Wireless Network Optimization by Perron-Frobenius Theory

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Abstract

A basic question in wireless networking is how to optimize the wireless network resource allocation for utility maximization and interference management. How can we overcome interference to efficiently optimize fair wireless resource allocation, under various stochastic constraints on quality of service demands? Network designs are traditionally divided into layers. How does fairness permeate through layers? Can physical layer innovation be jointly optimized with network layer routing control? How should large complex wireless networks be analyzed and designed with clearly-defined fairness using beamforming?

This monograph provides a comprehensive survey of the models, algorithms, analysis, and methodologies using a Perron-Frobenius theoretic framework to solve wireless utility maximization problems. This approach overcomes the notorious non-convexity barriers in these problems, and the optimal value and solution of the optimization problems can be analytically characterized by the spectral property of matrices induced by nonlinear positive mappings. It also provides a systematic way to derive distributed and fast-convergent algorithms and to evaluate the fairness of resource allocation. This approach can even solve several previously open problems in the wireless networking literature.

More generally, this approach links fundamental results in nonnegative matrix theory and (linear and nonlinear) Perron-Frobenius theory with the solvability of non-convex problems. In particular, it can solve a particular class of max-min problems optimally; for truly nonconvex problems, e.g., the sum rate maximization problem, it can even be used to identify polynomial-time solvable special cases or to enable convex relaxation for global optimization. We highlight the key aspects of the nonlinear Perron-Frobenius theoretic framework through several practical examples in MIMO wireless cellular, heterogeneous small-cell and cognitive radio networks.

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1

Wireless Network Optimization

1.1 Introduction

The demand for broadband mobile data services has grown significantly and rapidly in wireless networks. As such, many new wireless devices are increasingly operating in the wireless spectrum that are meant to be shared among many different users. Yet, the sharing of the spectrum is far from perfect. Due to the broadcast nature of the wireless medium, interference has become a major source of performance impairment. Current systems suffer from deteriorating quality due to a fixed resource allocation that does not adequately take interference into account.

As wireless networks become more heterogeneous and ubiquitous in our life, they also become more difficult to design and optimize. How should these large complex wireless networks be analyzed and designed with clearly-defined fairness and optimality in mind? In this regard, wireless network optimization has become an important tool to design resource allocation algorithms that can realize the untapped benefits of co-sharing wireless resources and to manage interference in wireless networks [56, 36, 69, 19, 23]. Without appropriate resource coordination, the wireless network may become unstable or may operate in a highly inefficient and unfair manner.

In wireless network optimization, the performance objective of a wireless transmission can be modeled by a nonlinear utility function that takes into account important wireless link metrics. Examples of these wireless metrics are the Signal-to-Interference-and-Noise Ratio (SINR), the Mean Square Error (MSE) or the transmission outage probability. The total utility function is then maximized over the joint solution space of all possible operating points in the wireless network. These operating points are realized in terms of the powers and interference at the physical link layer.

As such, wireless network optimization can be used to address engineering issues such as how to design wireless network algorithms or analyzing the tradeoffs between individual link performance and overall system performance. It can even be useful for understanding cross-layer optimization, for example, how these algorithms interact between different network layers, such as the physical and medium access control layers, in order to achieve provable efficiency for the overall system. It also sheds insights on how fairness permeates through the network layers when interference is dominant. This can open up new opportunities to jointly optimize physical layer innovation and other networking control mechanism that lead to more robust and reliable wireless network protocols.

1.2 Related Work

Due to the need to share limited wireless resources, fairness is an important consideration in wireless networks. Fairness is affected by the choice of the nonlinear utility functions of the wireless link metrics [56, 19, 21, 12]. In addition, fairness experienced by each user in the wireless networks is also affected by the channel conditions, multiuser interference, and other factors such as the wireless quality-of-service requirements. An example of such a requirement is the interference temperature constraints in cognitive radio networks that are essentially constraints imposed on the received interference for some users [40, 96, 70]. Another example is outage probability specification constraints in heterogeneous networks [45, 52]. As such, fairness can be provisioned by choosing an optimal operating point that is fair in some sense to all the users by an appropriate formulation of a wireless utility maximization. The main challenges in solving these wireless utility maximization problems come from the nonlinear and coupling dependency of link metrics on channel conditions and powers, as well as the interference among the users. In addition, these are nonconvex problems that are notoriously difficult to solve optimally. Moreover, designing scalable and distributed algorithms with low-complexity to solve these nonconvex problems is even harder.

In fact, there are several important considerations to algorithm design in wireless networks. First, algorithms have to adapt the wireless resources such as the transmit power and to overcome interference based on locally available information. This means that the algorithms have to be as distributed as possible. Second, the algorithms are practical to deploy in a decentralized manner, i.e., the algorithms have minimal or, preferably, no parameter tuning by a controller. Third, the algorithms have good convergence performance. This is especially important since wireless users can arrive and depart in a dynamic setting. Henceforth, wireless resources need to be adapted fast enough to converge to a new optimal operating point whenever the network conditions change. This can be particularly challenging for some kinds of wireless networks such as wireless cognitive radio networks due to the tight coupling in the transmit powers and the interference temperature constraints between the primary users and the secondary users [40, 96, 70]. Whatever the algorithms may be, the algorithm design methodology is intrinsically driven by the theoretical approach used in analyzing the optimization problems. Finding an appropriate theoretical approach to study wireless network optimization is thus important.

There are several work in the literature on tackling the nonconvexity hurdles in wireless network optimization. The authors in [20, 19, 23, 45] applied geometric programming to solve a certain class of nonconvex wireless utility maximization problems that can be transformed into convex ones. The authors in [12] studied the use of Gibbs sampling techniques to solve nonconvex utility maximization problems, but the optimality of the solutions cannot be guaranteed. The authors in

[21, 39, 54] tackled deterministic wireless utility maximization problems that involved rates and powers, but the proposed techniques could not handle stochastic constraints. In [77, 76, 72, 74, 16, 18, 97, 100, 99], the authors studied the max-min utility fairness problem in wireless networks using a particular form of nonlinear Perron-Frobenius theory [50], [55]. These works demonstrated that the optimal solution to various widely-studied max-min optimization problems, e.g., the max-min SINR and max-min rate problems, can be characterized analytically and, more importantly, can be efficiently computed by iterative algorithms that can be made distributed. We introduce and present some of these work using the nonlinear Perron-Frobenius theory approach in this monograph.

1.3 Why is the Perron-Frobenius Theory useful?

The Perron-Frobenius theory introduced in this monograph is a new theoretical framework for analyzing a class of nonconvex optimization problems for resource allocation in wireless networks. Essentially, this framework provides a convenient suite of theories and algorithms to solve a broad class of wireless network optimization problems optimally by leveraging on the recent developments of the nonlinear Perron-Frobenius theory in mathematics. When combined with optimization-theoretic approaches such as convex reformulation and convex relaxation, this nonlinear Perron-Frobenius theory framework enables the design of efficient algorithms with low complexity that are applicable to a wide range of wireless network applications. Let us first discuss a special case of this nonlinear Perron-Frobenius theory in the following.

In nonnegative matrix theory, the classical *linear Perron-Frobenius theorem* is an important result that concerns the eigenvalue problem of nonnegative matrices, and has many engineering applications [63, 35, 68, 64, 30]. Notably, the linear Perron-Frobenius Theorem has consistently proven to be a useful tool in wireless network resource allocation problems. Its application to power control in wireless networks has been widely recognized (see, e.g., [67, 80, 22]), and can be traced back to earlier work in [1, 60] on balancing the signal to interference ratio in satellite communication that was later adopted for wireless cellular networks in [2, 94, 31, 87, 92, 83, 66, 90, 10, 11] and wireless ad hoc networks [28]. In particular, it has been used in a total power minimization problem studied in [31, 92, 83, 66, 28], in which the Perron-Frobenius Theorem is used to ascertain the problem feasibility and the stability of power control algorithms proposed in [31, 83, 66, 28].

In the seminal work in [1] that first formulated and analyzed the signal to interference ratio balancing problem, the linear Perron-Frobenius theorem was used to derive the optimal solution analytically for this nonconvex problem. Subsequently, the same problem formulation was adopted in [60, 2, 94, 87] for designing power control algorithms for both satellite and wireless cellular communication networks that converge to the solution established in [1]. That the Perron-Frobenius theorem is fundamental is due to two facts. First, the problem parameters and optimization variables in wireless network optimization problems are mostly *nonnegative*. Second, it captures succinctly the unique feature of *competition* for limited resources among users, namely, *increasing the share of one decreases the shares of others* as well as *who is competing with whom*.

Another popular approach to tackle the nonconvexity hurdles in these wireless network optimization problems has been the use of geometric programming (see [27, 15, 20, 14] for an introduction) and its successive convex approximation as used by the authors in [45, 19, 23, 20]. The idea of the geometric programming approach is to reformulate the nonconvex problems as suitable classes of convex optimization problems (geometric programs) through a logarithmic change-of-variable trick. This leverages the inherent nonnegativity property. The geometric programs are then typically solved numerically by the interior-point method in a centralized fashion. In fact, geometric programming is closely related to the Perron-Frobenius theorem. For example, it can be used to establish the log-convexity property of the Perron-Frobenius eigenvalue [46, 61] (also see [15]).

The use of the linear Perron-Frobenius theorem in earlier work however has several limitations. They cannot address the general case (such as when we consider the thermal noise or general power constraints). In addition, the Perron-Frobenius theorem has not been used to systematically solve other broader nonconvex wireless network optimization problems beyond the power control optimization problems studied in [1, 60, 2, 94, 87, 31, 92, 83, 66, 28]. In fact, to overcome the specific challenges due to nonconvexity, it is imperative to consider more general (i.e., nonlinear) version of the Perron-Frobenius theory that can spawn new approaches to characterize optimality and analyze the equilibrium as well as designing distributed algorithms for wireless networks.

There are various mathematical advances in extending the linear Perron-Frobenius theorem to nonlinear ones. These include works that extend the Perron-Frobenius theorem for positive matrices to nonsmooth and nonlinear functions in the 1960s (e.g., see Chapter 16 in [5], [55]) for studying the dynamics of cone-preserving operators. The nonlinear Perron-Frobenius theory is now emerging as a rigorous and practically useful mathematical tool to solve a wide range of important engineering problems and applications [49, 50, 9, 3, 55]. In this monograph, we will introduce and illustrate how the nonlinear Perron-Frobenius theory can tackle several key challenging wireless network optimization problems following the work in [76, 79, 78, 72, 77, 16, 18, 17, 43, 97, 100, 99, 98, 53, 42, 73, 74]. Whenever applicable, we also highlight the connection to previous works that rely on the linear Perron-Frobenius theorem as special cases.

The following notation is used in this monograph. Boldface uppercase letters denote matrices, boldface lowercase letters denote column vectors, and $\mathbf{u} \geq \mathbf{v}$ denotes componentwise inequality between vectors $\mathbf{u}$ and $\mathbf{v}$. We also let $(\mathbf{By})_l$ denote the lth element of $\mathbf{By}$. Let $\mathbf{x} \circ \mathbf{y}$ denote the Schur product of the vectors $\mathbf{x}$ and $\mathbf{y}$, i.e., $\mathbf{x} \circ \mathbf{y} = [x_1 y_1, \ldots, x_L y_L]^\top$. Let $\|\mathbf{w}\|_\infty^{\mathbf{x}}$ be the weighted maximum norm of the vector $\mathbf{w}$ with respect to the weight $\mathbf{x}$, i.e., $\|\mathbf{w}\|_\infty^{\mathbf{x}} = \max_l w_l / x_l$, $\mathbf{x} > \mathbf{0}$. We write $\mathbf{B} \geq \mathbf{F}$ if $B_{ij} \geq F_{ij}$ for all i, j. The Perron-Frobenius eigenvalue of a nonnegative matrix $\mathbf{F}$ is denoted as $\rho(\mathbf{F})$, and the Perron right and left eigenvector of $\mathbf{F}$ associated with $\rho(\mathbf{F})$ are denoted by $\mathbf{x}(\mathbf{F}) \geq \mathbf{0}$ and $\mathbf{y}(\mathbf{F}) \geq \mathbf{0}$ (or simply $\mathbf{x}$ and $\mathbf{y}$ when the context is clear), respectively. The superscript $(\cdot)^\top$ denotes transpose. We denote $\mathbf{e}_l$ as the lth unit coordinate vector and $\mathbf{I}$ as the identity matrix.

1.4 System Model

In this section, we introduce the system models for the wireless network utility maximization problems considered in the monograph. There are primarily two different kinds of system models - one that considers a static transmission channel (i.e., frequency-flat fading) and one that considers stochastic channel fading. Whenever applicable, we will emphasize the system model to avoid confusion.

Let us first introduce the static transmission channel for modeling a wireless network by the Gaussian interference channel [24]. There are altogether L links or users (equivalently, transceiver pairs) that want to communicate with its desired receiver. Due to mutual interfering channels, each user treats the multiuser interference as noise, i.e., no interference cancellation. This is a commonly used model (in, e.g., [19, 23, 22]) to model many wireless networks such as the radio cellular networks and ad-hoc networks. Let us denote the transmit power for the lth user as p_l for all l. Assuming that a linear single-user receiver (e.g., a matched-filter) is used, the Signal-to-Interference-and-Noise-Ratio (SINR) for the lth user can be given by

$$\mathsf{SINR}_l(\mathbf{p}) = \frac{G_{ll}p_l}{\sum_{j \neq l} G_{lj}p_j + n_l}, \tag{1.1}$$

where G_{lj} are the channel gains from the transmitter j to the receiver l and n_l is the additive white Gaussian noise (AWGN) power for the lth receiver. For brevity, we collect the channel gains in the channel gain matrix $\mathbf{G}$, and the channel gains take into account propagation loss, spreading loss and other transmission modulation factors. Notice that the SINR is a function in terms of the transmit powers and, furthermore, it is always nonnegative since all the quantities involved in (1.1) are nonnegative. There are many other important wireless performance metrics that are also directly dependent on the achieved SINR. For example, assuming a fixed bit error rate at the receiver, the Shannon capacity formula can be used to deduce the achievable data rate of the lth link as [24]:

$$\log\left(1 + \mathsf{SINR}_l(\mathbf{p})\right) \quad \text{nats/symbol}. \tag{1.2}$$

Let us define a nonnegative square matrix $\mathbf{F}$ with the entries given by:

$$F_{lj} = \begin{cases} 0, & \text{if } l = j \\ \frac{G_{lj}}{G_{ll}}, & \text{if } l \neq j \end{cases} \tag{1.3}$$

and a vector

$$\mathbf{v} = \left(\frac{n_1}{G_{11}}, \frac{n_2}{G_{22}}, \ldots, \frac{n_L}{G_{LL}} \right)^\top . \tag{1.4}$$

Observe that $\mathbf{F}$ and $\mathbf{v}$ capture the normalized values of the cross-channel gain parameters and the background noise power respectively. They are regarded as given constant problem parameters and are useful for notations represented in a compact manner in this monograph.

Let us next introduce the system model with stochastic channel fading that builds on top of the static transmission channel model by taking into account more realistic wireless transmission features. One important feature is the stochastic channel fading that is typically modeled by a Rayleigh, a Ricean or a Nakagami distribution depending on the wireless environment [80, 47]. For example, Rayleigh fading is relevant to in-building coverage model and urban environments (where small cells are mostly deployed in a heterogeneous network).

Under stochastic channel fading, the power received from the jth transmitter at lth receiver is given by $G_{lj}R_{lj}p_j$ where G_{lj} models a constant nonnegative path gain and R_{lj} is a random variable to model the stochastic channel fading between the jth transmitter and the lth receiver. In particular, we assume that R_{lj} is independently distributed with unit mean. For example, under Rayleigh fading, the distribution of the received power from the jth transmitter at the lth receiver is exponential with a mean value $\mathbb{E}[G_{lj}R_{lj}p_j] = G_{lj}p_j$.

When there is stochastic channel fading, the Signal-to-Interference-Noise Ratio (SINR) at the lth receiver can be expressed as the following by using the above notations [45, 22]:

$$\mathsf{SINR}_l(\mathbf{p}) = \frac{R_{ll}p_l}{\sum_{j \neq l} F_{lj}R_{lj}p_j + v_l}. \tag{1.5}$$

Notice that (1.5) is a random variable that depends on the stochastic channel fading realization. In particular, this random variable in

(1.5) is also a function of the transmit powers (and should not be confused with (1.1) which has no direct physical meaning in the context of stochastic channel fading).

Now, the transmission from the lth transmitter to its receiver is successful if $\mathsf{SINR}_l(\mathbf{p}) \geq \beta_l$ (no outage), where β_l is a given threshold for reliable communication. An outage occurs at the lth receiver whenever $\mathsf{SINR}_l(\mathbf{p}) < \beta_l$. We express this outage probability of the lth user by

$$P(\mathsf{SINR}_l(\mathbf{p}) < \beta_l). \tag{1.6}$$

Notice that the transmit powers are typically coupled together through the various wireless performance metric functions for any particular user. For example, the transmit powers of different users are coupled in (1.1) and (1.6) when the channel has frequency-flat fading and stochastic fading respectively. Adapting the transmit powers directly influences the wireless performance metrics. As such, in the wireless network optimization problems studied in this monograph, the transmit power vector $(p_1, \ldots, p_L)^\top$ is the main optimization variable of interest.

In addition, the transmit powers in wireless networks are typically constrained. This is modeled by a power constraint set $\mathcal{P}$ that can be due to resource budget consideration [80]. For example, in a cellular uplink system, we often have individual power constraints, i.e.,

$$\mathcal{P} = \{\mathbf{p} \mid \mathbf{p} \geq \mathbf{0},\, p_l \leq \bar{p} \;\; \forall\, l\}. \tag{1.7}$$

Power constraints can also be used for interference management. Let us give an example of interference management in wireless heterogeneous networks. Say, in a wireless heterogeneous network, there are two different user type - the small cell users and the macrocell users. A basic premise imposed on small cells in wireless heterogeneous networks is that the following two conditions are satisfied [52]:

1. A small cell user receives adequate levels of transmission quality within the small cell.

2. The small cell users do not cause unacceptable levels of interference to the macrocell users.

To satisfy the second condition above, a possibility is to explicitly impose power constraints on the small cell users. Let us illustrate using an example of a single macrocell user and multiple small cells in [52]. Assume that there is no fading between this single macrocell receiver and all the small cell users. This assumption holds only in this paragraph for illustration purpose. Let us denote this macrocell user by the index 0 and the small cell users by indices $1, \ldots, L$. The macrocell user transmits with a fixed power P_0, where $P_0 \geq \gamma_0 v_0$, i.e., the macrocell user can satisfy the SINR threshold γ_0 even when there is no interference from the small cells. In the presence of small cells' interference, the SINR of this macrocell user has to satisfy $\frac{P_0}{\sum_{j=1}^{L} F_{0j} p_j + v_0} \geq \gamma_0$, which can be rewritten as a single power constraint to yield

$$\mathcal{P} = \left\{ \mathbf{p} \mid \mathbf{p} \geq \mathbf{0}, \sum_{j=1}^{L} F_{0j} p_j \leq (P_0/\gamma_0 - v_0) \right\} \tag{1.8}$$

that must be satisfied by the transmit powers of all the small cell users. Note that (1.8) is feasible when $P_0 \geq \gamma_0 v_0$. In general, a feasible power constraint of the form $\mathbf{a}^\top \mathbf{p} \leq 1$ for some positive constant vector $\mathbf{a}$ can be used to model interference management requirements. Notice that this example of an interference management constraint is also applicable to other wireless applications such as the cognitive radio networks with the primary user and secondary user types [40, 96, 70].

Now, there are many different possible ways to satisfy the first condition above on the adequate levels of transmission quality. In this monograph, we examine some of these different ways that in fact also relate to how this monograph is organized in the following.

We first begin with the mathematical preliminaries on the Perron-Frobenius theorem and the nonlinear Perron-Frobenius theory in Chapter 2, and then introduce how these theories are used to solve various optimization problems in subsequent chapters. In the first part of Chapter 3, we study the optimization of the *max-min weighted SINR* using (1.1) for a static channel model. In the second part of Chapter 3, we study the optimization of the *worst-case outage probability* using (1.6), i.e., minimizing the maximum outage probability when there is stochastic channel fading, to provision a minimum adequate level of fairness for all the users. These two problems only involve simple power constraints such as that given in (1.8). In Chapter 4, we study more general utility functions to capture the satisfaction level of transmission quality in different kinds of wireless networks, and also to consider a broader class of nontrivial power constraint sets to model resource constraints and interference management requirements. The Perron-Frobenius theory suggests that the unique equilibrium that results from the competition for resources in the optimization problems in Chapters 3 and 4 is a meaningful one. In Chapter 5, we study more general nonconvex optimization problems involving the achievable data rate using (1.2) and show how the Perron-Frobenius theory can be a useful mathematical tool to tackle nonconvexity. We also highlight the open issues in these various wireless network optimization problems and finally conclude the monograph in Chapter 6.

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