A Review of Point Cloud Registration Algorithms for Mobile Robotics

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Contents

Abstract

The topic of this review is geometric registration in robotics. Registration algorithms associate sets of data into a common coordinate system. They have been used extensively in object reconstruction, inspection, medical application, and localization of mobile robotics. We focus on mobile robotics applications in which point clouds are to be registered. While the underlying principle of those algorithms is simple, many variations have been proposed for many different applications. In this review, we give a historical perspective of the registration problem and show that the plethora of solutions can be organized and differentiated according to a few elements. Accordingly, we present a formalization of geometric registration and cast algorithms proposed in the literature into this framework. Finally, we review a few applications of this framework in mobile robotics that cover different kinds of platforms, environments, and tasks. These examples allow us to study the specific requirements of each use case and the necessary configuration choices leading to the registration implementation. Ultimately, the objective of this review is to provide guidelines for the choice of geometric registration configuration.

1

Twenty years of Iterative Closest Point (ICP): The Legacy

The scope of this work is to present *registration* algorithms and their use in mobile robotics. Registration algorithms associate sets of data into a common coordinate system by minimizing the alignment error. This allows to integrate data from different sources into a bigger model.

Although they can be quite an abstract and technical concept, registration solutions already had an impact on the artistic field and popular culture. Photographers proficiently use image registration to build photograph composites achieving different looks-and-feels. The Brenizer method is an exemplary technique that is applied to achieve dramatic depth of field using panoramic image stitching (Figure 1.1 - Top). Another example is High Dynamic Range (HDR) photographs, where multiple images at different exposure levels need to be precisely overlaid to retrieve details in shaded and highlighted areas (Figure 1.1 - Bottom). Nowadays, even the latest cellphones have the capacity to build panoramic images from a series of pictures taken based on a visual guidelines that direct the user to move the camera viewfinder at the optimal position for the next picture. As for the specific case of 3D mapping application, cinematographers are depicting possible uses of registration algorithms in several recent science fiction movies. For

instance, in the remake of *Total Recall* (Colombia Pictures, 2012), an armed intervention team employed an array of hundreds of tiny cameras in a dangerous room leading to a 3D reconstruction of the area used to monitor potential threats within couple of seconds. Another closely related potential application was the used by a geologist of flying drones carrying laser rangefinders to explore an alien facility in *Prometheus* (Twentieth Century Fox, 2012).
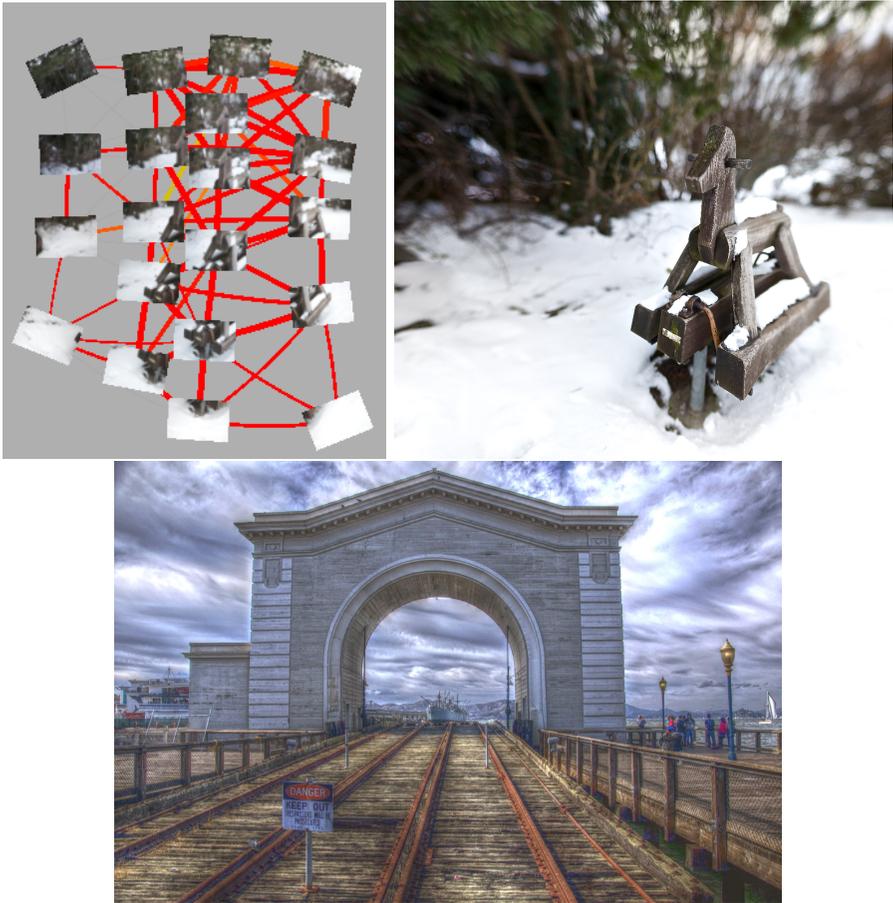
Figure 1.1: Example of image registrations used in photography. *Top*: Brenizer method using the open source software Hugin to stitch multiple images. *Bottom*: HDR composite of the San Francisco harbor using the open source software Luminance HDR to overlay three images.

More at the research level, current applications include: robotic exploration in harsh environments, organ reconstruction to improve medical diagnostics and object reconstruction for reverse engineering. Although registration using 2D images can be part of the same group of solutions, we focus on systems where depth information is already available (e.g., from laser rangefinders) and is mainly used for resolving misalignment error. We refer to the latter type as *geometric registration*[1]. However, some parallels with image registrations will be made throughout this work when relevant.

A simplified example of geometric registration is illustrated in Figure 1.2. A scene with a large tree, a lamppost and a bench was scanned using a laser rangefinder from two different poses. As laser points are indistinguishable, only their location information is available to resolve the alignment error. In that example, the point cloud in light green and with the horizontal ground is used as our fixed reference coordinates. Figure 1.2-*Left* shows the starting position of the two scans. The overlaid point cloud in dark blue has a misalignment error shifting it to the left with a tilt angle. This initial misalignment is represented with dark red lines in Figure 1.2-*Middle*. Although all individual points are similar, their proximity to other points gives enough information to automatically align the two point clouds (Figure 1.2-*Right*).
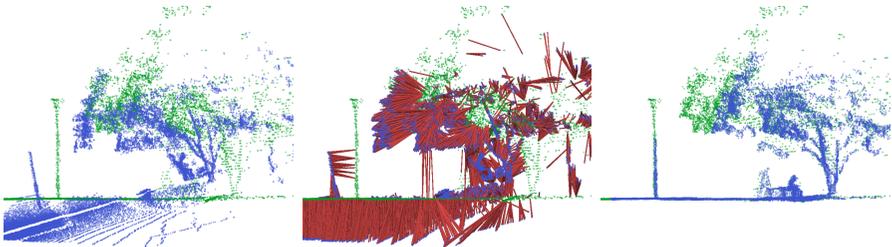
Figure 1.2: Examples of geometric registration between a reference point cloud (light green points) and a reading point cloud (dark blue points). *Left*: Initial position of the two point clouds. *Middle*: Alignment error (dark red lines). *Right*: Final alignment of the two point clouds.

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[1]In general, image registration often has access to *labelled* points, which is less the case for geometric registration, either in 2D or 3D.

1.1 Early Solutions

As an interesting historical note, in an early publication by Hurley and Cattell [1962], registration is presented as an *Orthogonal Procrustes* problem. The name Procrustes refering to a bandit from the Greek mythology who made his victims fit on his bed by either stretching their limbs or cutting them off. Theseus eventually defeated Procrustes using the same violent procedure (Figure 1.3). Nowadays, the reference to the *Orthogonal Procrustes* problem is not often used in the scientific literature, but it illustrates well the idea.

Figure 1.3: Theseus *adjusting* Procrustes to the size of his bed. Photograph provided by Marie-Lan Nguyen / Wikimedia Commons.

While working more specifically on 3D-shape primitives, Faugeras and Hebert [1986] defined closed-form distances to minimize point-to-point and plane-to-plane alignment error. The proposed method solved translation and rotation as a two-step procedure. Later, a solution proposed by Walker et al. [1991] resolved together rotation and translation error using dual quaternions. The registration problem concretizes itself further in a survey of geometric matching algorithms and geometric representations for point sets, curves, surfaces, volumes, and their respective space-time trajectories [Besl, 1988]. At this time, the main ex-

perimental validation was using Computer-aided design (CAD) models with simple shapes. The first mention of the name ICP[2] was proposed by Besl and McKay [1992]. They expressed the problem as follows:

> "Given 3-D data in a sensor coordinate system, which describes a data shape that may correspond to a model shape, and given a model shape in a model coordinate system in a different geometric shape representation, estimate the optimal rotation and translation that aligns, or registers, the model shape and the data shape minimizing the distance between the shapes and thereby allowing determination of the equivalence of the shapes via a mean-square distance metric."

In their work, the proof of the solution convergence is demonstrated under the assumption that the number of associated points, or their weight, remains constant. Unless two identical shapes are registered together, outliers that are not present in both shapes need to be identified. This problems is observed by Champleboux et al. [1992] while developing early registration solutions for medical applications. They report failures when wrong initial transformations are used in combination with scans having low overlap ratio. During the same years, Chen and Medioni [1991] work with dense laser scans of statues and, shortly later, scans of tooth mockups [Chen and Medioni, 1992]. They propose a registration solution based on the minimization of point-to-plane alignment errors, which is still quite often used nowadays.

Even though a large volume of theoretical works was published on advanced geometric primitives (e.g., planes, curves, quadrics), Zhang [1994] states that primitives derived from points are too sensitive to noise and are not stable in moving systems with current (1994) sensing capabilities. Thus, he concludes that points were more reliable. Zhang [1994] pioneers the idea of using ICP-based solutions for outdoor robotic applications. He proposes a generic framework for symmetric match, but considers only one direction of registration as an

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[2]In the remainder of this review, ICP and geometric registration have the same generic meaning.

approximation to save computation costs. He highly emphasizes the importance of removing spurious pairs and gives the first characterization of fast subsampling solutions. In addition, he highlighted the fact that outlier rejection is required for robotic applications, and that the proof of ICP convergence stated by Besl and McKay [1992] cannot hold for most of the robotics applications. In the outlook section of his work, he already mentions the use of uncertainty on the initial alignment, based on Kalman filters and Mahalanobis distance, and the need to handle dynamic elements.

1.2 Division and Explosion of the Field

Within only two years, four main application types already emerged from the possibilities to register 3D point clouds: object reconstructions [Chen and Medioni, 1991], non-contact inspections [Besl and McKay, 1992], medical and surgery support [Champleboux et al., 1992] and autonomous vehicle navigation [Zhang, 1993]. Publications in specialized journals for computer vision, robotics and medical imaging slowly divided the types of *interesting* problems to be solved. We can still read in current literature that the credits for being the first article to provide a solution differ from authors in different fields.

The field of registration crystalized with its first survey on medical image registration covering the years 1993 to 1998 [Maintz and Viergever, 1998]. It took 12 years for a specialized survey of 3D registration in computer vision to appear [Bowyer et al., 2006]. This work intends to be the first large scale review adapted for Robotics application.

ICP is a popular algorithm due to its simplicity: its general idea is easy to understand and to implement. However, the basic algorithm only works well in ideal cases. This led to hundreds[3] of variations around the original algorithm that were published on numerous different experimental scenarios (see Figure 1.4). This highlights both the usefulness of ICP and the difficulty to find a single versatile version.

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[3] Close to 450 papers based on IEEE Xplore and around 1350 based on Scopus, between 1992 and 2013.
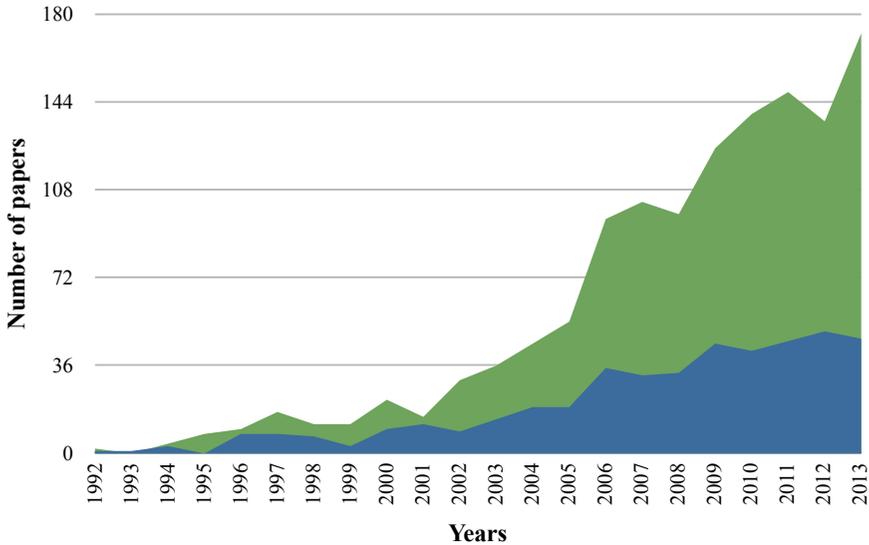
Figure 1.4: Evolution of the number of publications over the years. Results were obtained for the keywords *Iterative Closest Point* appearing in the abstract or the title of publications. The dark blue area is computed based on IEEE Xplore database and the light green area from the Scopus database.

In Figure 1.4, one can observe an increasing number of publications appearing around the year 2000. In robotics, this coincides with the advent of a 2D solution for pose estimations demonstrated with a SICK rangefinder [Lu and Milios, 1997] and of the basis of Simultaneous Localization and Mapping (SLAM) algorithms [Thrun et al., 1998]. Prior to the arrival of the SICK LMS-200 in robotics [Pfister et al., 2002], most of the sensors used were custom-made. This situation renders experiments difficult to replicate by other researchers. In those years, 2D lasers appeared as a viable solution for navigation over sonars, which were traditionally used [Thrun et al., 1998]. The 3D real-time applications were then not accessible due to high computation costs leading to an increased research focus toward 2D solutions for autonomous navigation, while other fields continued in 3D. At the same time in computer vision, the seminal work of Rusinkiewicz and Levoy [2001] on ICP algorithm comparisons led to significant progress in the scan

registration field. The experiments employed simulated 3D scans, highlighting different spatial constraints and sensor noises. Results mainly focused on the rapidity of convergence and the final precision of different solutions helping to select more appropriate solutions in further applications.

With the arrival of more standard sensors, researchers in robotics pushed the 2D registration algorithms so they could deal with larger environments with faster platforms [Bosse et al., 2004] and 3D slowly came back [Nüchter et al., 2004]. Since no comparison framework exists, the selection of an appropriate variant for particular experimental conditions is difficult. This is a major challenge because registration is at the front-end of the mapping pipeline, and the arbitrary nature of its selection affects the results of all subsequent steps of more advanced robotic tasks. Even the early work of Eggert et al. [1998] highlights the difficulty to compare with other solutions given the lack of metric over common data sets. In their survey, Maintz and Viergever [1998] point the fact that proper accuracy studies are just starting; the problem being that the results provided are too specific. In addition, they highlight the imprecise use of the terms accuracy, precision and robustness. They suggest to set up public databases and validation protocols, but foresee logistic, costs and efforts as incoming problem to those solutions.

Recently, the demand for a stronger experimental methodology in robotics was also stressed by Amigoni et al. [2009]. The authors survey different SLAM publications to highlight proper evaluation metrics that are applied to SLAM algorithms. Three principles of an experimental methodology in science (i.e, comparison, reproducibility/repeatability and justification/explanation) are translated in requirements for stronger SLAM results. As stated in their paper, a sound methodology should allow researchers to gain an insight about *intrinsic* (e.g., computational time, parameters used, parameter behaviors) and *extrinsic* (e.g., accuracy, precision) quantities. The authors report that, even though comparisons between algorithms are present in SLAM publications, very few researchers can reuse the same protocol and directly compare their results without having to re-implement other solutions.

With the introduction of the Microsoft Kinect in 2010, another wave of publications is expected, similar to what was observed following the widespread utilization of SICK rangefinders. The Kinect is a handheld camera sensor connected via USB to a computer that produces both depth and color readings. Such RGB-D sensors augment accessibility to object modeling and indoor mapping research [Henry et al., 2012]. This also opens the door to a mix of hybrid algorithms using features and descriptors without the need of expertise in sensor calibration. RGB-D cameras have different characteristics than laser-based sensors, such as a higher density of points at a higher frequency but covering a more restricted Field of View (FoV). A smaller FoV means less time to compute the registration before the sensor trajectory reduce the overlap to an unusable range. Having access to a higher frame rate with an optimized ICP solution shows that hand-waved sensor trajectory was trackable with real-time constraints [Pomerleau et al., 2011]. The Velodyne HDL-64E, first commercialized for the DARPA Urban Challenge in 2007, optimized its FoV to cover the expected trajectory of a ground vehicle. To cope with the high speed of a car, the sensor delivered a high data rate at $1.3\,\mathrm{M}$ points per second, bring the real-time constraint to another level. Those two sensors were the latest publication catalysts for the field of registration in mobile robotics, field often modulated by the development of new hardwares.

1.3   Algorithm Overview

The aim of geometric registration is to be able to represent a shape, called `reading`, in the same coordinate frame as another, called `reference`. This is equivalent to finding the transformation of `reading` that best aligns it to `reference`.

A shape $\mathcal{S}$ is a set of points including both geometric and non-geometric information. Geometric information is affected by a spatial transformation; this part of the dimension of a point will be called a *feature*. Features are typically coordinates of points, surface normals or tangent vectors. Non-geometric information is not affected by spatial

transformation; this part of the dimension of a point will be called a *descriptor*. Descriptors can be color, temperature, identifiers, etc.

Most algorithms actually apply some filters on the shapes in order to help the registration. There are mainly two uses of such filters. The first one is to remove some points that do not bring any valuable information for the registration. As the complexity of the algorithm is linear in the number of points, reducing this number can have a significant impact on the time of registration. The second use of filters can be to add information to the point. The typical example is the inference of local structural properties of the shape, such as normal information or curvature. This information, which is usually not present in the raw sensor data, can allow for better registration through a more precise association of the points, or the computation of the error to minimize.

More formally, let $^{\mathbb{A}}\mathcal{P}$ be the shape representing `reading` in a coordinate frame $\mathbb{A}$ and $^{\mathbb{B}}\mathcal{Q}$ the shape representing `reference` in its coordinate frame $\mathbb{B}$. The aim of registration is to estimate the transformation $^{\mathbb{B}}_{\mathbb{A}}\mathcal{T}$ by minimizing an error function $\text{error}(\mathcal{P}', \mathcal{Q})$:

$$^{\mathbb{B}}_{\mathbb{A}}\hat{\mathcal{T}} = \arg\min_{\mathcal{T}} \left(\text{error}\left(\mathcal{T}\left(^{\mathbb{A}}\mathcal{P}\right), {}^{\mathbb{B}}\mathcal{Q} \right) \right) \qquad (1.1)$$

where $\mathcal{T}(\mathcal{S})$ is the application of the geometric transformation $\mathcal{T}$ to the shape $\mathcal{S}$.

One specificity of geometric registration is that the error function is computed on pairs of points that have been associated between the two shapes. The classical association is done by finding the closest point in `reference` of each point in `reading`. Ideally the association should be between points that, when the two shapes are aligned, are the closest in position. This problem is called *data association, point matching, correspondence finding* depending on the literature. Association solving can be done purely on the features but can also be improved by using the descriptors.

Formally, let $\mathcal{M} = \text{match}(\mathcal{P}, \mathcal{Q}) = \{(\boldsymbol{p}, \boldsymbol{q}) : \boldsymbol{p} \in \mathcal{P}, \boldsymbol{q} \in \mathcal{Q}\}$ be the set of matches between $\mathcal{P}$ and $\mathcal{Q}$. The error function is then of the form:

$$\text{error}(\mathcal{P}, \mathcal{Q}) = \sum_{(\boldsymbol{p}, \boldsymbol{q}) \in \mathcal{M}} \text{d}(\boldsymbol{p}, \boldsymbol{q}).$$

In order to make this error function more robust, *outliers* are some-
times identified and removed from the list of matches. In addition,
weights $\mathcal{W} = \text{outlier}(\mathcal{M}) = \{w(\boldsymbol{p}, \boldsymbol{q}) : \forall (\boldsymbol{p}, \boldsymbol{q}) \in \mathcal{M}\}$ can be associated
to the matches so as to increase or decrease their influence in the error
function:

$$\text{error}(\mathcal{P}, \mathcal{Q}) = \sum_{(\boldsymbol{p}, \boldsymbol{q}) \in \mathcal{M}} w(\boldsymbol{p}, \boldsymbol{q}) \, \text{d}(\boldsymbol{p}, \boldsymbol{q}).$$

It is clear that minimizing this error function with an ideal associ-
ation yields the best estimate for $_{\mathbb{A}}^{\mathbb{B}}\mathcal{T}$ (Equation 1.1). However, unless
the descriptors are discriminative enough (as with visual descriptors),
the association can generally not be perfectly solved. The idea of ICP
is that even with an imperfect association, minimizing the error yields
a better estimates that, in turn, allows for better association. Con-
cretely, the idea is to build a sequence of transformations $_{i-1}^{i}\mathcal{T}$ that
are successively applied to $\mathcal{P}$. At a given iteration, a set of matches
$\mathcal{M}_i$ is computed from the given relative position of the points. Then,
based of those matches, a new transformation $_{i}^{i+1}\mathcal{T}$ is computed by
minimizing the error:

$$_{i}^{i+1}\mathcal{T} \leftarrow \underset{\mathcal{T}}{\arg\min} \left(\text{error} \left(\mathcal{T} \left(^{i}\mathcal{P}' \right), \mathcal{Q}' \right) \right). \tag{1.2}$$

Finally, the estimate of the transformation between the two original
shapes is the composition of all intermediary transformations:

$$_{\mathbb{A}}^{\mathbb{B}}\hat{\mathcal{T}} = \left(\underset{i}{\bigcirc} \, _{i-1}^{i}\mathcal{T} \right) \circ \mathcal{T}_{init} \tag{1.3}$$

where $\underset{i}{\bigcirc} \, _{i-1}^{i}\mathcal{T} = \cdots \circ \, _{2}^{3}\mathcal{T} \circ \, _{1}^{2}\mathcal{T}$ is the iterative composition of the
transformations, and $\mathcal{T}_{init}$ an initial transformation.

The generic procedure is summarized in Algorithm 1 and shown as
a chart in Figure 1.5.

1.4   Overview of the Review

ICP is a framework where multiple variations and algorithms can be
used to resolve geometric registration problems. In the light of this
large corpus of work related to ICP and more generally to geometric

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Algorithm 1 Summary of ICP algorithm.

Require: $^\mathbb{A}\mathcal{P}$ ▷ `reading`

Require: $^\mathbb{B}\mathcal{Q}$ ▷ `reference`

Require: $\mathcal{T}_{init}$ ▷ initial transformation

  $^\mathbb{A}\mathcal{P}' \leftarrow \text{datafilter}(^\mathbb{A}\mathcal{P})$ ▷ data filters

  $^\mathbb{B}\mathcal{Q}' \leftarrow \text{datafilter}(^\mathbb{B}\mathcal{Q})$ ▷ data filters

  $_{i-1}{}^i\mathcal{T} \leftarrow \mathcal{T}_{init}$

  repeat

    $^i\mathcal{P}' \leftarrow {}_{i-1}{}^i\mathcal{T}\left(^{i-1}\mathcal{P}'\right)$ ▷ move `reading`

    $\mathcal{M}_i \leftarrow \text{match}(^i\mathcal{P}', \mathcal{Q}')$ ▷ associate points

    $\mathcal{W}_i \leftarrow \text{outlier}(\mathcal{M}_i)$ ▷ filter outliers

    $^{i+1}_i\mathcal{T} \leftarrow \underset{\mathcal{T}}{\arg\min}\left(\text{error}\left(\mathcal{T}\left(^i\mathcal{P}'\right), \mathcal{Q}'\right)\right)$

  until convergence

Ensure: $^\mathbb{B}_\mathbb{A}\hat{\mathcal{T}} = \left(\bigcirc_i {}_{i-1}{}^i\mathcal{T}\right) \circ \mathcal{T}_{init}$
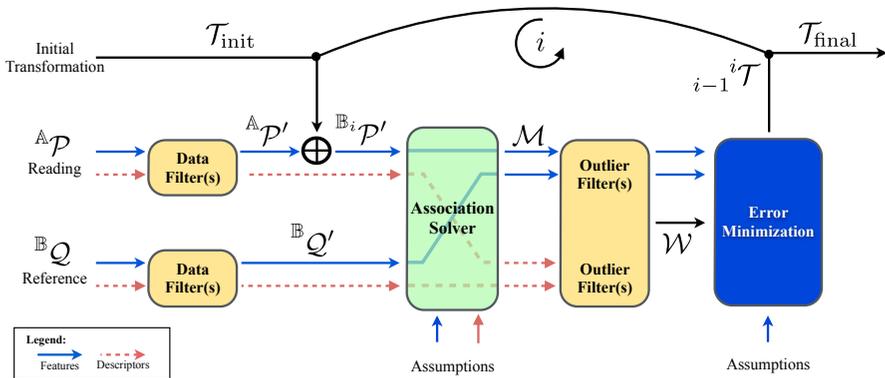
Figure 1.5: Generic scheme proposed for registration algorithms.

registration, we present a general framework to classify the existing solutions. We believe that after 20 years of *new* registration algorithms, it is time to evaluate what works best for which robotic systems. Therefore, our contributions aim at strengthening the current methodology and bring deeper analysis of current solutions. The timing is appropriate for such study given that computational power is now sufficient to support registration on embedded systems in real-time [Pomerleau et al., 2011]. Also, new advancements in electronics have improved the accuracy and speed of sensors. Improvements in battery technology have enabled longer autonomous operation time. Most importantly, researchers face a plethora of solutions from which a definition of usable solutions can be out of reach. This situation impedes the robotic field to progress on algorithms that rely on registration (e.g., path planning, autonomous exploration).

This review addresses this problem and is structured in two main sections:

Section 2 presents a literature review of different solutions with the aim to express ICP solutions in a common framework and validate our generic scheme proposed in Figure 1.5.

Section 3 describes case studies using five different robotic platforms. The requirements of each application are explained with some insight on how to tune parameters for specific applications. Those applications cover Search & Rescue activities, industrial inspection, shore monitoring and autonomous driving.

All sections close with a discussion in addition to a short summary. The main observations of those sections are recapitulated in Section 4 along with final remarks.

References

Francesco Amigoni, Monica Reggiani, and Viola Schiaffonati. An insightful comparison between experiments in mobile robotics and in science. *Autonomous Robots*, 27(4):313–325, August 2009.

Leopoldo Armesto, Javier Minguez, and Luis Montesano. A generalization of the metric-based Iterative Closest Point technique for 3D scan matching. In *Robotics and Automation, 2010. Proceedings of the IEEE International Conference on*, pages 1367–1372, 2010.

K S Arun, T S Huang, and S D Blostein. Least-Squares Fitting of Two 3-D Point Sets. *Pattern Analysis and Machine Intelligence, IEEE Transactions on*, 9(5):698–700, 1987.

Sunil Arya and David Mount. Approximate nearest neighbor queries in fixed dimensions. In *Discrete Algorithms, 1993. Proceedings of the 4th Annual ACM-SIAM Symposium on*, pages 271–280. Department of Computer Science, University of Maryland, College Park, Maryland, 20742 Department of Computer Science and Institute for Advanced Computer Studies, University of Maryland, College Park, Mary- land, 20742, 1993.

R Benjemaa and F Schmitt. Fast global registration of 3D sampled surfaces using a multi-z-buffer technique. In *3-D Digital Imaging and Modeling, 1997. Proceedings of the International Conference on Recent Advances in*, pages 113–120, 1997.

R Bergevin, M Soucy, H Gagnon, and D Laurendeau. Towards a general multi-view registration technique. *Pattern Analysis and Machine Intelligence, IEEE Transactions on*, 18(5):540–547, 1996.

P Besl and H McKay. A method for registration of 3-D shapes. *Pattern Analysis and Machine Intelligence, IEEE Transactions on*, 14(2):239–256, February 1992.

P J Besl. Geometric modeling and computer vision. In *Proceedings of the IEEE*, pages 936–958, 1988.

Michael Bosse and R Zlot. Map Matching and Data Association for Large-Scale Two-dimensional Laser Scan-based SLAM. *The International Journal of Robotics Research*, 27(6):667–691, June 2008.

Michael Bosse and R Zlot. Keypoint design and evaluation for place recognition in 2D lidar maps. *Robotics and Autonomous Systems*, 57(12):1211–1224, 2009a.

Michael Bosse and Robert Zlot. Continuous 3D scan-matching with a spinning 2D laser. In *Robotics and Automation, 2009. Proceedings of the IEEE International Conference on*, pages 4312–4319, 2009b.

Michael Bosse, Paul Newman, John Leonard, and Seth Teller. Simultaneous Localization and Map Building in Large-Scale Cyclic Environments Using the Atlas Framework. *The International Journal of Robotics Research*, 23 (12):1113–1139, December 2004.

Kevin W Bowyer, Kyong Chang, and Patrick Flynn. A survey of approaches and challenges in 3D and multi-modal 3D+2D face recognition. *Computer Vision And Image Understanding*, 101(1):1–15, January 2006.

A Censi. Scan matching in a probabilistic framework. In *Robotics and Automation, 2006. Proceedings of the IEEE International Conference on*, pages 2291–2296, 2006.

A Censi. An ICP variant using a point-to-line metric. In *Robotics and Automation, 2008. Proceedings of the IEEE International Conference on*, pages 19–25, 2008.

G Champleboux, S Lavallee, R Szeliski, and L Brunie. From accurate range imaging sensor calibration to accurate model-based 3D object localization. In *Computer Vision and Pattern Recognition, 1992. Proceedings of the 1992 IEEE Computer Society Conference on*, pages 83–89, 1992.

Y Chen and G Medioni. Object modeling by registration of multiple range images. In *Robotics and Automation, 1991. Proceedings of the IEEE International Conference on*, pages 2724–2729. IEEE Comput. Soc. Press, April 1991.

Yang Chen and Gérard Medioni. Object modelling by registration of multiple range images. *Image And Vision Computing*, 10(3):145–155, April 1992.

D Chetverikov, D Svirko, D Stepanov, and P Krsek. The Trimmed Iterative Closest Point algorithm. In *Pattern Recognition, 2002. Proceedings of the 16th International Conference on*, pages 545–548, 2002.

Francis Colas, Srivatsa Mahesh, François Pomerleau, Ming Liu, and Roland Siegwart. 3D path planning and execution for search and rescue ground robots. In *Intelligent Robots and Systems (IROS), 2013 IEEE/RSJ International Conference on*. IEEE, 2013.

J Diebel, K Reutersward, S Thrun, J Davis, and R Gupta. Simultaneous localization and mapping with active stereo vision. In *Intelligent Robots and Systems, 2004. Proceedings of the IEEE/RSJ International Conference on*, pages 3436–3443 vol.4, 2004.

S Druon, M J Aldon, and A Crosnier. Color constrained ICP for registration of large unstructured 3d color data sets. In *Information Acquisition, 2006. Proceedings of the IEEE International Conference on*, pages 249–255. IEEE, 2006.

David W Eggert, Adele Lorusso, and Robert B Fisher. Estimating 3-D rigid body transformations: A comparison of four major algorithms. *Machine Vision and Applications*, 9(5-6):272–290, 1997.

David W Eggert, Andrew W Fitzgibbon, and Robert B Fisher. Simultaneous Registration of Multiple Range Views for Use in Reverse Engineering of CAD Models. *Computer Vision And Image Understanding*, 69(3):253–272, February 1998.

N Fairfield and D Wettergreen. Evidence grid-based methods for 3D map matching. In *Robotics and Automation, 2009. Proceedings of the IEEE International Conference on*, pages 1637–1642, May 2009.

O D Faugeras and M Hebert. The Representation, Recognition, and Locating of 3-D Objects. *The International Journal of Robotics Research*, 5(3):27–52, September 1986.

J Feldmar and N Ayache. Locally affine registration of free-form surfaces. In *Computer Vision and Pattern Recognition, 1994. Proceedings of the IEEE Computer Society Conference on*, pages 496–501, 1994.

Jacques Feldmar and Nicholas Ayache. Rigid, affine and locally affine registration of free-form surfaces. *International Journal of Computer Vision*, 18 (2):99–119, May 1996.

O Fluck, C Vetter, W Wein, A Kamen, B Preim, and R Westermann. A survey of medical image registration on graphics hardware. *Computer Methods and Programs in Biomedicine*, 104(3):e45–e57, December 2011.

H Gagnon, M Soucy, R Bergevin, and D Laurendeau. Registration of multiple range views for automatic 3-D model building. In *Computer Vision and Pattern Recognition, 1994. Proceedings of the IEEE Computer Society Conference on*, pages 581–586, 1994.

N Gelfand, L Ikemoto, S Rusinkiewicz, and M Levoy. Geometrically stable sampling for the ICP algorithm. In *3-D Digital Imaging and Modeling, 2003. Proceedings of the Fourth International Conference on*, pages 260–267, 2003.

G Godin, M Rioux, and R Baribeau. Three-dimensional registration using range and intensity information. In *Videometric III. Proceedings of SPIE Conference on*, pages 279–290, 1994.

Steven Gold, Anand Rangarajan, Chien-Ping Lu, Suguna Pappu, and Eric Mjolsness. New algorithms for 2D and 3D point matching: pose estimation and correspondence. *Pattern Recognition*, 31(8):1019–1031, August 1998.

Giorgio Grisetti, C Stachniss, and W Burgard. Improving Grid-based SLAM with Rao-Blackwellized Particle Filters by Adaptive Proposals and Selective Resampling. In *Robotics and Automation, 2005. Proceedings of the IEEE International Conference on*, pages 2432–2437, 2005.

Giorgio Grisetti, S Grzonka, C Stachniss, P Pfaff, and W Burgard. Efficient estimation of accurate maximum likelihood maps in 3D. In *Intelligent Robots and Systems, 2007. Proceedings of the IEEE/RSJ International Conference on*, pages 3472–3478, 2007.

P Henry, M Krainin, E Herbst, X Ren, and D Fox. RGB-D mapping: Using Kinect-style depth cameras for dense 3D modeling of indoor environments. *The International Journal of Robotics Research*, 31(5):647–663, April 2012.

Gregory Hitz, François Pomerleau, Marie-Eve Garneau, Cedric Pradalier, Thomas Posch, Jakob Pernthaler, and Ronald Siegwart. Autonomous Inland Water Monitoring: Design and Application of a Surface Vessel. *Robotics & Automation Magazine, IEEE*, 19(1):62–72, March 2012.

Gregory Hitz, François Pomerleau, Francis Colas, and Roland Siegwart. State Estimation for Shore Monitoring Using an Autonomous Surface Vessel. In *International Symposium on Experimental Robotics (ISER)*, pages 1–15, Marrakech/Essaouira, June 2014.

Berthold K P Horn. Closed-form solution of absolute orientation using unit quaternions. *Journal of the Optical Society of America A*, 4(4):629, 1987.

Berthold K P Horn, Hugh M Hilden, and Shahriar Negahdaripour. Closed-form solution of absolute orientation using orthonormal matrices. *Journal of the Optical Society of America A*, 5(7):1127–1135, 1988.

John R Hurley and Raymond B Cattell. The Procrustes program: Producing direct rotation to test a hypothesized factor structure. *Behavioral Science*, 7(2):258–262, April 1962.

A E Johnson and Sing Bing Kang. Registration and integration of textured 3-D data. In *3-D Digital Imaging and Modeling, 1997. Proceedings of the International Conference on Recent Advances in*, pages 234–241, 1997.

T Jost and H Hügli. Fast ICP algorithms for shape registration. In *Pattern Recognition, 2002. Proceedings of the 24th DAGM German Symposium on*, pages 91–99. Pattern Recognition Group, Institute of Microtechnology, University of Neuchâtel, Breguet 2, CH-2000 Neuchâtel, Switzerland, 2002.

T Jost and H Hugli. A multi-resolution scheme ICP algorithm for fast shape registration. In *3D Data Processing Visualization and Transmission, 2002. Proceedings of the First International Symposium on*, pages 540–543, 2002.

T Jost and H Hugli. A multi-resolution ICP with heuristic closest point search for fast and robust 3D registration of range images. In *3-D Digital Imaging and Modeling, 2003. Proceedings of the Fourth International Conference on*, pages 427–433, 2003.

D Kim. A Fast ICP Algorithm for 3-D Human Body Motion Tracking. *Signal Processing Letters, IEEE*, 17(4):402–405, 2010.

G J M Kruijff, M Janicek, S Keshavdas, B Larochelle, H Zender, N J J M Smets, T Mioch, M A Neerincx, J van Diggelen, Francis Colas, M Liu, F Pomerleau, Roland Siegwart, V Hlavac, T Svoboda, T Petricek, M Reinstein, K Zimmerman, F Pirri, M Gianni, P Papadakis, A Sinha, P Balmer, N Tomatis, R Worst, T Linder, H Surmann, V Tretyakov, H Surmann, S Corrao, S Pratzler-Wanczura, and M Sulk. Experience in System Design for Human-Robot Teaming in Urban Search and Rescue. In *Field and Service Robotics*, pages 1–14, Matsushima, Japan, July 2012.

Vladimír Kubelka, Lorenz Oswald, François Pomerleau, Francis Colas, Tomáš Svoboda, and Michal Reinstein. Robust data fusion of multi-modal sensory information for mobile robots. *Journal of Field Robotics*, in press, 2014.

P Kumari, R Shrestha, and B Carter. Registration of LiDAR data through stable surface matching. In *Geoinformatics, 2009. Proceedings of the 17th International Conference on*, pages 1–5, 2009.

P Lamon, S Kolski, and Roland Siegwart. The SmartTer-a vehicle for fully autonomous navigation and mapping in outdoor environments. In *Proceedings of CLAWAR*, 2006.

Ce Li, Jianru Xue, Shaoyi Y Du, and Nanning Zheng. A Fast Multi-Resolution Iterative Closest Point Algorithm. In *Pattern Recognition, 2010. Proceedings of the Chinese Conference on*, pages 1–5, 2010.

Yonghuai Liu. Automatic Range Image Registration in the Markov Chain. *Pattern Analysis and Machine Intelligence, IEEE Transactions on*, 32(1): 12–29, 2010.

D Lowe. Distinctive image features from scale-invariant keypoints. *International Journal of Computer Vision*, 60(2):91–110, 2004.

F Lu and E Milios. Globally Consistent Range Scan Alignment for Environment Mapping. *Autonomous Robots*, 4(4):333–349, 1997.

M Magnusson, Andreas Nüchter, C Lorken, A Lilienthal, and J Hertzberg. Evaluation of 3D registration reliability and speed - A comparison of ICP and NDT. In *Robotics and Automation, 2009. Proceedings of the IEEE International Conference on*, pages 3907–3912, 2009.

J B Antoine Maintz and Max A Viergever. A survey of medical image registration. *Medical Image Analysis*, 2(1):1–36, March 1998.

P Markelj, D Tomaževič, B Likar, and F Pernuš. A review of 3D/2D registration methods for image-guided interventions. *Medical Image Analysis*, 16(3):642–661, April 2012.

T Masuda. Generation of geometric model by registration and integration of multiple range images. In *3-D Digital Imaging and Modeling, 2001. Proceedings of the Third International Conference on*, pages 254–261, 2001.

T Masuda, K Sakaue, and N Yokoya. Registration and integration of multiple range images for 3-D model construction. In *Pattern Recognition, 1996. Proceedings of the 13th International Conference on*, pages 879–883 vol.1, 1996.

C R Jr Maurer, G B Aboutanos, B M Dawant, R J Maciunas, and J M Fitzpatrick. Registration of 3-D images using weighted geometrical features. *Medical Imaging, IEEE Transactions on*, 15(6):836–849, 1996.

G Medioni, C K Tang, and M S Lee. Tensor voting: Theory and applications. In *Reconnaissance des formes et Intelligence Artificielle, 2000. Proceedings of the Conference on*, 2000.

Helena Mitasova, Margery F Overton, Juan José Recalde, David J Bernstein, and Christopher W Freeman. Raster-Based Analysis of Coastal Terrain Dynamics from Multitemporal Lidar Data. *Journal of Coastal Research*, 252:507–514, March 2009.

E Mortensen, Hongli Deng, and L Shapiro. A SIFT descriptor with global context. In *Computer Vision and Pattern Recognition, 2005. Proceedings of the IEEE Computer Society Conference on*, pages 184–190 vol. 1, 2005.

Andreas Nüchter, H Surmann, K Lingemann, J Hertzberg, and S Thrun. 6D SLAM with an application in autonomous mine mapping. In *Robotics and Automation, 2004. Proceedings of the IEEE International Conference on*, pages 1998–2003 Vol.2, 2004.

Andreas Nüchter, K Lingemann, J Hertzberg, and H Surmann. 6D SLAM with approximate data association. In *Advanced Robotics, 2005. Proceedings of the 12th International Conference on*, pages 242–249, 2005.

Andreas Nüchter, K Lingemann, and J Hertzberg. Cached k-d tree search for ICP algorithms. In *3-D Digital Imaging and Modeling, 2007. Proceeding of the Sixth International Conference on*, pages 419–426, 2007.

Ye Pan, Bo Dai, and Qicong Peng. Fast and robust 3D face matching approach. In *Image Analysis and Signal Processing, 2010. Proceedings of the Second International Conference on*, pages 195–198, 2010.

S T Pfister, K L Kriechbaum, S I Roumeliotis, and J W Burdick. Weighted range sensor matching algorithms for mobile robot displacement estimation. In *Robotics and Automation, 2002. Proceedings of the IEEE International Conference on*, 2002.

J P W Pluim, J B A Maintz, and M A Viergever. Mutual-information-based registration of medical images: a survey. *Medical Imaging, IEEE Transactions on*, 22(8):986–1004, 2003.

François Pomerleau, Francis Colas, François Ferland, and F Michaud. Relative motion threshold for rejection in ICP registration. *Field and Service Robotics*, pages 229–238, 2010.

François Pomerleau, Stéphane Magnenat, Francis Colas, Ming Liu, and Roland Siegwart. Tracking a depth camera: Parameter exploration for fast ICP. In *Intelligent Robots and Systems, 2011. Proceedings of the IEEE/RSJ International Conference on*, pages 3824–3829, 2011.

François Pomerleau, Andreas Breitenmoser, Ming Liu, Francis Colas, and Roland Siegwart. Noise Characterization of Depth Sensors for Surface Inspections. In *Applied Robotics for the Power Industry, 2012. Proceedings of the Second International Conference on*, pages 1–6, Zurich, Switzerland, August 2012a.

François Pomerleau, M. Liu, Francis Colas, and Roland Siegwart. Challenging Data Sets for Point Cloud Registration Algorithms. *The International Journal of Robotics Research*, September 2012b.

François Pomerleau, Francis Colas, Roland Siegwart, and Stéphane Magnenat. Comparing ICP Variants on Real-World Data Sets. *Autonomous Robots*, 34(3):133–148, February 2013.

François Pomerleau, Philipp Krüsi, Paul Furgale, and Roland Siegwart. Long-term 3D map maintenance in dynamic environments. In *2014 IEEE International Conference on Robotics and Automation (ICRA 2014)*, pages 1–8, February 2014.

K Pulli. Multiview registration for large data sets. In *3-D Digital Imaging and Modeling, 1999. Proceedings of the Second International Conference on*, pages 160–168, October 1999.

L Reyes, G Medioni, and E Bayro. Registration of 3D points using geometric algebra and tensor voting. *International Journal of Computer Vision*, 2007.

S Rusinkiewicz and M Levoy. Efficient variants of the ICP algorithm. In *3-D Digital Imaging and Modeling, 2001. Proceedings of the Third International Conference on*, pages 145–152, 2001.

J Salvi, C Matabosch, D Fofi, and J Forest. A review of recent range image registration methods with accuracy evaluation. *Image And Vision Computing*, 2007.

C Schutz, T Jost, and H Hugli. Multi-feature matching algorithm for free-form 3D surface registration. In *Pattern Recognition, 1998. Proceedings of the 14th International Conference on*, pages 982–984 vol.2, 1998.

Aleksandr Segal, Dirk Haehnel, and Sebastian Thrun. Generalized-ICP. In *Robotics: Science and Systems V. Proceedings of the Conference on*, page 21. Stanford University, 2009.

L Silva, O Bellon, and K Boyer. Precision range image registration using a robust surface interpenetration measure and enhanced genetic algorithms. *Pattern Analysis and Machine Intelligence, IEEE Transactions on*, 27(5): 762–776, 2005.

C Stewart, Chia-Ling Tsai, and B Roysam. The dual-bootstrap iterative closest point algorithm with application to retinal image registration. *Medical Imaging, IEEE Transactions on*, 22(11):1379–1394, 2003.

E Stumm, A Breitenmoser, François Pomerleau, C Pradalier, and Roland Siegwart. Tensor-voting-based navigation for robotic inspection of 3D surfaces using lidar point clouds. *The International Journal of Robotics Research*, 31(12):1465–1488, November 2012.

Fabien Tâche, Wolfgang Fischer, Gilles Caprari, Roland Siegwart, Roland Moser, and Francesco Mondada. Magnebike: A magnetic wheeled robot with high mobility for inspecting complex-shaped structures. *Journal of Field Robotics*, 26(5), May 2009.

Fabien Tâche, François Pomerleau, W Fischer, Gilles Caprari, F Mondada, R Moser, and Roland Siegwart. MagneBike: Compact magnetic wheeled robot for power plant inspection. In *Applied Robotics for the Power Industry, 2010. Proceedings of the First International Conference on*, pages 1–2, 2010.

Fabien Tâche, François Pomerleau, Gilles Caprari, Roland Siegwart, Michael Bosse, and Roland Moser. Three-Dimensional Localization for the MagneBike Inspection Robot. *Journal of Field Robotics*, 28(2):180–203, 2011.

Sebastian Thrun, Wolfram Burgard, and Dieter Fox. A Probabilistic Approach to Concurrent Mapping and Localization for Mobile Robots. *Machine Learning*, 31(1/3):29–53, 1998.

C Tsai, C Li, G Yang, and K Lin. The Edge-Driven Dual-Bootstrap Iterative Closest Point Algorithm for Registration of Multimodal Fluorescein Angiogram Sequence. *Medical Imaging, IEEE Transactions on*, 29(3):636–649, 2010.

T Tuytelaars and K Mikolajczyk. A survey on local invariant features. *Foundations and Trends in Computer Graphics and Vision*, 2008.

Michael W Walker, Lejun Shao, and Richard A Volz. Estimating 3-D location parameters using dual number quaternions. *CVGIP: Image Understanding*, 54(3):358–367, November 1991.

O Wulf, Andreas Nüchter, J Hertzberg, and B Wagner. Benchmarking urban six-degree-of-freedom simultaneous localization and mapping. *Journal of Field Robotics*, 25(3):148–163, 2008.

Kai M Wurm, Armin Hornung, Maren Bennewitz, Cyrill Stachniss, and Wolfram Burgard. OctoMap: A Probabilistic, Flexible, and Compact 3D Map Representation for Robotic Systems. In *Workshop on Best Practice in 3D Perception and Modeling for Mobile Manipulation, 2010 (ICRA)*, 2010.

H Yoshitaka, K Hirohiko, O Akihisa, and Y Shin'ichi. Mobile Robot Localization and Mapping by Scan Matching using Laser Reflection Intensity of the SOKUIKI Sensor. In *Industrial Electronics, 2006. Proceedings of the IEEE 32nd Annual Conference on*, pages 3018–3023, 2006.

Zhengyou Zhang. Point matching for registration of free-form surfaces. *Lecture Notes in Computer Science*, 719:460–467, May 1993.

Zhengyou Zhang. Iterative point matching for registration of free-form curves and surfaces. *International Journal of Computer Vision*, 13(2):119–152, October 1994.

T Zinsser, J Schmidt, and H Niemann. A refined ICP algorithm for robust 3-D correspondence estimation. In *Image Processing, 2003. Proceedings of the IEEE International Conference on*, pages II–695–8 vol.3, 2003.

Barbara Zitová and Jan Flusser. Image registration methods: a survey. *Image And Vision Computing*, 21(11):977–1000, 2003.

R Zlot and Michael Bosse. Place recognition using keypoint similarities in 2D lidar maps. *Experimental Robotics*, 54:363–372, 2009.