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Boston — Delft

Foundations and Trends® in Optimization

Published, sold and distributed by:
now Publishers Inc.
PO Box 1024
Hanover, MA 02339
United States
Tel. +1-781-985-4510
www.nowpublishers.com
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Outside North America:
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PO Box 179
2600 AD Delft
The Netherlands
Tel. +31-6-51115274

The preferred citation for this publication is

K. Benidis and Y. Feng and D. P. Palomar. *Optimization Methods for Financial Index Tracking: From Theory to Practice*. Foundations and Trends® in Optimization, vol. 3, no. 3, pp. 171–279, 2018.

ISBN: 978-1-68083-465-9

Full text available at: http://dx.doi.org/10.1561/2400000021

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Foundations and Trends® in Optimization, 2018, Volume 3, 4 issues. ISSN paper version 2167-3888. ISSN online version 2167-3918. Also available as a combined paper and online subscription.

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Optimization Methods for Financial Index Tracking: From Theory to Practice

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ABSTRACT

Index tracking is a very popular passive investment strategy. Since an index cannot be traded directly, index tracking refers to the process of creating a portfolio that approximates its performance. A straightforward way to do that is to purchase all the assets that compose an index in appropriate quantities. However, to simplify the execution, avoid small and illiquid positions, and large transaction costs, it is desired that the tracking portfolio consists of a small number of assets, i.e., we wish to create a sparse portfolio.

Although index tracking is driven from the financial industry, it is in fact a pure signal processing problem: a regression of the financial historical data subject to some portfolio constraints with some caveats and particularities. Furthermore,

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Konstantinos Benidis, Yiyong Feng and Daniel P. Palomar (2018), "Optimization Methods for Financial Index Tracking: From Theory to Practice", Foundations and Trends® in Optimization: Vol. 3, No. 3, pp 171–279. DOI: 10.1561/2400000021.

the sparse index tracking problem is similar to many sparsity formulations in the signal processing area in the sense that it is a regression problem with some sparsity requirements. In its original form, sparse index tracking can be formulated as a combinatorial optimization problem. A commonly used approach is to use mixed-integer programming (MIP) to solve small sized problems. Nevertheless, MIP solvers are not applicable for high-dimensional problems since the running time can be prohibiting for practical use.

The goal of this monograph is to provide an in-depth overview of the index tracking problem and analyze all the caveats and practical issues an investor might have, such as the frequent rebalancing of weights, the changes in the index composition, the transaction costs, etc. Furthermore, a unified framework for a large variety of sparse index tracking formulations is provided. The derived algorithms are very attractive for practical use since they provide efficient tracking portfolios orders of magnitude faster than MIP solvers.

1

Introduction

1.1 What Is a Financial Index?

An index is a number that represents the aggregate value of a group of items. In particular, a financial index is composed of a collection of assets, such as stocks or bonds, which captures the value of a specific market or a segment of it. A stock or a bond market index is effectively equivalent to a hypothetical portfolio of assets in the sense that we cannot invest directly on it, i.e., an index is not a financial instrument that we can trade. In Section 1.3 we will analyze the various ways we can gain practical access to an index.

The value of a financial index depends on all the underlying assets that compose it. However, the significance of each asset, or in other words its relative weight in the index, is different. There are two basic types of financial indices:

1. Capitalization-weighted (cap-weighted): the assets are weighted based on the ratio of their capitalization[1] to the overall capitalization of the assets that compose the index. The index value is

[1]Capitalization refers to the number of outstanding shares multiplied by share price.

Table 1.1: Example of composition of index.

Stock	Shares	Price	Cap.	Price-weighted	Cap-weighted
1	100	\$20	\$2,000	0.25	0.4
2	50	\$60	\$3,000	0.75	0.6
Total:		\$80	\$5,000	1.0	1.0

proportional to the weighted average of the capitalization of the underlying assets.

2. Price-weighted: the assets are weighted based on the ratio of their price to the sum of all of the prices of the assets that compose the index. The index value is proportional to the weighted average of the prices of the underlying assets.

To clarify the above, consider a simple example where an index is composed by only two stocks, as shown in Table 1.1. Observe that the weights of the two stocks can be very different depending on the type of index. The value of a price-weighted index would be proportional to $20 \times 0.25 + 60 \times 0.75 = 50$ and of a cap-weighted index to $2{,}000 \times 0.4 + 3{,}000 \times 0.6 = 2{,}600$.

In practice, when an index is introduced it is common to set its value to a round number such as 100 or 1,000. Therefore, the actual value of the index has to be divided by a number which is known as the index divisor. Going back to our example, if this was the first day of the index a possible divisor for the price-weighted and cap-weighted version could be \$0.5 and \$2.6, giving an initial value of 100 and 1,000 points, respectively.

Although the cap-weighted and the price-weighted indices are the most common types, the are several other variations of weighted indices. For example, an index can be equal-weighted (or unweighted), where all the assets have exactly the same importance, or volume-weighted, where the weight is based on the traded volume of the assets during some period. Another example is the Tokyo Stock Price Index (TOPIX), which transitioned from a weighting system based on the outstanding shares of each company to a weighting system based on the shares available for trading (free float).

Table 1.2: List of well known indices and their type.

Index	Type
Standard & Poor's 500 (S&P 500)	cap-weighted
Dow Jones Industrial Average (DJIA)	price-weighted
NASDAQ Composite	cap-weighted
Hang Seng Index (HSI)	cap-weighted
Financial Times Stock Exchange 100 (FTSE 100)	cap-weighted
Russell 2000	cap-weighted

In order for an index to be consistent over time it should be adjusted to capture corporate actions that affect market capitalization, such as additional share issuance, dividends and restructuring events such as mergers or spin-offs. Additionally, to remain indicative of the market that the index represents, the underlying assets that compose the index change frequently. To prevent all these corporate actions and changes in the index composition from affecting its value, the divisor of an index is adjusted appropriately so its value remains constant.

A list of well known indices is presented in Table 1.2. Since most of the major indices are cap-weighted (with the most important exception being the Dow Jones Industrial Average (DJIA) index), in all the numerical experiments we will mainly focus on the cap-weighted type, however, the algorithms in principle work for any type of index.

1.2 Why Track an Index?

Fund managers follow two basic investment strategies: active and passive. In active management strategies, the fund managers assume that the markets are not perfectly efficient and through their expertise and superior prediction methods they hope to add value by choosing high performing assets. On the contrary, the passive management strategies are based on the assumption that the market cannot be beaten in the long run. The passive managers have less flexibility and their role is to conform to a closely defined set of criteria.

Analysis of historical data has shown that the majority of the actively managed funds do not outperform the market in the long run [9, 62].

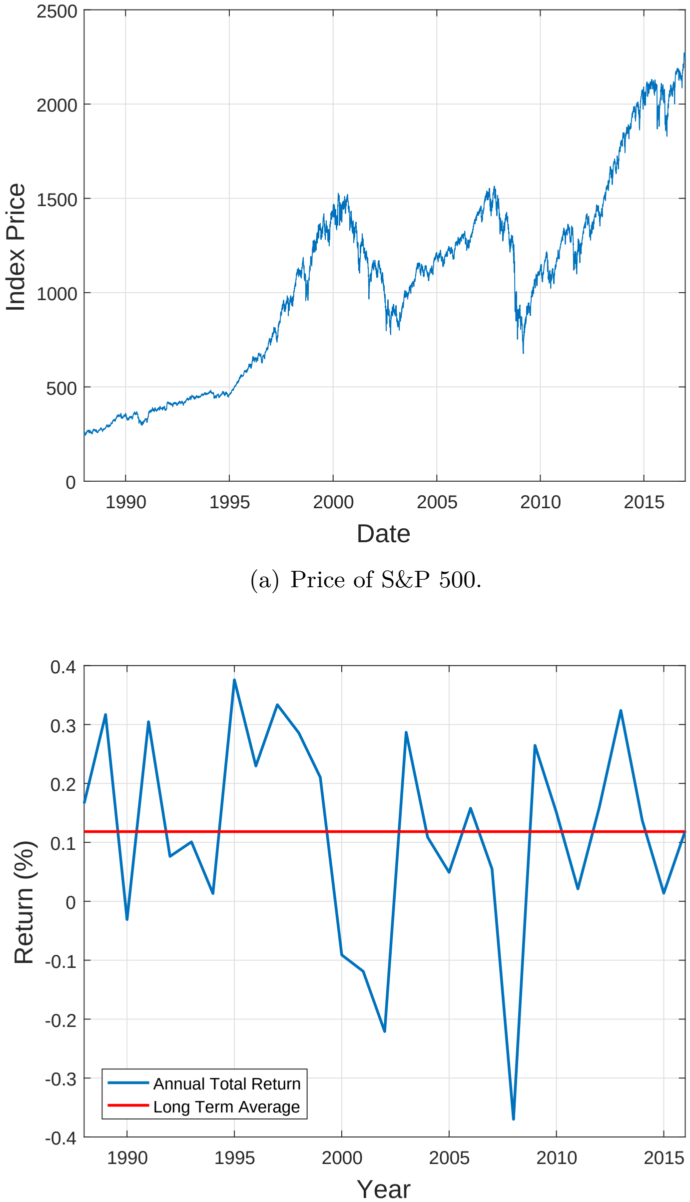
Furthermore, the stock markets have historically risen and therefore reasonable returns can be obtained without the active management's risk. These reasons have prompted the investor's interest into more passive management strategies. Index tracking, also known as index replication, is one of the most popular passive portfolio management strategies. It refers to the problem of reproducing the performance of a market index.

Apart from the direct gains that an investor could have by tracking an index, the index based exchange traded funds (ETFs), which are effectively index tracking portfolios (see Section 1.3.2), have been used widely for hedging purposes [2, 40, 3]. That is, an investor tries to minimize the risk of an investment with an index that is correlated to that investment by taking an appropriate long or short position on the index.

The Standard & Poor's 500 (S&P 500) is one of the world's best known (cap-weighted) indices and one of the most commonly used benchmarks for the stock market. To this end, we will use S&P 500 for illustration purposes throughout this monograph. Figure 1.1 illustrates the performance of S&P 500 over the last three decades. In Figure 1.1(b) we observe that the average annual return of the index is approximately 11%. This is a great average return considering the fact that it includes the 2000-2002 dot-com bubble and the severe crisis of 2008 where the market lost 37% of its value. Of course, an investor that was tracking this index would not have on average an 11% profit per year. This is due to various reasons such as trading costs, inflation, etc. Nevertheless, a reasonable return could be achieved by following this passive strategy.

1.3 Index Tracking

As we have already mentioned, it is not possible to trade an index directly. In order to gain access to an index we need to use other financial instruments such as options, futures, and exchange traded funds (ETFs), or create a portfolio of assets that tracks closely a given index.



(a) Price of S&P 500.

(b) Annual returns of S&P 500.

Figure 1.1: Performance of the index S&P 500 for the period 1988 - 2016.

1.3.1 Options and Futures

An option is a financial derivative since its value is linked to the price of something else. The holder of an option contract has the right, but not the obligation, to buy or sell an underlying asset at a set price on (e.g., European option) or before (e.g., American option) the expiration date of the option. Of particular interest are the index options, which give the right to buy or sell the value of an underlying index. However, note that index options are always cash settled, i.e., no actual stocks are bought or sold. Index options can be used to gain profit from general index movements or for hedging risks in a portfolio. There have been many works on the pricing of index option contracts and on their volatility estimation, for example see [32, 27, 26, 24].

An index future contract is a financial derivative that gives the holder the obligation to purchase an index at a particular price on a specified date in the future. If on that specified date the price of the index has surpassed the price that is agreed in the contract, then the holder makes a profit, and the seller suffers a loss. Futures differ from options in that a futures contract is considered an obligation, while an option is considered a right that may or may not be exercised. Index futures are a very popular way of investing in an index and many works have focus on analyzing their pricing and their relationship to the underlying index [28, 69, 46, 7, 76].

Both index options and index futures are derivative products that do not track the value of an index explicitly but rather their value is associated to the index value.

1.3.2 ETFs

Another popular way to engage in index tracking is to purchase an exchange traded fund (ETF). An ETF is like a stock but its value tracks closely a given index, e.g., see SPDR[2]. It is constructed either by using derivative products, leading to synthetic ETFs, or the underlying components of the index, leading to physical ETFs. Many physical ETFs

[2] SPDR funds are a family of ETFs. The name is an acronym for the first member of the family, the Standard & Poor's Depositary Receipts, which was later renamed to SPDR S&P 500 (ticker SPY).

use all the underlying assets of the index they are tracking, e.g., the Standard and Poor's Depositary Receipts (ticker SPY) based on the S&P 500 and the Nasdaq 100 Trust Shares (ticker QQQ) based on the Nasdaq 100. However, there are also many ETFs using a sparse construction, where representative sampling, with 80-95% of the underlying securities being used, or aggressive sampling, with only a tiny percentage being used [37, 64].

An ETF, unlike options and futures, tracks the value of an index explicitly.

1.3.3 Tracking Portfolios

Finally, we can again track the value of an index explicitly by constructing a portfolio of assets or derivatives whose value follows the value of the given index. The construction of such a tracking portfolio is important for several reasons. First, it is the building block of an ETF, i.e., in order to issue an ETF we need first to construct the corresponding portfolio that this ETF will represent. In addition, not all indices or market sectors have an ETF associated with them. Therefore, a tracking portfolio can be used to explicitly track an index where an ETF does not exist. Finally, having the tools to create such portfolios gives us the flexibility to include any partial information that we may possess or even create portfolios that try to beat the value of an index, instead of using some predetermined financial instruments that we have no freedom on adjusting.

Now, let us introduce some notation that we will use extensively throughout the monograph. Assume that an index is composed of N assets. We denote by $\mathbf{r}^b = [r_1^b, \dots, r_T^b]^\top \in \mathbb{R}^T$ and $\mathbf{X} = [\mathbf{r}_1, \dots, \mathbf{r}_T]^\top \in \mathbb{R}^{T \times N}$ the (arithmetic) net returns of the index and the N assets in the past T days, respectively, with $\mathbf{r}_t \in \mathbb{R}^N$ denoting the net returns of the N assets at the t-th day. Further, $\mathbf{b}_t \in \mathbb{R}^N_{++}$ denotes the normalized benchmark index weights at the t-th day, such that $\mathbf{b}_t^\top \mathbf{1} = 1$ and $\mathbf{r}_t^\top \mathbf{b}_t = r_t^b$. The prices of the assets at the t-th day are denoted by $\mathbf{p}_t = [p_{t1}, \dots, p_{tN}]^\top$ and the number of shares of each asset as $\mathbf{n}_t = [n_{t1}, \dots, n_{tN}]^\top$. The designed portfolio is denoted by $\mathbf{w}_t \in \mathbb{R}^N_+$, with $\mathbf{w}_t^\top \mathbf{1} = 1$.

Full Replication

The most straightforward manner to create a tracking portfolio $\mathbf{w}_t \in \mathbb{R}^N$ is by buying appropriate quantities of all the assets that compose the index, i.e., by choosing $\mathbf{w}_t = \mathbf{b}_t$. This technique is known as full replication and it requires that the true index construction weights $\mathbf{b}_t$ are available. Following this approach, a perfect tracking can be achieved.

The full replication technique has several drawbacks. First, the execution of such a portfolio may be involved since it may consist of thousands of stocks. Second, a portfolio consisting of all the assets may incorporate too many small and illiquid stocks. This translates into higher risk to investors since an illiquid stock is hard to sell if we are looking to exit and moreover it increases the costs due to slippage. Furthermore, allocating capital to all the assets increases significantly the commission fees since every asset is associated with a separate transaction. These drawbacks become more severe as we increase the rebalancing frequency of our tracking portfolio. Finally, the benchmark portfolio weight vector $\mathbf{b}_t$ and all its changes (the benchmark weight vector is consistently rebalanced by the indices providers) can be very expensive to obtain. For example, in 2006 the index sponsors S&P, Dow Jones, MSCI, and FTSE earned total revenues of $1.66 billion from the ETF providers and therefore the ETF providers were even thinking of cutting these costs by setting up their own market indices[3].

Sparse Index Tracking

A natural way to deal with the problems caused by the full replication is to use a small number of assets to (approximately) replicate an index. This leads to the construction of a sparse[4] index tracking portfolio [45, 10]. A sparse portfolio simplifies the execution of the portfolio and tends to avoid illiquid stocks that usually correspond to the assets with small weights in an index, since in a sparse setting most of these assets are

[3] See "ETF providers float idea of setting up their own market indices" published in Financial Times on 2017-05-24.

[4] If we use only a small number of assets, only a small number of weights will be nonzero, i.e., the portfolio will be sparse.

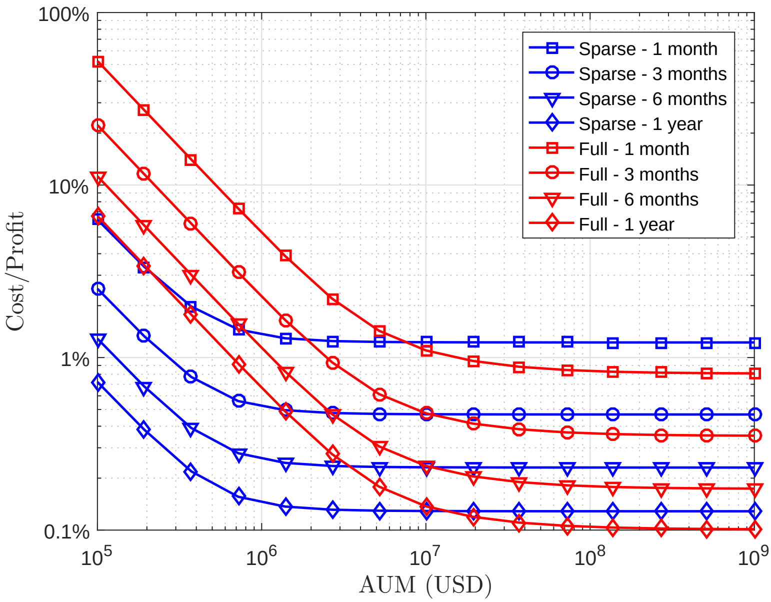


Figure 1.2: Commission fees as a percentage of profit for sparse (40 assets) and full portfolios for different AUM and rebalancing frequencies.

discarded. Furthermore, since only a small number of assets is used, the transaction costs are reduced significantly due to the reduction of the fixed (minimum) costs in the commission fees.

Now, in order to verify the advantages of sparse portfolios, let us get a more quantitative idea on the commission fees reduction that we can achieve. Consider two tracking portfolios of the index S&P 500 for a 5-year period (2011-2015): the first is sparse[5], composed of only 40 assets, while the second uses all the (approximately) 500 assets of the index. Figure 1.2 illustrates the commission fees of the two portfolios as a percentage of the profit, for a range of rebalancing frequencies and assets under management (AUM)[6]. For simplicity we do not have any leverage (i.e., leverage = 1).

[5]This portfolio was constructed using the algorithms presented in Chapter 4.

[6]AUM, also known as net asset value (NAV), measures the total market value of all the financial assets that a financial institution manages.

The first thing to observe is that there is a threshold AUM (around \$10 million) where the commission fees become equal for the two portfolios. For smaller AUM the sparse portfolio has significantly less fees while for larger AUM the full portfolio has slightly smaller fees. Although it is not straightforward why we get such a behavior, it has a simple explanation. Note that the commission fees of each transaction (given by (2.4)) depend on the number of shares Δn we buy or sell, with a fixed minimum fee. For small AUM, the number of shares Δn is not that large and therefore the fixed fee is dominating. For a sparse portfolio we need to pay this fixed fee only for a few assets whereas for a full portfolio this cost becomes significant. As the AUM grows, the fixed cost becomes less important since it is dominated by the large amount of shares we trade. Therefore, the difference of the commission fees between the two portfolios becomes more narrow. Finally, for large AUM the full portfolio has less commission fees. This is because the rebalancing of a sparse portfolio can result in a different composition of assets. This means we need to sell all the holding shares of one asset and buy many shares of another whereas for full portfolios we do not need to make such severe changes. Of course, the change in the asset composition in sparse portfolios is true for lower AUM as well. However, this effect is mitigated in lower AUM as the fixed costs are dominant.

In general, the change in the composition of the holding portfolio during rebalancing can be controlled by including a turnover penalty as we will see in Section 3.5. To illustrate the benefit of the turnover penalty, we consider the same setting as in Figure 1.2 and we focus only on the 3-month rebalancing frequency. Apart from the sparse and full portfolios, we design one more sparse portfolio that has a turnover penalty, i.e., we penalize the changes in the portfolio after rebalancing. In Figure 1.3 we observe how this penalization reduces further the transaction costs and makes the sparse portfolio with turnover penalty more cost effective that the full replication portfolio even for large AUM.

As we will see in Section 4.6, the turnover penalization can be controlled by a tuning parameter. However, we should keep in mind that there is a tradeoff between the reduction of costs and the tracking error, i.e., by enforcing only small changes in a portfolio during rebalancing can lead to a larger tracking error.

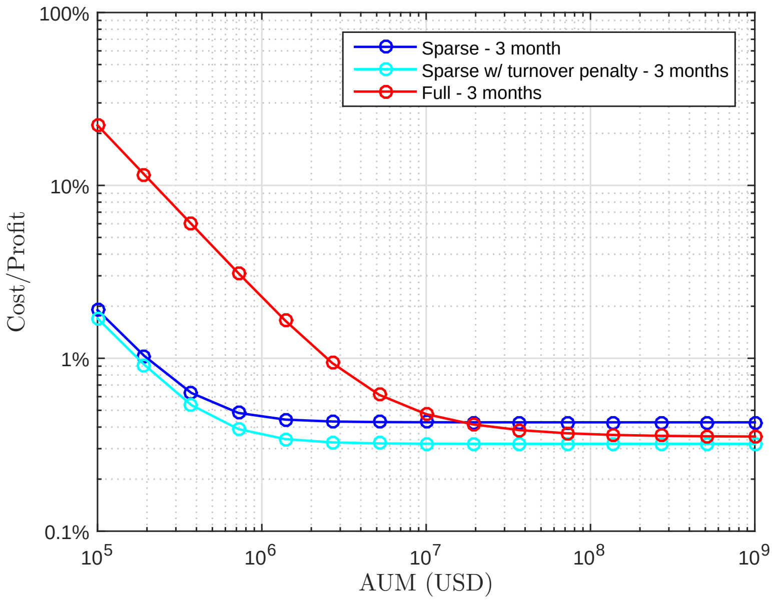


Figure 1.3: Commission fees as a percentage of profit for sparse (40 assets), sparse with turnover penalty, and full portfolios for different AUM and a rebalancing frequency of 3 months.

1.4 Goal

Due to the importance of sparse tracking portfolios, our main focus will be on deriving such portfolios with the goal of tracking an index as efficiently as possible. It is worth mentioning that in general index tracking portfolios are not efficient in the sense that they do not lie on the efficient frontier as defined by Markowitz [53]. This is expected since the goal of a sparse tracking portfolio algorithm is to find an optimal tradeoff between tracking error and sparsity and not between return and risk.

Further, the analysis and the algorithms derived in the monograph assume decisions for a single-period, i.e., the portfolio derivations do not incorporate information about future trades. For multi-period trading please refer to [16] and references therein.

Finally, although index tracking is not a real-time application, the construction time should be reasonable given the fact that extensive backtesting should be made before deploying an index tracking strategy, which requires the construction of many portfolios for a given algorithm.

1.5 Outline

The abbreviations and the notation used throughout the monograph are provided on pages 89 and 90, respectively.

In Chapter 2 we present two basic challenges we face when we engage in index tracking, namely the need for a frequent rebalancing of a tracking portfolio due to the constant changes in an index, and the transaction costs that are associated with a portfolio. As we will see, these two challenges form a natural tradeoff.

In Chapter 3 we introduce the sparse index tracking problem in its general form and we discuss the various tracking error functions and possible constraints that one could impose. We further analyze existing methods that produce sparse tracking portfolios and their drawbacks.

In Chapter 4 we derive algorithms for the sparse index tracking problem. We consider various tracking error functions and constraints. All of the possible problem variations boil down to the same effective problem that we need to solve iteratively until the algorithms converge.

In Chapter 5 we provide numerical experiments that show the performance of the derived algorithms. For illustration purposes we use the indices S&P 500 and Russell 2000.

Finally, Chapter 6 concludes the monograph.

1.6 Software

Many of the derived algorithms can be found in the R [59] software package ***sparseIndexTracking*** [12], which is available in CRAN.

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