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Foundations and Trends® in Robotics

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Outside North America:
now Publishers Inc.
PO Box 179
2600 AD Delft
The Netherlands
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The preferred citation for this publication is P. Abichandani, H. Benson and M. Kam, Mathematical Programming Approaches for Multi-Vehicle Motion Planning: Linear, Nonlinear, and Mixed Integer Programming, Foundations and Trends® in Robotics, vol 2, no 4, pp 261–338, 2011.

ISBN: 978-1-60198-722-8

Foundations and Trends® in Robotics

Volume 2 Issue 4, 2011

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Foundations and Trends® in Robotics, 2011, Volume 2, 4 issues. ISSN paper version 1935-8253. ISSN online version 1935-8261. Also available as a combined paper and online subscription.

Foundations and Trends® in
Robotics
Vol. 2, No. 4 (2011) 261–338

DOI: 10.1561/2300000025

Mathematical Programming Approaches for Multi-Vehicle Motion Planning: Linear, Nonlinear, and Mixed Integer Programming

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Abstract

Real world Multi-Vehicle Motion Planning (MVMP) problems require the optimization of suitable performance measures under an array of complex and challenging constraints involving kinematics, dynamics, collision avoidance, and communication connectivity. The general MVMP problem is thus formulated as a Mathematical Programming (Optimization) problem. In this monograph, we present a Mathematical Programming (MP) framework that captures the salient features of the general MVMP problem. To demonstrate the use of MP for the formulation and solution of MVMP problems, we examine in detail four representative works and summarize several other related ones.

Following this conceptual discussion, we provide a step-by-step demonstration of how to formulate, solve, and experimentally validate an MP problem that represents an MVMP. Finally, we discuss the advantages, technical challenges, and limitations of this framework. As solution algorithms and their implementations in solvers continue to develop, we anticipate that MP solution techniques will be applied to an increasing number of MVMP problems, and that the framework, formulations, and experimental approach presented here may serve as a guide for future MVMP research.

Contents

1

The Future is Bright . . . and Driverless

In 2011–2012, Google demonstrated autonomous ground-based mobility in an urban environment. By August 2012, the Google Driverless Car, similar to the one shown in Figure 1.1, had logged more than 300,000 miles in the state of California [134]. Sebastian Thrun, the lead developer of the car described the broader impacts of this technology as follows, "we could change the capacity of highways by a factor of two or three if we didn't rely on human precision on staying in the lane — improve body position and therefore drive a little bit closer together on a little bit narrower lanes, and do away with all traffic jams on highways" [132]. Recently, Newman and others at Oxford University have demonstrated driverless cars with similar capabilities [35]. Several states in the United States have already passed legislation that allow driver's licenses being issued to driverless cars [106].

During the same period, Kumar et al. demonstrated multiple quadrotors operating in an indoor workspace and maintaining formations that translate and rotate with time. This is an example where multiple autonomous vehicles coordinate to achieve a collective task [96].

Fig. 1.1 Google's driverless car had logged more than 300,000 miles in the state of California by August 2012.

These and many other developments are harbingers of a future where multiple autonomous vehicles will become an all pervasive concept with applications that improve our standard of living and lifestyles. Such vehicles will lead to a reduction in the number of accidents and commute times, and improved fuel efficiency. These, in turn, will drive down the cost of public transportation, logistics, and supply chain management, thereby allowing transportation of personnel and goods to previously unreachable locations in record times.

While exciting, such autonomous vehicle-based applications come with their own challenges. For example, vehicles may need to plan their motions in real-time while in transit. This necessitates that the autonomous decision making be dynamic and efficient. There are kinematic challenges as well, e.g., cars cannot slide sideways to parallel park and a fixed-wing aircraft has a nonzero turn radius. Design limitations such as finite battery and fuel capacity add to the complexity of the situation.

Furthermore, if multiple vehicles are operating in a common workspace (e.g., urban environments, highways, airways), some shared knowledge of their behavior is important. To obtain this knowledge, either one needs to have sensors or the vehicles need to communicate with each other or with central entities that update other vehicles with the latest information about their intentions as and when needed.

The biggest challenges, however, will be the commercial viability of such vehicles and adoption by the general public. Specifically, we need to address issues of energy efficiency and safety.

- Efficiency requirements dictate that these vehicles use as little fuel as possible, to travel as far as possible, as quickly as possible.
- Safety requirements dictate that these vehicles be capable of avoiding obstacles and not collide with each other.

Despite all these challenges, the current propensity of innovation in this space points to a future that is indeed bright...and driverless!

1.1 What's in it for the Readers?

In the following discussions, the readers will be introduced to a rigorous and systematic treatment of these challenges and a family of technical approaches that will assist in addressing them. Specifically, we will ground our discussions in the idea of mathematical programming as applied to Multiple-Vehicle Motion Planning (MVMP) and present the material in the following order:

- In Section 2, we start by formally defining the MVMP problem and its most common variants, such as path planning, trajectory planning, and path coordination problems. We then present the basic elements for modeling MVMP systems in detail. These elements include vehicle kinematics, dynamics, path primitives, and communication models. We also provide an introduction to the mathematical programming framework that will be used to model MVMP problems, relevant solution algorithms, modeling environments, and solvers.
- A general MP based framework that captures the salient features of MVMP problems is introduced in Section 3. We start

this discussion with a review of existing literature of MVMP using MP by focusing on four representative papers that best demonstrate the application of this framework to a range of MVMP scenarios. Each of the four papers provides various model components, which are presented in great detail. Key analyses performed in these papers have been independently derived and presented in the Appendices.

- In Section 4, readers are provided a step-by-step demonstration on how to formulate and solve a distributed MVMP involving autonomous unmanned ground vehicles operating in an indoor environment under communication connectivity constraints. Experimental approaches utilized in the literature to validate MP based MVMP formulations are documented.
- Finally, in Section 5, we discuss the technical challenges and limitations of this framework and present future directions of this research.

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