of-freedom region), proving the interesting fact that two-unicast layered networks only admit 1, 3/2 or 2 sum degrees of freedom. Moreover, only a small fraction of the nodes (more precisely, those that remain in the condensed network) may be required to perform more sophisticated non-linear operations. We conclude the chapter by discussing future research directions and possible extensions to non-layered scenarios and to networks with more than two source-destination pairs, i.e., K-unicast networks, for $K > 2$.

As it turns out, for more than two users, fully characterizing the degrees of freedom is a fairly difficult challenge. We make progress in this direction by focusing on $K \times K \times K$ wireless networks, i.e., two-hop K-user networks with K relays. As a natural generalization of $2 \times 2 \times 2$ wireless networks, it makes sense to ask whether the new opportunities provided by a unified approach to relaying and interference management extend to these settings. How much of the interference experienced by the destinations can be effectively neutralized? What $K \times K \times K$ topologies are more amenable to interference management? How can techniques such as manageable interference and the outer bounds that go beyond the cut-set bound obtained in Chapter 3 can be extended to $K \times K \times K$ networks?

These questions are the topic of Chapter 4. We first consider fully-connected $K \times K \times K$ wireless networks. Previously, the best known coding scheme for the setup with $K > 2$ consisted of assigning each relay to a source-destination pair, and viewing the network as the concatenation of two single-hop networks, essentially decoupling relaying and interference management. Under such an approach, the coding performance is limited by the amount of interference management that can be performed in a K-user single-hop network and $1/2$ a degree of freedom per user can be achieved. Our first result in this chapter is a new coding scheme, called *Aligned Network Diagonalization* (AND), that exploits the potential of the relays for interference management in order to substantially outperform the previous state of the art. In fact, we show that when the relays operate according to AND, they manage to neutralize all the interference experienced by the destinations, proving that $K \times K \times K$ wireless networks have K degrees of freedom (under both fast-fading and slow-fading scenarios). This result also settles the degrees of freedom of layered networks with fully connected hops. Therefore, next we consider relaxing the assumption of fully connected hops. By introducing a new general bound on the degrees of freedom of K-unicast networks, which improves over the cut-set bound, and by extending the AND scheme to certain non-fully connected topologies, we obtain a graph-theoretic characterization of manageable interference for $K \times K \times K$ topologies. We conclude the chapter by discussing extensions to networks with more than two hops and ideas to improve the proposed outer bounds.

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