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Biomedical Image Reconstruction: From the Foundations to Deep Neural Networks

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Biomedical Image Reconstruction: From the Foundations to Deep Neural Networks

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ABSTRACT

This tutorial covers biomedical image reconstruction, from the foundational concepts of system modeling and direct reconstruction to modern sparsity and learning-based approaches.

Imaging is a critical tool in biological research and medicine, and most imaging systems necessarily use an image reconstruction algorithm to create an image; the design of these algorithms has been a topic of research since at least the 1960's. In the last few years, machine learning-based approaches have shown impressive performance on image reconstruction problems, triggering a wave of enthusiasm and creativity around the paradigm of learning. Our goal is to unify this body of research, identifying common principles and reusable building blocks across decades and among diverse imaging modalities.

We first describe system modeling, emphasizing how a few building blocks can be used to describe a broad range of imaging modalities. We then discuss reconstruction algorithms, grouping them into three broad generations. The

first are the classical direct methods, including Tikhonov regularization; the second are the variational methods based on sparsity and the theory of compressive sensing; and the third are the learning-based (also called data-driven) methods, especially those using deep convolutional neural networks. There are strong links between these generations: classical (first-generation) methods appear as modules inside the latter two, and the former two are used to inspire new designs for learning-based (third-generation) methods. As a result, a solid understanding of all three generations is necessary for the design of state-of-the-art algorithms.

List of Abbreviations

ADMM alternating direction method of multipliers

CCD charge-coupled device

CG conjugate gradient

CNN convolutional neural network

CT computed tomography

DCT discrete cosine transform

ET electron tomography

FBP filtered back projection

FFT fast Fourier transform

GPU graphics processing unit

i.i.d. independent and identically distributed

ISTA iterative shrinkage and thresholding

MAP maximum a posteriori

MMSE minimum mean square error

MRI magnetic resonance imaging

MSE mean squared error

PDF probability distribution function

PET positron emission tomography

PSF point spread function

RKHS reproducing kernel Hilbert space

SGD stochastic gradient descent

SIM structured-illumination microscopy

SNR signal-to-noise ratio

SPECT single-photon emission computed tomography

SSIM structural similarity index

TCIA The Cancer Imaging Archive

TV total variation

USC-SIPI University of Southern California Signal and Image Processing Institute

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