
**Source Coding:
Part I of Fundamentals of
Source and Video Coding**

Source Coding: Part I of Fundamentals of Source and Video Coding

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Abstract

Digital media technologies have become an integral part of the way we create, communicate, and consume information. At the core of these technologies are source coding methods that are described in this monograph. Based on the fundamentals of information and rate distortion theory, the most relevant techniques used in source coding algorithms are described: entropy coding, quantization as well as predictive and transform coding. The emphasis is put onto algorithms that are also used in video coding, which will be explained in the other part of this two-part monograph.

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1

Introduction

The advances in source coding technology along with the rapid developments and improvements of network infrastructures, storage capacity, and computing power are enabling an increasing number of multimedia applications. In this monograph, we will describe and analyze fundamental source coding techniques that are found in a variety of multimedia applications, with the emphasis on algorithms that are used in video coding applications. The present first part of the monograph concentrates on the description of fundamental source coding techniques, while the second part describes their application in modern video coding.

The block structure for a typical transmission scenario is illustrated in Figure 1.1. The *source* generates a signal s . The *source encoder* maps the signal s into the bitstream b . The bitstream is transmitted over the *error control channel* and the received bitstream b' is processed by the *source decoder* that reconstructs the decoded signal s' and delivers it to the *sink* which is typically a human observer. This monograph focuses on the source encoder and decoder parts, which is together called a source codec.

The error characteristic of the digital channel can be controlled by the *channel encoder*, which adds redundancy to the bits at the source

2 Introduction

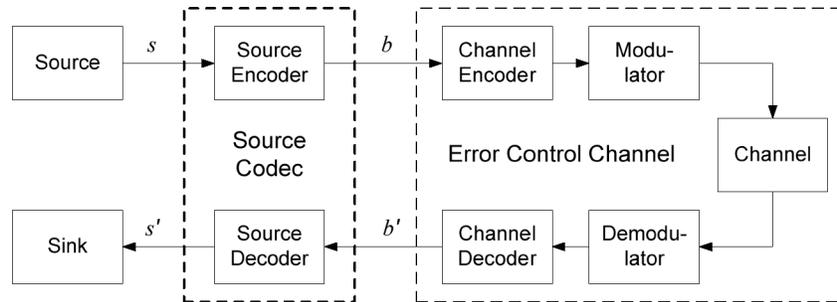


Fig. 1.1 Typical structure of a transmission system.

encoder output b . The *modulator* maps the channel encoder output to an analog signal, which is suitable for transmission over a physical *channel*. The *demodulator* interprets the received analog signal as a digital signal, which is fed into the *channel decoder*. The channel decoder processes the digital signal and produces the received bitstream b' , which may be identical to b even in the presence of channel noise. The sequence of the five components, channel encoder, modulator, channel, demodulator, and channel decoder, are lumped into one box, which is called the error control channel. According to Shannon's basic work [63, 64] that also laid the ground to the subject of this text, by introducing redundancy at the channel encoder and by introducing delay, the amount of transmission errors can be controlled.

1.1 The Communication Problem

The basic communication problem may be posed as conveying source data with the highest fidelity possible without exceeding an available bit rate, or it may be posed as conveying the source data using the lowest bit rate possible while maintaining a specified reproduction fidelity [63]. In either case, a fundamental trade-off is made between bit rate and signal fidelity. The ability of a source coding system to suitably choose this trade-off is referred to as its *coding efficiency* or *rate distortion performance*. Source codecs are thus primarily characterized in terms of:

- *throughput of the channel*: a characteristic influenced by the transmission channel bit rate and the amount of protocol

and error-correction coding overhead incurred by the transmission system; and

- *distortion of the decoded signal*: primarily induced by the source codec and by channel errors introduced in the path to the source decoder.

However, in practical transmission systems, the following additional issues must be considered:

- *delay*: a characteristic specifying the start-up latency and end-to-end delay. The delay is influenced by many parameters, including the processing and buffering delay, structural delays of source and channel codecs, and the speed at which data are conveyed through the transmission channel;
- *complexity*: a characteristic specifying the computational complexity, the memory capacity, and memory access requirements. It includes the complexity of the source codec, protocol stacks, and network.

The practical source coding design problem can be stated as follows:

Given a maximum allowed delay and a maximum allowed complexity, achieve an optimal trade-off between bit rate and distortion for the range of network environments envisioned in the scope of the applications.

1.2 Scope and Overview of the Text

This monograph provides a description of the fundamentals of source and video coding. It is aimed at aiding students and engineers to investigate the subject. When we felt that a result is of fundamental importance to the video codec design problem, we chose to deal with it in greater depth. However, we make no attempt to exhaustive coverage of the subject, since it is too broad and too deep to fit the compact presentation format that is chosen here (and our time limit to write this text). We will also not be able to cover all the possible applications of video coding. Instead our focus is on the source coding fundamentals of video

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coding. This means that we will leave out a number of areas including implementation aspects of video coding and the whole subject of video transmission and error-robust coding.

The monograph is divided into two parts. In the first part, the fundamentals of source coding are introduced, while the second part explains their application to modern video coding.

Source Coding Fundamentals. In the present first part, we describe basic source coding techniques that are also found in video codecs. In order to keep the presentation simple, we focus on the description for one-dimensional discrete-time signals. The extension of source coding techniques to two-dimensional signals, such as video pictures, will be highlighted in the second part of the text in the context of video coding. Section 2 gives a brief overview of the concepts of probability, random variables, and random processes, which build the basis for the descriptions in the following sections. In Section 3, we explain the fundamentals of lossless source coding and present lossless techniques that are found in the video coding area in some detail. The following sections deal with the topic of lossy compression. Section 4 summarizes important results of rate distortion theory, which builds the mathematical basis for analyzing the performance of lossy coding techniques. Section 5 treats the important subject of quantization, which can be considered as the basic tool for choosing a trade-off between transmission bit rate and signal fidelity. Due to its importance in video coding, we will mainly concentrate on the description of scalar quantization. But we also briefly introduce vector quantization in order to show the structural limitations of scalar quantization and motivate the later discussed techniques of predictive coding and transform coding. Section 6 covers the subject of prediction and predictive coding. These concepts are found in several components of video codecs. Well-known examples are the motion-compensated prediction using previously coded pictures, the intra prediction using already coded samples inside a picture, and the prediction of motion parameters. In Section 7, we explain the technique of transform coding, which is used in most video codecs for efficiently representing prediction error signals.

Application to Video Coding. The second part of the monograph will describe the application of the fundamental source coding techniques to video coding. We will discuss the basic structure and the basic concepts that are used in video coding and highlight their application in modern video coding standards. Additionally, we will consider advanced encoder optimization techniques that are relevant for achieving a high coding efficiency. The effectiveness of various design aspects will be demonstrated based on experimental results.

1.3 The Source Coding Principle

The present first part of the monograph describes the fundamental concepts of source coding. We explain various known source coding principles and demonstrate their efficiency based on one-dimensional model sources. For additional information on information theoretical aspects of source coding the reader is referred to the excellent monographs in [4, 11, 22]. For the overall subject of source coding including algorithmic design questions, we recommend the two fundamental texts by Gersho and Gray [16] and Jayant and Noll [40].

The primary task of a source codec is to represent a signal with the minimum number of (binary) symbols without exceeding an “acceptable level of distortion”, which is determined by the application. Two types of source coding techniques are typically named:

- *Lossless coding*: describes coding algorithms that allow the exact reconstruction of the original source data from the compressed data. Lossless coding can provide a reduction in bit rate compared to the original data, when the original signal contains dependencies or statistical properties that can be exploited for data compaction. It is also referred to as *noiseless coding* or *entropy coding*. Lossless coding can only be employed for discrete-amplitude and discrete-time signals. A well-known use for this type of compression for picture and video signals is JPEG-LS [35].
- *Lossy coding*: describes coding algorithms that are characterized by an irreversible loss of information. Only an approximation of the original source data can be reconstructed from

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the compressed data. Lossy coding is the primary coding type for the compression of speech, audio, picture, and video signals, where an exact reconstruction of the source data is not required. The practically relevant bit rate reduction that can be achieved with lossy source coding techniques is typically more than an order of magnitude larger than that for lossless source coding techniques. Well known examples for the application of lossy coding techniques are JPEG [33] for still picture coding, and H.262/MPEG-2 Video [34] and H.264/AVC [38] for video coding.

Section 2 briefly reviews the concepts of probability, random variables, and random processes. Lossless source coding will be described in Section 3. Sections 5–7 give an introduction to the lossy coding techniques that are found in modern video coding applications. In Section 4, we provide some important results of rate distortion theory, which will be used for discussing the efficiency of the presented lossy coding techniques.

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