Energy in Robotics

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Abstract

Energy and energy exchange govern interactions in the physical world. By explicitly considering the energy and power in a robotic system, many control and design problems become easier or more insightful than in a purely signal-based view. We show the application of these energy considerations to robotics; starting from the fundamental aspects, but, most importantly, continuing to the practical application to robotic systems. Using the theory of Port-Hamiltonian Systems as a fundamental basis, we show examples concerning energy measurement, passivity and safety. Control by interconnection covers the shaping and directing of energy inside the controller algorithms, to achieve desired behaviour in a power-consistent manner. This idea of control over the energy flows is extended to the physical domain. In their mathematical description and analysis, the boundary between controller and robot disappears and everything is an interconnected system, driven by energy exchange between its parts.

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1

Introduction

The physical world is governed by energy.
From the kinetic energy in a speeding car to the first law of thermodynamics, energy is the *lingua franca* in all physical domains. It is a coordinate-independent description of the energetic state of a system.

Interactions are almost exclusively[1] characterised by energy exchange.
From a battery, through an electric motor—via the magnetic fields—to the mechanical system of a robot: the power or exchanged energy can be traced across all these physical domains. While a car speeds up because the engine applies a torque on the wheels through a set of transmissions, this effort is really a means of pouring energy from the petrol or battery into the kinetic energy of the car as a moving mass.

Many applications in robotics are concerned with energy: the amount of kinetic energy in the robot (e.g. for safety issues), a periodic motion—oscillation—with a certain amplitude (i.e. total energy), energy-efficiency objectives, and storing and releasing energy in springs for explosive motions are some examples.

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[1] Certain interactions, like ideal constraints, can influence motion without energy exchange.

By not solely considering signals, but rather the energy in robotic systems explicitly, more insight can be gained, control problems may become easier and a "feel" for the actual physical processes emerges. This energy-based perspective need not focus on only the control system, nor only on the description of the physical robot. We present a holistic, energy-based view of robotic systems: **Energy in Robotics**. To achieve this holistic view, we shall address the following topics:

1. Energy-based formulation of physical systems: Port-Hamiltonian System theory.
2. Passivity and stability in robotic systems.
3. Measurement and control of energy flowing through actuators.
4. Energy-based controller design: energy shaping and energy routing in the controller.
5. Energy-based system design: shaping the energy flows in a physical robotic system.

The use of energy in robotics is broader than just these topics: there are for example energy-based navigation methods; and in control theory there is a strong link between Lyapunov's stability theorem and energy. The focus of this paper is on the cyber-physical interaction: the study and control of energy flows between the physical system and the controller.

1.1 Port-Hamiltonian systems

Hamiltonian mechanics is a theory of classical mechanics similar to Lagrangian mechanics. The classical canonical formulation is described by a set of equations governing the *Hamiltonian*:

$$\begin{aligned} \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} &= -\frac{\partial \mathcal{H}}{\partial \boldsymbol{q}} \\ \frac{\mathrm{d}\boldsymbol{q}}{\mathrm{d}t} &= +\frac{\partial \mathcal{H}}{\partial \boldsymbol{p}} \\ \mathcal{H} &= T + V. \end{aligned} \tag{1.1}$$

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$\mathcal{H}$ is the Hamiltonian, the sum of kinetic T and potential energy V, i.e. the *total internal energy* of the system; $\boldsymbol{q}$ and $\boldsymbol{p}$ are the generalised coordinates and momenta, respectively. A generalised coordinate is e.g. a position, or charge displacement in the electrical domain. Mechanical momentum is e.g. $p = mv$; in the electrical domain it is the state variable of a inductor, the magnetic flux.

Hamiltonian mechanics are suitable for energy-based modelling and control: the total energy $\mathcal{H}$ is expressed explicitly in the equations.

Example 1.1. A simple example of a physical system described with Hamiltonian mechanics is the mass-spring oscillator. The position $\boldsymbol{q}$ is the spring deflection; momentum $\boldsymbol{p}$ is the momentum of the mass, $\boldsymbol{p} = m \cdot v$. With kinetic energy $T = p^2/(2m)$ (mass m) and potential energy $V = q^2/(2C)$ (C is the compliance of the spring, the inverse of its stiffness) the dynamic equations become:

$$\begin{aligned}\mathcal{H} &= \frac{p^2}{2m} + \frac{q^2}{2C} \\ \frac{\mathrm{d}p}{\mathrm{d}t} &= -\frac{q}{C} \\ \frac{\mathrm{d}q}{\mathrm{d}t} &= \frac{p}{m}.\end{aligned} \tag{1.2}$$

Of course, in the equation for p we recognise $\dot{p} = F$, Newton's second law; in this case $m\dot{v} = Kq$. The equation for q is the obvious $\dot{q} = v$. ⟨*example end*⟩

This example shows that energy is explicitly modelled: when solving the equations one will see the energy flow between T and V. In this closed system without friction, the total energy $\mathcal{H}$ is conserved.

In robotics, however, there is always interaction: between mechanical parts, across domains through transducers, and with the environment. For this interaction, the sub-systems must be interconnected. This interconnection can be described by so-called *power ports*: interfaces that transfer energy between elements, domains, systems. A power port is always a pair of variables whose pairing characterises the power exchange, e.g. force and velocity or voltage and current.

In port-Hamiltonian systems theory, a common representation is the causal Poisson framework representation, which is an input-state-output representation. In this representation, all the states like q and p are collected in a single state vector which may even be a combination of generalised moments and displacements and indicated as x:

$$\begin{aligned} \dot{x} &= [J(x) - R(x)]\frac{\partial \mathcal{H}}{\partial x}(x) + g(x)u \qquad && x \in \mathcal{X}, u \in \mathbb{R}^m \\ y &= g^\top(x)\frac{\partial \mathcal{H}}{\partial x}(x), && y \in \mathbb{R}^m \end{aligned} \tag{1.3}$$

where $J(x) = -J^\top(x)$, $R(x) = R^\top(x) \geq 0$. J is an internal interconnection matrix; R is a resistive structure. g represents the interconnection, and therefore effect, of the port variables on the state variables—and vice versa.

The matrix J is a *power-continuous* interconnection by its skew-symmetry, whereas R models pure resistive losses of the system, as can be seen by taking the time derivative of the Hamiltonian:

$$\begin{aligned} \dot{\mathcal{H}}(x) &= \frac{\partial \mathcal{H}}{\partial x}^\top(x) \cdot \dot{x} \\ &= \frac{\partial \mathcal{H}}{\partial x}^\top(x)\Big[J(x) - R(x)\Big]\frac{\partial \mathcal{H}}{\partial x}(x) + \frac{\partial \mathcal{H}}{\partial x}^\top(x) \cdot g(x)u \\ &= -\frac{\partial \mathcal{H}}{\partial x}^\top(x)R(x)\frac{\partial \mathcal{H}}{\partial x}(x) + y^\top u, \end{aligned} \tag{1.4}$$

which is the power supplied through the port $y^\top u$, minus the power lost to friction, quadratic on $R(x)$.

Example 1.2. Consider the mass-spring-damper system in Figure 1.1: it does not have an external interaction port, so $g(x) \equiv 0$, hence the Hamiltonian should change only with the quadratic $R(x)$ term of (1.4).

The state vector comprises p and q as in Example 1.1; the damping force $F_\mathrm{b} = b \cdot v = b \cdot p/m$ is modelled in the R matrix.

$$\begin{aligned} \mathcal{H}(x) &= \frac{p^2}{2m} + \frac{q^2}{2C} \\ \begin{pmatrix} \dot{p} \\ \dot{q} \end{pmatrix} &= \left[\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} b & 0 \\ 0 & 0 \end{pmatrix}\right]\begin{pmatrix} p/m \\ q/C \end{pmatrix} \end{aligned} \tag{1.5}$$

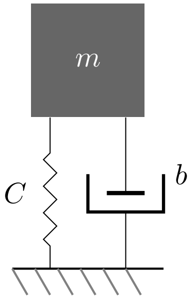

Figure 1.1: A mass-spring-damper system. (Example 1.2)

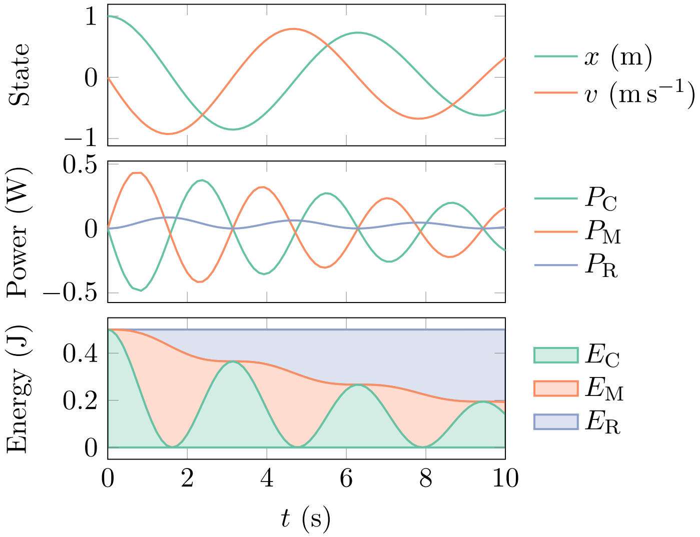

Figure 1.2: Simulation of the mass-spring-damper system of Figure 1.1. Energy flows back and forth between the spring and mass, and is dissipated in the damper. (Example 1.2)

Figure 1.2 shows a simulation of this example system, with $C = 1\,\mathrm{m\,N^{-1}}$, $b = 0.1\,\mathrm{N\,s\,m^{-1}}$, $m = 1\,\mathrm{kg}$, $x(0) = (0 \quad 1\,\mathrm{m})^\top$. Especially the plot of the energy shows how the Hamiltonian $(E_\mathrm{M} + E_\mathrm{C})$ decreases with the energy dissipated in the damper, as expected from (1.4). (E_M and E_C are the first and second term of the Hamiltonian of (1.5); E_R is the energy dissipated by the damper, given by $\int F_\mathrm{b} v \,\mathrm{d}t = \int b v^2 \,\mathrm{d}t$.) ⟨*example end*⟩

Example 1.3. An example of a system with an external port is the sliding mass, with an actuator applying a force on it, as in Figure 1.3. The only state is the momentum p. Choosing F as the input determines $g(x) = 1$ and the dynamic equations are:

$$\begin{aligned} \mathcal{H}(x) &= \frac{p^2}{2m} \\ \dot{x} = \dot{p} &= \left[(0) - (b)\right] \cdot \frac{p}{m} + (1)F \\ y &= (1)^\top \frac{p}{m}. \end{aligned} \tag{1.6}$$

The choice for F as input has made $y = p/m = v$, such that the product of input and output is power and this is indeed a *power port.*

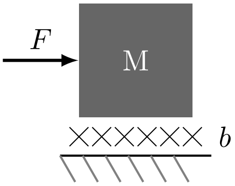

Figure 1.3: A mass sliding on a surface with friction, with a port to the environment: the actuator force. (Example 1.3)

Simulation results of this system (with $m = 1\,\mathrm{kg}$, $b = 0.5\,\mathrm{N\,s\,m^{-1}}$, $F = 0.5\,\mathrm{N}\mathbb{1}(t-1)$) are shown in Figure 1.4. The difference between the power injected by the actuator ($P_\mathrm{F} = v \cdot F$) and the power lost in friction ($P_\mathrm{R} = bv^2$), shaded in the middle graph, is exactly equal to the time derivative of the Hamiltonian, $\dot{E}_\mathrm{M} = P_\mathrm{M}$. ⟨*example end*⟩

Finally, the port of the Port-Hamiltonian System is an interface: the system can be connected to other systems through this power port. The *interconnection* between two or more PHS is described by a Dirac structure, which is a power-continuous coupling of the port variables. In fact, the mass-spring-damper of Example 1.2 can be viewed—and modelled—as three PHS, one for each element, interconnected by a Dirac structure, as in Figure 1.5. The interconnection of Port-Hamiltonian Systems is again a Port-Hamiltonian System, with a Hamiltonian that is the sum of the two systems' Hamiltonians and a new internal interconnection matrix J that incorporates the (old, external) Dirac structure.

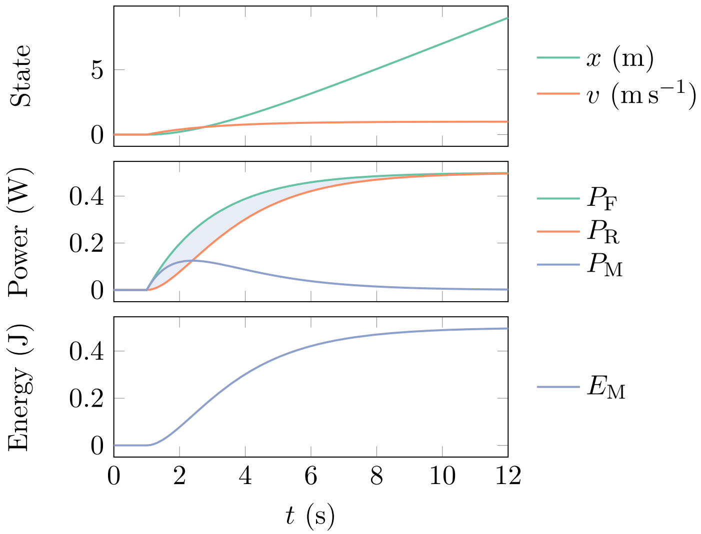

Figure 1.4: Simulation of the sliding mass in Figure 1.3. The difference between the power supplied through the port, P_F, and the power lost to friction, P_R, is equal to the time derivative of the Hamiltonian E_M. (Example 1.3)

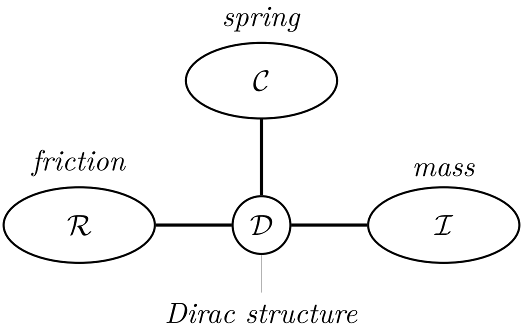

Figure 1.5: A Dirac structure is a power-continuous interconnection between Port-Hamiltonian Systems. This figure shows the system of Figure 1.1 as three interconnected elements, or systems.

An excellent introductory overview of Port-Hamiltonian Systems Theory can be found in van der Schaft and Jeltsema (2014).

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