Robust Estimation and Applications in Robotics

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Contents

Abstract

Solving estimation problems is a fundamental component of numerous robotics applications. Prominent examples involve pose estimation, point cloud alignment, or object tracking. Algorithms for solving these estimation problems need to cope with new challenges due to an increased use of potentially poor low-cost sensors, and an ever growing deployment of robotic algorithms in consumer products which operate in potentially unknown environments. These algorithms need to be capable of being robust against strong nonlinearities, high uncertainty levels, and numerous outliers. However, particularly in robotics, the Gaussian assumption is prevalent in solutions to multivariate parameter estimation problems without providing the desired level of robustness.

The goal of this tutorial is helping to address the aforementioned challenges by providing an introduction to robust estimation with a particular focus on robotics. First, this is achieved by giving a concise overview of the theory on M-estimation. M-estimators share many of the convenient properties of least-squares estimators, and at the same time are much more robust to deviations from the Gaussian model assumption. Second, we present several example applications where M-Estimation is used to increase robustness against nonlinearities and outliers.

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Introduction

Parameter estimation is the problem of inferring the value of a set of parameters through a set of noisy observations. Many tasks in robotics are formulated as an estimation problem. Most notable examples involve odometry, simultaneous localization and mapping (SLAM), or calibration. In case of odometry, the parameters often involve the sequence of robot poses and locations of landmarks that were seen (as in Leutenegger et al. (2015)). This is also true for SLAM, where additionally a map is built that can be used for later relocalization. For calibration, the estimated quantites usually involve the pose of a sensor and some of its internal parameters, e.g. the focal length of a camera lens. Since observations are subject to noise, the parameter estimate will always be afflicted with some level of uncertainty.

To model uncertainty, sensor and system noise are usually characterized by a probability distribution, one of the most common distributions being the Gaussian. Assuming Gaussian noise models leads to convenient simplifications due to its analytical properties and compact mathematical representation. Theoretically, the *central limit theorem* (CLT) is the main justification for the use of the Gaussian distribution.[1] The CLT can be applied in applica-

[1] The Gaussian distribution arises as the limit distribution of a sum of arbitrary independent, identically distributed random variables with finite variance.

tions where random variables are generated as the sum of many independent random variables. This assumption is known as the *hypothesis of elementary errors* and discussed in more detail in Fischer (2011). There are also several computational properties that make the Gaussian distribution an attractive choice. Namely, the fact that any linear combination of Gaussian random variables is Gaussian, and that the product of Gaussian likelihood functions is itself Gaussian. These properties allow additive Gaussian noise to be easily integrated into the parameter estimation framework of linear systems, where variables are assumed to be jointly Gaussian-distributed.[2]

Unfortunately, there is a tendency to invoke the Gaussian in situations where there is little evidence about whether or not it is applicable. Although the CLT provides a justification, to some extent and in some situations, the use of the Gaussian is rarely motivated by the nature of the actual stochastic process that generates the noise. There are situations that arise in practice which violate the CLT conditions. Many real-world systems contain strongly non-linear dynamics that destroy Gaussianity, since a non-linear transformation of a Gaussian random variable is not generally Gaussian-distributed. In certain applications the noise is multiplicative rather than additive, and the Gaussian assumption is inadequate due to the nature of the process.

The success of parameter estimation hinges on the assumptions placed on the noise distribution. Assuming a Gaussian distribution might still be a reasonable approximation even in the presence of non-linearity or non-additive noise, provided that the non-linearity is mild and the noise level is low. However, as these effects increase, there is neither a theoretical justification nor a practical advantage for using methods that rely on this assumption. If the Gaussian assumption is violated, then the parameter estimate may be misleading, which leads to the possibility of drawing incorrect conclusions about the parameter.

Outliers are a common type of a non-Gaussian phenomenon. An outlier may stem from hidden factors or characteristics that are intrinsic to the problem, but are tedious or otherwise impractical to model. Systems that rely on high-quality parameter estimates, such as robots, are especially sensitive to outliers. In certain cases, outliers can cause the system to fail catastrophically

[2]There are a number of other properties motivating the use of the Gaussian distribution. An introductory discussion of these properties can be found in Kim and Shevlyakov (2008).

to the point where a full recovery is no longer possible. For instance, a SLAM solution is vulnerable to false data associations, which may introduce strong biases or even lead to divergence in filter estimates.

Least-squares estimators are particularly prone to bias, outliers, or non-Gaussian noise. The squared-error loss is extremely sensitive, and its performance quickly degrades in the presence of these effects. The reason for this is that the estimator is an unbounded function of the residuals. From a probabilistic perspective, the Gaussian distribution is light-tailed, *i.e.* the tails of the Gaussian account for a very small fraction of the probability mass. This essentially rules out the possibility that an observation is wrong. Therefore, when a large discrepancy arises between the bulk of the observations and an outlier, the parameter estimate becomes an unrealistic compromise between the two.

The main goal of this tutorial is to make robust statistical tools accessible to the robotics community. Specifically, to provide the basis necessary for addressing the problems described above using M-estimators. Hence the contributions of this tutorial are twofold. On one hand, it provides an introduction to robust statistics that only requires preliminary knowledge of probability theory. In particular, the notion of random variables, probability distributions, probability density functions, and multi-variate linear regression are assumed to be known to the reader. On the other hand, this tutorial includes examples of robotics applications where robust statistical tools make a difference. It also includes corresponding Matlab scripts, and discusses how robust statistics improves parameter estimation in these examples.

The remainder of this tutorial is structured as follows. Chapter 2 gives an overview of the history and development of robust statistics and briefly discusses introductory material and existing applications in robotics. Chapter 3 starts with an overview of the challenges of non-linear least-squares estimation, and motivates the use of robust statistics for tackling some of these challenges. It also introduces basic concepts such as *loss functions*, and *iteratively re-weighted non-linear least-squares*. Chapter 4 describes qualitative and quantitative criteria for characterizing the robustness of M-estimators and provides definitions of concepts such as estimator bias, the influence function and the breakdown point are found here. Chapter 5 presents example applications that illustrate the advantage of using robust estimation in robotics.

Specifically, robust approaches to pose graph optimization, parameter estimation under non-Gaussian noise, and state-estimation in the presence of outliers and biases. Finally, chapter 6 concludes with a discussion of further reading and applications of robust statistics to robotics.

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