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Contagion Source Detection in Epidemic and Infodemic Outbreaks: Mathematical Analysis and Network Algorithms

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Contagion Source Detection in Epidemic and Infodemic Outbreaks: Mathematical Analysis and Network Algorithms

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ABSTRACT

The rapid spread of infectious diseases and online rumors share similarities in terms of their speed, scale, and patterns of contagion. Although these two phenomena have historically been studied separately, the COVID-19 pandemic has highlighted the devastating consequences that simultaneous crises of epidemics and misinformation can have on the world. Soon after the outbreak of COVID-19, the World Health Organization launched a campaign against the COVID-19 Infodemic, which refers to the dissemination of pandemic-related false information online that causes widespread panic and hinders recovery efforts. Undoubtedly, *nothing spreads faster than fear.*

Networks serve as a crucial platform for viral spreading, as the actions of highly influential users can quickly render others susceptible to the same. The potential for contagion in epidemics and rumors hinges on the initial source, underscoring the need for rapid and efficient digital contact

tracing algorithms to identify superspreaders or Patient Zero. Similarly, detecting and removing rumor mongers is essential for preventing the proliferation of harmful information in online social networks. Identifying the source of large-scale contagions requires solving complex optimization problems on expansive graphs. Accurate source identification and understanding the dynamic spreading process requires a comprehensive understanding of surveillance in massive networks, including topological structures and spreading veracity. Ultimately, the efficacy of algorithms for digital contact tracing and rumor source detection relies on this understanding.

This monograph provides an overview of the mathematical theories and computational algorithm design for contagion source detection in large networks. By leveraging network centrality as a tool for statistical inference, we can accurately identify the source of contagions, trace their spread, and predict future trajectories. This approach provides fundamental insights into surveillance capability and asymptotic behavior of contagion spreading in networks. Mathematical theory and computational algorithms are vital to understanding contagion dynamics, improving surveillance capabilities, and developing effective strategies to prevent the spread of infectious diseases and misinformation.

1

Introduction

1.1 Epidemics and Rumors

The spreading of epidemics and rumors on networks share many important features [28], [29], [36], [58]. The underlying network interaction cannot be directly observed and often has to be implicitly inferred from macroscopic phenomena. Driven by the same human collective crowd behavior, these network dynamics can lead to common network effects like the small-world phenomenon and percolation thresholds. The study of epidemics and rumor spreading is thus an important part of applied probability theory and graph theory related to the analysis of the evolution of large systems arising in networks. Even though the process of spreading information in online social networks differs from that of disease epidemics, the proliferation of fake news and recent disinformation campaigns in online social networks has emerged in recent years as a formidable cybersecurity threat that can have catastrophic real-world consequences like a pandemic [45], [92], [135], [149].

Though epidemics and rumor spreading have been separately studied in the past with a longer history for the stochastic theory of epidemic spreading, the COVID-19 pandemic has been the first of simultaneous global crises in which both the epidemic and overabundance of mis-

information devastatingly wreak havoc on the world. The COVID-19 pandemic is the first pandemic in history in which humans rely heavily on the Internet and online social networks to stay connected amidst the prolonged lockdown and social distancing measures in place. It has also spawned an epidemic of online misinformation, undermining the efficacy of online social networks that humans crucially rely on and disrupting public health risk communications. Shortly after the COVID-19 pandemic started, the World Health Organization (WHO) declared war against the COVID-19 Infodemic, which is the viral spreading of pandemic-related misinformation or disinformation in social media [54].

Spreading processes are dynamic cascading phenomena where the action of some users increases the susceptibility of other users to the same; this results in the successive spread of a disease virus or rumor from an initial few users to a much larger set of users [28], [29], [36], [58]. When a new infectious virus spreads, public healthcare authorities want to identify persons who may have come into contact with an infected person and to trace close social contacts in order to stop ongoing transmission or reduce the spread of infection. When rumors like false treatment for the COVID-19 disease spread in online social networks, this can prevent humans from adopting the right behaviors to reduce the COVID-19 pandemic risk. Once misinformation morphed into disinformation attacks, it can be disruptive and deadly. These simultaneous crises require both public healthcare and cyber security experts to work together to fight infodemics by identifying sources of misinformation.

An objective of interests is to unravel the dynamical spreading process to root out the malicious source quickly, accurately, and reliably with only limited observation data of infected nodes in the network. Just like epidemic countermeasures like digital contact tracing and policies to identify *Patient Zero* in an outbreak,¹ building resilience to catastrophic viral misinformation is of huge importance to a safe and functioning cyberspace because of the highly-connected online social networks.

¹Contact tracing apps based on the Bluetooth wireless radio standard are arguably one of the defining technologies for surveillance during the COVID-19 pandemic [65], [91].

1.1. Epidemics and Rumors

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To accurately detect or predict the causation of contagion in large networks, it is crucial to identify the superspreaders and the origin of disease viruses. Similarly, it is essential to determine who is spreading rumors and disinformation to cause division and influence decisions among users. This raises questions about the provenance of such information [92], [100], [150]. Additionally, the implications of network surveillance and the response to contain a contagion must be considered. With the emergence of new communication platforms, new avenues for spreading misinformation and disinformation arise. Identifying the source of contagion can have far-reaching consequences, such as timely responses to the next pandemic or promoting a safe cyberspace.

Numerous fundamental questions remain unanswered in the statistical inference of infection sources in networks. The theory of stochastic processes over large networks is still evolving, and the computational aspects of estimation and detection in networks have not yet been systematically examined, with source identification understood only in the simplest graph topology cases. It is remarkable that even though human social interaction or online social networks are not designed with the intention of spreading a payload (such as an infectious disease virus or rumor) as rapidly as possible, the process of viral spreading over large networks is not fully comprehended [92], [100], [150]. Are there specific network structures, quantifiable measures of user influence that promote viral spreading? If so, what particular features could aid in the development of better digital contact tracing strategies or interventions to counter the spread of malicious rumors?

As we strive to comprehend the spread of contagions across large networks, it is crucial to recognize the potential for cross-pollination of ideas between different types of networks, each with distinct interaction graph structures, initial nodes, and nature of user interactions [28], [29], [36], [58]. For instance, in [6], researchers proposed intervention strategies based on a generative model of viral misinformation spread using infectious disease spreading dynamics. Moreover, when network topology abstraction is sufficiently random, it may provide insights into network phenomena based on percolation theory, as noted in [38].

1.2 Propagated Epidemics and Contact Tracing

Tracing the origins of propagated epidemics can be traced back to the investigation of the 1854 London cholera epidemics by John Snow (1813–1858), who is widely recognized as a pioneer of modern epidemiology [5], [49], [50]. His work in tracking the source of the cholera outbreak was a significant breakthrough in epidemiological research. By creating detailed dot distribution maps of household deaths due to cholera, Snow was able to identify the source of the epidemic - a water pump located in Broad Street, Golden Square. Snow's methodical tracing effort was one of the earliest applications of inferential statistics to the study of epidemics [5], [49], [50]. Additionally, his heroic intervention in persuading the parish's vestrymen to remove the water pump symbolizes one of the earliest examples of public health action. It is important to note that Snow's contribution to epidemiology was not only a significant scientific achievement but also a landmark event in the history of public health. The removal of the water pump resulted in the rapid cessation of the cholera epidemic, saving countless lives and laying the foundation for modern epidemiological research.

Nowadays, epidemiologists agree that it is necessary to employ contact tracing to stop an infectious disease from spreading: Once a person has been diagnosed as infected, public health authorities fan out to trace the recent contacts of this person for the purpose of monitoring or quarantine. This process repeats if one of those contacts exhibits symptoms until all the contacts who have been exposed are out of circulation. Contact tracing can be effective in the early stage of an epidemic. However, the COVID-19 pandemic had revealed severe deficiencies in public health protection due to asymptomatic infections. Prior study [22] shows that asymptomatic infections need to be considered in analyzing the spread of the disease. The COVID-19 disease is highly contagious, wide-ranging with long incubation periods and transmissible within 6 feet. Its speed and scale of infection had overwhelmed most contact tracing capabilities which are labor-intensive, cost-inefficient and very slow [45], [86]. A new public health innovation, *digital contact tracing*, then came to the scene. Digital contact tracing leverages a plethora of mobile apps to contact trace people and to provide exposure notifications [8], [17], [46], [65], [82], [91], [94], [95].

1.3. Disinformation and Rumor Source Detection

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Current contact tracing practices focus primarily on finding recent contacts of index cases, while overlooking the source of origin. In fact, source inference is an important factor that explains the initial success of backward contact tracing adopted by countries like Japan and Australia in the early days of the COVID-19 pandemic [12], [17], [86], showing that, whenever there is a sudden outbreak, tracing transmission events rather than infectious individuals can efficiently and effectively prevent infection waves.

There are several challenging unsolved problems in digital contact tracing [17], [81], [91], [139]. First, what is the fundamental relationship between infectiousness and the agility of contact tracing? Can contact tracing be faster than the spreading of an infectious disease? Second, how to quadruple the speed of contact tracing? Can backward contact tracing complement forward contact tracing to find Patient Zero or the superspreaders accurately? Third, can we design disease surveillance networks so as to provide timely prediction and early warning capability to automate digital contact tracing upon the arrival of future epidemics?

1.3 Disinformation and Rumor Source Detection

Online social networks like Twitter, Facebook, and YouTube are critical online platforms for spreading news and the diffusion of all kinds of information. They can however cause misinformation and disinformation to spread faster and more rampantly than the traditional “word-of-mouth” mechanism [3], [15], [52], [62], [92], [96], [98], [110], [135], [149], [159]. In fact, false news spreads faster than the truth in a Twitter network [150]. Misinformation is inaccurate or unreliable information that is spread regardless of an intent to mislead. On the other hand, disinformation is intentionally-fabricated misinformation (e.g., hoax news) that is spread with the intent to influence people to make certain decisions or to further an agenda. A malicious rumor monger can now “infect” people across geographical regions on a massive scale faster than ever before. Online rumors, misinformation, and disinformation can thus disrupt livelihood and have serious real-world repercussions.

Recent examples are political mobilization messages spreading in social media that sparked off waves of demonstrations and protests in

the Middle East (dubbed the “Arab Spring” or the Twitter revolution) in 2010-2012. In 2013, a bogus Tweet that the White House was attacked went viral after it was sent out by the Associated Press Twitter account that was hacked [11]. This incident momentarily crashed the stock market, demonstrating how online disinformation can cause flash crash and allowing computer hackers to profiteer in the process. A similarly severe incident happened in 2020 when computer hackers seized control of dozens of Twitter accounts belonging to high-profile users like Barack Obama and Elon Musk to tweet out a “double your bitcoin” scam, which went viral quickly. Eventually, this cryptocurrency scam led to a theft of bitcoins worth more than US \$110,000 before all the scam messages were removed. Such Internet frauds and cybersecurity threats will be more widespread, especially when bots are recruited to sow discord to amplify the spread of disinformation.

Nations worldwide now recognize that the spread of misinformation and disinformation is an imminent cybersecurity threat that should be seriously addressed by law enforcement agencies [130]. However, the distinction between harmless misinformation and disinformation is often blurred. Moreover, rapid advances in deepfake technologies can make hoax news look legitimate and further exacerbate the situation. Rooting out rumor mongers and dispelling disinformation of increased scale and impact will be part of a timely and practical defense strategy that can offer intellectually deep insight to the science of networks.

What can cause the viral spreading of rumors or disinformation? One factor is semantics [52], [101]. For example, hoaxes and prank threats such as bomb threats are considered more serious but are likely short-lived as they can be quickly debunked. On the other hand, some rumors might swirl longer in social media (e.g., workplace rumors like layoffs or the inefficacy of certain pandemic measures) [52], [101], [126], [147]. Another factor is the principle of homophily in which humans have a tendency to associate with similar others, leading to cognitive bias typically known as the “echo chamber” effect [56], [62], [159]. The element of surprise can also affect rumor viscosity as people will tend to spread the information.

1.4 Overview of the Monograph

This monograph provides an overview of the surveillance of contagion sources in networks that find applications in digital contact tracing and rumor source detection to combat epidemics and infodemics, respectively. Given data that embeds both network topological structure (e.g., knowing who is connected to whom) and relational patterns on how a disease virus or rumor propagates, the objective is to answer the fundamental question: *how to unravel stochastic spreading processes in the network to find the initial outbreak source quickly, accurately and reliably with high confidence by exploiting the topological and statistical properties of networks.*

The contagion source detection problem was first studied in the seminal work [131]–[133]. Mathematically, the problem is: Given a snapshot observation of the contagion graph (showing how “infected” users are connected), who is the contagion source of the spreading? This problem is formulated as a maximum likelihood estimation problem over graphs and then solved exactly for special cases of degree-regular trees with infinite underlying graphs using a new form of network centrality called rumor centrality. Since then, it has spawned a huge literature on contagion source detection with various extensions such as random trees in [37], [53], to multiple sources in [69], [70], [105], [106], [108], [109], [112], [144], [167], to probabilistic sampling in [77], [120] and detection with multiple observations in [35], [153], belief propagation [40], general graphs with irregularity [154], [155], [165] and the implication of probabilistic spreading models and different graph topological features on solving the contagion source detection problem [4], [40], [84], [103], [114], [127], [137], [155], [168].

Different types of network centrality defined on vertices can resolve different types of network problems. Rumor centrality [132] is designed to solve the contagion source detection problem on infinite-size regular tree networks optimally (cf. Section 3.2). The vertex with the maximum rumor centrality is called the rumor center of a tree graph, and the rumor center was proved to be the same as the distance center [131], and the graph centroid of the tree [142], [163], [164]. Furthermore, it was shown in [72], [73], [85] that the graph centroid is almost surely central in the

limit of the random growth process of infection on an underlying infinite regular graph. Aside from the distance centrality, another distance-based centrality, the Jordan center, was proposed to solve the contagion source detection in different scenarios [111], [112]. Dynamic influence due to stochastic spreading and opinion dynamics in online social networks can be characterized by the harmonic influence centrality in [1], [148] and the Shapley centrality in [21]. The protection centrality in [2] and relative centrality in [18] measure how important a set of vertices in a network is with respect to other vertices at the gatekeeper level and community level respectively. Querying this contagion source in a large graph with cost constraints and query complexity has been analyzed in [25], [93], [127]. Centrality measures related to the eigenvectors of the network topology are also important in the study of stochastic processes over large graphs [31], [57], [76], [124].

The bibliography included in this monograph seeks to encompass as many contributions as possible, aiming to provide a balanced overview of the key results and methodologies. Although the monograph may not be a perfect summary of the state-of-the-art (see related surveys in [71], [160] before 2018), it aims to serve as an imperfect yet informative summary, providing a rough illustration of the existing literature in the last 15 years and with relevance to the COVID-19 pandemic and infodemic. We survey the various work in this field with a particular focus on the intricate interplay between contagion source detection and mathematical tools like graph theory, probability theory, combinatorics, and algorithm design for statistical inference in the context of large networks.

This monograph provides a comprehensive overview of contagion source detection problem along with a problem-solving approach called “*network centrality as statistical inference*” that expounds a systematic approach to analyze inferential statistical problems in networks with applications to digital contact tracing and rumor source detection. The framework presented in this work establishes a connection between network centrality and the solution of challenging optimization problems that involve complex combinatorial constraints arising from the interaction of a stochastic process with the underlying network. By leveraging an appropriate network centrality, which induces a metric on each

1.4. Overview of the Monograph

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graph node, it is possible to obtain compact measures that quantify the importance of nodes and accurately capture the optimality of stochastic optimization. This framework also enables the utilization of graph algorithm techniques to address these problems effectively [59], [145].

We will discuss how the “*network centrality as statistical inference*” approach can be useful to the graph algorithm design that comes with performance guarantees, computational complexity, detection accuracy, and to address the “big data” regime in which the contagion graph can be very large (as is the case in the COVID-19 pandemic and infodemic). Designing scalable algorithms that uncover the contagion source accurately by leveraging network science and mathematical tools will be important to prevent future pandemics (e.g., ‘Disease X’ pandemic and infodemic) given the enhanced human connectivity on a global scale. We will conclude with open issues and several promising research directions to address the challenges of surveillance of spreading in networks.

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