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Sound Field Estimation: Theories and Applications

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Contents

Sound Field Estimation: Theories and Applications

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ABSTRACT

The spatial information of sound plays a crucial role in various situations, ranging from daily activities to advanced engineering technologies. To fully utilize its potential, numerous research studies on spatial audio signal processing have been carried out in the literature. Sound field estimation is one of the key foundational technologies that can be applied to a wide range of acoustic signal processing techniques, including sound field reproduction using loudspeakers and binaural playback through headphones. The purpose of this monograph is to present an overview of sound field estimation methods. After providing the necessary mathematical background, two different approaches to sound field estimation will be explained. This monograph focuses on clarifying the essential theories of each approach, while also referencing state-of-the-art developments. Finally, several acoustic signal processing technologies will be discussed as examples of the application of sound field estimation.

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1

Introduction

1.1 Background

Sound is one of the most commonly used media in all kinds of human activities, including human (or human–robot) communication, the analysis of materials and environments, and art-related activities. In many of these situations, the spatial information of sound plays an essential role as well as temporal information. For example, in the localization of sound sources, humans benefit from the interaural difference between sound signals received by both ears without depending much on the temporal waveform of the source signal, and further theoretical and experimental investigations demonstrated the additional effectiveness of head movement during sound source localization [76], [79], [86], [88]. Such importance of the spatial information of sound has stimulated the widespread research and development of audio signal processing technologies for the analysis and control of spatial acoustics.

As a direct attempt to obtain the spatial information of sound, sound field estimation, also called sound field reconstruction, measurement, or recording depending on the context, has been a fundamental technique under intensive investigation. The purpose of sound field estimation is to estimate the spatio-temporal distribution of sound pressure within a target region from the data obtained by multiple sensors, *i.e.*, microphones. In this context, it should be emphasized that the parameters related to the process of generating a sound field, such as the positions of sound sources or the strength of reverberation, are not subject to estimation; a sound field can be directly represented as it is without strong assumptions on or simplifications of these factors. By combining other signal processing applications, sound field estimation enables more than just the reconstruction of the sound pressure at an arbitrary position (see Figure 1.1). One such example is binaural reproduction [2], [8], [25], [33], [55], [69], which aims to reproduce the sound that someone would hear if they were present in the target sound field, including the complex effects of reflection and diffraction caused by the head. Using this technique, one can appreciate, for example, an orchestra music recorded in a concert hall anywhere by a headphone with a high (ideally complete) degree of fidelity. Compared with the direct playback of binaural signals recorded with a dummy head, the binaural reproduction from the estimated sound field allows for various post-processing steps *after* the recording, such adapting individual head-related transfer functions (HRTFs) and rendering with head tracking. There are also other applications, such as sound field synthesis using multiple loudspeakers [3], [10], [13], [19], [20], [66], [82], [92], spatial active noise control [16], [42], [52], [94], [95], the analysis or visualization of room acoustic condition [39], [62], [70], [78].

A sound field can be essentially regarded as a scalar field, *i.e.*, a function from the space and time/frequency variables to sound pressure. However, the estimation of a sound field is distinguished from a simple estimation or interpolation of a function in the common context of machine learning in several aspects. The most distinctive aspect of the sound field estimation problem is the existence of the constraint due to the physical properties of a sound field. This constraint is described typically by the acoustic wave equation or the Helmholtz equation [90]. In addition, there are also distinctive characteristics in the observation of a sound field. First, unlike a common interpolation problem of a function, the observation of a sound field is not necessarily limited to the sampling of sound pressure. This is because a microphone generally has a nonuniform frequency response or directivity. Actually, a specific

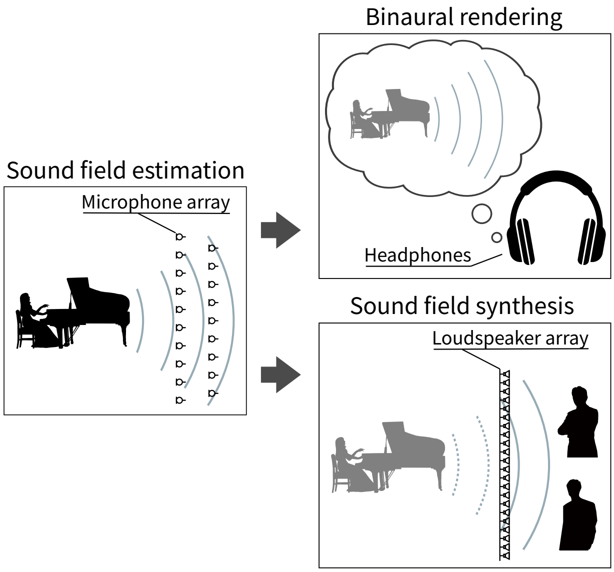

Figure 1.1: Sound field estimation and its applications.

(nonuniform) directivity can even have its advantages over the uniform directivity in some cases, which will be described in Section 4 in detail. On the other hand, the observation of a sound field can be regarded generally as a linear time-invariant system, regardless of the frequency response or directivity of the microphone. This linear time-invariant property makes it easy to analyze an observation and estimation of a sound field in the frequency domain. Thus, how to deal with these generalities and specificities in a technically tractable way is and will always be a critical problem in sound field estimation.

From the historical viewpoint, the exact origin of sound field estimation methods is difficult to identify, to the best of our knowledge. In 1985 at the latest, the pioneering idea of sound field estimation referred to as the near-field acoustic holography was investigated in the field of audio engineering by Maynard *et al.* [54], which was also developed as a sound field synthesis technique called the wave field synthesis [10], although similar theories were developed earlier for the optical field [50], [91]. Their approach was based on the Kirchhoff–Helmholtz integral theorem and Rayleigh's formula, both of which are boundary integral representations that relate a sound field—or, more precisely, a solution of the Helmholtz equation—to its values on a given surface. The use of these theorems has so far formed the basis for many sound field estimation methods [9], [20], [32]. Ambisonics, developed by Gerzon [28] (referred to as "periphony" therein), is another type of pioneering approach to sound field estimation. The original work by Gerzon [28] dealt with the directional component of a sound at one position, not a sound field in the direct form. However, it was later redeveloped as the higher-order ambisonics with theoretical and practical improvements [1], [20], [56], [66], [67], where a sound field was explicitly reconstructed from the signals observed by a spherical microphone array. In the higher-order ambisonics, a sound field is analyzed via its local expansion using the spherical wave functions, which is seemingly different from the approaches based on the Kirchhoff–Helmholtz integral theorem and Rayleigh's formula.

However, both approaches basically require the boundary measurement of the sound field by microphones with specific directivities, and indeed they can be interpreted in a unified manner, as pointed out by Daniel *et al.* [20] and Poletti [66]. These unified theories are also called acoustic holography [90]. In 2003, a new approach was proposed by Laborie *et al.* [49], which no longer requires boundary measurement but allows arbitrary positions and directivities of the microphones. The main idea of this approach lies in the vector and matrix representations of the sound field and observation, respectively, and the same or a similar idea is used in many of the current sound field estimation methods [66], [71], [81]. Whereas all methods mentioned above do not rely on any specific assumptions on the target sound field, several methods that utilize prior information on a target sound field, such as the approximate source direction [83] or sparsity of the source distribution [5], [12], [57], [58], [85], have also been proposed to improve the estimation accuracy. Moreover, several studies on learning-based sound field estimation have been conducted in recent years [18], [46], [51], [73] to further improve performance, the details of which are beyond the scope of this work.

Practical situations in sound field estimation have also changed significantly over the last few decades. In many of the early studies, the estimation methods were demonstrated primarily through numerical simulations, with few exceptions [9], [54] because of the difficulties in the implementation of a large number of microphones and analog-to-digital converters with a large number of synchronized channels. In recent years, however, the practical implementation and commercialization of microphone arrays having a large number of channels have been realized [25], [34] owing to the development of multichannel analog-to-digital converters and the improvement of the miniaturization and integration technologies for microphones. One example of a commercially available microphone array is the em64 Eigenmike®, which is a spherical microphone array of 84 mm diameter equipped with 64 microphones. Such an industrial background implies that sound field estimation techniques and their applications mentioned above are now available in almost any situation, in principle, and studies on sound field estimation methods are gaining increasing attention towards further performance improvement.

1.2 Purpose of This Monograph

The purpose of this tutorial is to provide basic and advanced theories on sound field estimation, focusing on how the physical constraints of sound fields are incorporated into the estimation methods, as well as to introduce several signal processing applications of sound field estimation. In this monograph, the sound field estimation methods are grouped into two types: one with boundary measurement and the other with discrete measurement, and they are described with the results of numerical experiments. It should be noted that the latter is not a simple generalization of the former because they also differ in terms of their background theories. Even though the estimation methods with discrete measurement have an advantage in practical feasibility, those with boundary measurement are also beneficial in terms of their rich theoretical implications. For the full understanding of these approaches, we provide the required mathematics, especially on the Helmholtz equation, with due care of technical correctness. Our main focus lies in the fundamental ideas of the above approaches, but the state-of-the-art methods are also discussed briefly with extensive references. Finally, three applications of sound field estimation are presented: binaural sound reproduction, sound field synthesis with loudspeakers, and active noise cancellation.

1.3 Outline of This Monograph

This monograph is organized as follows. We begin in Section 2 with the definition of the sound field estimation problem of interest in this monograph. Section 3 provides mathematical preliminaries used in later sections. Readers who are not interested in theoretical details can skip this section and refer to it later if required. In Sections 4 and 5, we describe two different approaches to sound field estimation based on boundary measurement and discrete measurement, respectively. We present signal processing applications of sound field estimation in Section 6, and finally in Section 7, we conclude this monograph.

1.4 Symbols and Notations

Basic mathematical symbols and notations are listed in Table 1.1. By convention, $\mathbb{R}^{n\times 1}$ and $\mathbb{C}^{n\times 1}$ are regarded as identical to $\mathbb{R}^n$ and $\mathbb{C}^n$, respectively ($n \in \mathbb{N}$); for instance, for $\boldsymbol{a} \in \mathbb{C}^n$, $\boldsymbol{a}^\mathsf{T}$ and $\boldsymbol{a}^\mathsf{H}$ are $1 \times n$ matrices. Similarly, $\mathbb{R}^1$ and $\mathbb{C}^1$ are regarded as identical to $\mathbb{R}$ and $\mathbb{C}$, respectively. For an inner product space over $\mathbb{C}$, the inner product is defined such that it is antilinear with respect to the first variable and linear with respect to the second variable.

Table 1.1: Notations

Numbers:	
$\mathbb{N}$	set of natural numbers (including 0)
$\mathbb{Z}$	set of integers
$\mathbb{R}$	set of real numbers
$\mathbb{C}$	set of complex numbers
i	imaginary unit in $\mathbb{C}$
z^*	complex conjugate of $z \in \mathbb{C}$
$[\![a, b]\!]$	set of all integers between a and b included $(a, b \in \mathbb{Z})$
Linear algebra $(\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\},\ m, n \in \mathbb{N})$:	
$\mathbb{K}^n$	n-dimensional coordinate space over $\mathbb{K}$
$\mathbb{K}^{m\times n}$	set of $m \times n$ matrices over $\mathbb{K}$
$\boldsymbol{A}^\mathsf{T}$	transpose of $\boldsymbol{A} \in \mathbb{K}^{m\times n}$
$\boldsymbol{A}^\mathsf{H}$	conjugate transpose of $\boldsymbol{A} \in \mathbb{K}^{m\times n}$
$\boldsymbol{A}^{-1}$	inverse of $\boldsymbol{A} \in \mathbb{K}^{n\times n}$ (if exists)
$\delta_{a,b}$	Kronecker's delta $(a, b \in \mathbb{Z})$
Vector analysis:	
$\boldsymbol{x} \cdot \boldsymbol{y}$	dot product between $\boldsymbol{x} \in \mathbb{R}^3$ and $\boldsymbol{y} \in \mathbb{R}^3$
$\lVert\boldsymbol{x}\rVert$	Euclidean norm of $\boldsymbol{x} \in \mathbb{R}^3$
$\mathbb{S}_2$	unit sphere in $\mathbb{R}^3$
$\mathrm{SO}(3)$	rotation group over $\mathbb{R}^3$
$\mathcal{C}_n(\Omega)$	set of nth continuously differentiable functions from an open set $\Omega \subseteq \mathbb{R}^3$ to $\mathbb{C}$ $(n \in \mathbb{N})$
∇	vector differential operator (gradient)
Δ	Laplace operator
$\partial\Omega$	topological boundary of $\Omega \subseteq \mathbb{R}^3$
$B(\boldsymbol{r}, R)$	open ball centered at $\boldsymbol{r} \in \mathbb{R}^3$ with radius $R \in (0, \infty)$
$\overline{B}(\boldsymbol{r}, R)$	closed ball centered at $\boldsymbol{r} \in \mathbb{R}^3$ with radius $R \in (0, \infty)$
V	volume measure (3-dimensional Lebesgue measure)
S	surface measure (2-dimensional Hausdorff measure)

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