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# Min-Max Framework for Majorization-Minimization Algorithms in Signal Processing Applications: An Overview

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# Min-Max Framework for Majorization-Minimization Algorithms in Signal Processing Applications: An Overview

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## ABSTRACT

This monograph presents a theoretical background and a broad introduction to the **Min-Max Framework for Majorization-Minimization** (MM4MM), an algorithmic methodology for solving minimization problems by formulating them as min-max problems and then employing majorization-minimization. The monograph lays out the mathematical basis of the approach used to reformulate a minimization problem as a min-max problem. With the prerequisites covered, including multiple illustrations of the formulations for convex and non-convex functions, this work serves as a guide for developing MM4MM-based algorithms for solving non-convex optimization problems in various areas of signal processing. As special cases, we discuss using the

majorization-minimization technique to solve min-max problems encountered in signal processing applications and min-max problems formulated using the Lagrangian. Lastly, we present detailed examples of using MM4MM in ten signal processing applications such as phase retrieval, source localization, independent vector analysis, beamforming and optimal sensor placement in wireless sensor networks. The devised MM4MM algorithms are free of hyper-parameters and enjoy the advantages inherited from the use of the majorization-minimization technique such as monotonicity.

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**Keywords:** Conjugate function; min-max problem; majorization-minimization; non-convex optimization.

# 1

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## Introduction

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The Majorization Minimization (MM) method, with its roots in the 1970s, is a generalization of the classical Expectation-Maximization (EM) method [33]. MM was earlier known by different names like space-alternating generalized EM (SAGE) [39], optimization transfer [6], and iterative majorization [56], and it was mainly used in the field of statistics [10], [29], [55], [56], [68] and image processing [30]–[32], [46], [47], [66], [101]. MM is a general algorithmic framework for convex and non-convex problems [59], [68]. [59], [67], [69], [132], [133] triggered the attention of researchers from the statistics and signal processing fields to the MM methodology. The work [59], in particular, sparked research interest in using the MM algorithmic framework for solving non-convex, non-smooth optimization problems encountered in applications such as compressive sensing [13], [17], [73], [107], covariance estimation [109]–[111], [123], non-negative matrix factorization [40]–[42], [113], sequence design [104]–[106], [115], [127], localization in sensor networks [26], [61], [82], and image processing [11], [43], [44], [70], [83], [125].

We will always abbreviate majorization-minimization as MM. However, the abbreviation MM4MM has different (but related) meanings depending on the context:

- In Section 3.2.1, we use a min-max formulation to derive an MM algorithm and there  $\text{MM4MM} = \text{min-max}$  for MM.
- In Section 3.2.2, we swap the min and max operators and therefore  $\text{MM4MM} = \text{max-min}$  for MM.
- Finally in Section 4, we get the min-max formulation for free (without any need for a max formulation of the function to be minimized), consequently there  $\text{MM4MM} = \text{MM}$  for min-max.

## 1.1 MM Summary

Employing MM involves two basic steps: majorizing the objective function to create a surrogate function and then minimizing the surrogate function iteratively to solve the min problem. Consider the following optimization problem:

$$\min_{\mathbf{x} \in \mathbb{X}} f(\mathbf{x}), \quad (1.1)$$

where  $\mathbb{X} \subseteq \mathbb{R}^n$  is a non-empty convex set and  $f(\mathbf{x}): \mathbb{X} \rightarrow \mathbb{R}$  is a continuous function that is bounded below. To solve (1.1) using MM, a suitable surrogate function  $f_s(\mathbf{x} | \mathbf{x}^t)$  needs to be constructed such that given a feasible point  $\mathbf{x}^t \in \mathbb{X}$ , the following inequality holds:

$$f_s(\mathbf{x} | \mathbf{x}^t) \geq f(\mathbf{x}) + c_t \quad \forall \mathbf{x} \in \mathbb{X}. \quad (1.2)$$

The constant  $c_t \in \mathbb{R}$  ensures that equality is satisfied in (1.2) at  $\mathbf{x} = \mathbf{x}^t$ . In other words, the surrogate function  $f_s(\mathbf{x} | \mathbf{x}^t)$  majorizes  $f(\mathbf{x})$  and is equal to  $f(\mathbf{x}^t) + c_t$  at  $\mathbf{x} = \mathbf{x}^t$ .

At the second step, the surrogate function is minimized yielding the next iterate point of the algorithm, that is,

$$\mathbf{x}^{t+1} = \arg \min_{\mathbf{x} \in \mathbb{X}} f_s(\mathbf{x} | \mathbf{x}^t). \quad (1.3)$$

This implies,

$$f_s(\mathbf{x}^{t+1} | \mathbf{x}^t) \leq f_s(\mathbf{x} | \mathbf{x}^t) \quad \forall \mathbf{x} \in \mathbb{X}. \quad (1.4)$$

Using (1.4) and (1.2), we get:

$$\begin{aligned} f(\mathbf{x}^{t+1}) &\leq f_s(\mathbf{x}^{t+1} | \mathbf{x}^t) - c_t \\ &\leq f_s(\mathbf{x}^t | \mathbf{x}^t) - c_t = f(\mathbf{x}^t). \end{aligned} \quad (1.5)$$

From (1.5) it follows that the sequence of function values  $\{f(\mathbf{x}^t)\}$  is monotonically non-increasing. What plays a crucial role in developing a computationally efficient MM algorithm for an optimization problem is the construction of a suitable surrogate function. The surrogate function needs to be such that it follows the shape of the objective function as closely as possible and the computational cost of minimizing it is low. To construct a surrogate function, different techniques can be used such as Jensen's inequality, arithmetic-geometric mean inequality, and linearizing a concave function using a first-order Taylor expansion. Many of these techniques are discussed in [112], which also provides a general overview of the MM algorithmic framework along with applications of MM in signal processing, communication and machine learning. The MM technique has been used to solve a wide variety of optimization problems from areas such as signal processing [1], [25], [62], [91], communication [3], [28], [48], [65], [85], radar and sonar [7], [36], [98], [124], machine learning and computer vision [16], [22], [45], [57], [72], [121], [129], image recovery [18], [75], [90], intelligent transportation systems [58], [80], [130], graph learning [37], [63], [64], [71], [77], [92], [128], biomedical signal processing [23], [34], [78], [79], and neuroimaging applications [49], [52], [131].

## 1.2 Need for MM4MM

There exist minimization problems for which surrogate functions are difficult to derive or, even if such functions can be found, they are not convenient to deal with from a computational standpoint. Indeed for functions in  $\mathbf{x} \in \mathbb{R}^n$  like  $\mathbf{x}^T \mathbf{A} \mathbf{x} \log(\mathbf{x}^T \mathbf{A} \mathbf{x})$  where  $\mathbf{A} \succ \mathbf{0}$ , and  $(\log \|\mathbf{x} - \mathbf{a}\|)^2$  where  $\mathbf{a} \in \mathbb{R}^n$  finding a suitable surrogate is difficult. On the other hand, for  $\log|\mathbf{X}^T \boldsymbol{\Sigma}^{-1} \mathbf{X}|$  (where  $\mathbf{X} \in \mathbb{R}^{n \times m}$  and  $\boldsymbol{\Sigma} \in \mathbb{S}_{++}^n$ ) a surrogate can be found using a first-order Taylor expansion (below  $\mathbf{Y} = \mathbf{X}^T \boldsymbol{\Sigma}^{-1} \mathbf{X}$ ):

$$\log|\mathbf{Y}| \leq \log|\mathbf{Y}_t| + \text{Tr}(\mathbf{Y}_t^{-1}(\mathbf{Y} - \mathbf{Y}_t)). \quad (1.6)$$

However, the cost of computing  $\mathbf{Y}_t^{-1} = (\mathbf{X}_t^T \boldsymbol{\Sigma}^{-1} \mathbf{X}_t)^{-1}$  at each iteration is usually high.

To deal with cases such as those above, the proposed MM4MM framework expresses the function  $f(\mathbf{x})$  as maximum of an augmented function  $g(\mathbf{x}, \mathbf{z})$ , where  $\mathbf{z}$  is an auxiliary variable:

$$f(\mathbf{x}) = \max_{\mathbf{z}} g(\mathbf{x}, \mathbf{z}). \quad (1.7)$$

We call this representation of  $f(\mathbf{x})$  the *max formulation*. Finding the function  $g(\mathbf{x}, \mathbf{z})$  requires ingenuity. This function should be such that suitable surrogate function(s) for the non-convex term(s) of  $g(\mathbf{x}, \mathbf{z})$  are easy to obtain, making it possible to come up with an efficient MM algorithm. The problem of minimizing  $f(\mathbf{x})$  is reformulated as a min-max problem with  $g(\mathbf{x}, \mathbf{z})$  as the objective function:

$$\min_{\mathbf{x}} \max_{\mathbf{z}} g(\mathbf{x}, \mathbf{z}). \quad (1.8)$$

The problem in (1.8) can be solved using MM as we explain in Section 3. We show how the proposed MM4MM algorithmic framework can be used to find suitable surrogate functions for the examples mentioned in the previous paragraph in Section 5.3 (for  $\mathbf{x}^T \mathbf{A} \mathbf{x} \log(\mathbf{x}^T \mathbf{A} \mathbf{x})$ ), Section 5.4 (for  $(\log \|\mathbf{x} - \mathbf{a}\|)^2$ ), and Section 5.5 (for  $\log|\mathbf{X}^T \Sigma^{-1} \mathbf{X}|$ ).

One of the first algorithms using the basic ideas of the MM4MM framework is the PDMM (Primal-Dual Majorization Minimization) [38] algorithm that solves the inverse problem of phase retrieval for Poisson noise. [95] proposes a special case of MM4MM algorithm using a Lagrangian min-max formulation of the problem of E-optimal experiment design (see Section 4.1). Also, [99] uses the max formulation of the objective function and then employs MM for the problem of total variation filtering. However, this problem is convex, and thus [99] uses the max formulation only to deal with non-differentiability of  $\ell_1$  norm penalty. More details on the applications in [4], [38], [86], [95]–[97], [99], [114] and on the way in which these works use the MM4MM framework can be found in Section 5.

### 1.3 Organization

Before describing the Min-Max framework for MM, we present some preliminary results in Sections 2 and 3.1. In Section 2, we describe the max formulation for convex and non-convex functions, along with ten

illustrative examples for functions which are found in various signal processing problems. The minimax theorem is discussed in Section 3.1. These sections cover the prerequisites for the MM4MM framework, which is described in Section 3.2. Section 4 describes two special cases of the MM4MM framework. The bulk of the monograph is Section 5 in which we present detailed derivations of the MM4MM algorithms for ten signal processing applications.

## 1.4 Notation

Italic letters ( $x$ ), lower case bold letters ( $\mathbf{x}$ ) and upper case bold letters ( $\mathbf{X}$ ) denote scalars, vectors and matrices respectively.  $|x|$  is the absolute value of  $x$  and  $|\mathbf{x}|$  is the element-wise absolute value of vector  $\mathbf{x}$ . The  $i$ th element of a vector  $\mathbf{x}$  is denoted  $x_i$ .  $\mathbf{x}_i$  denotes the  $i$ th column and  $X_{ij}$  the  $(i, j)$ th element of the matrix  $\mathbf{X}$ .  $\mathbf{I}$  is the identity matrix and  $\mathbf{0}$  the zero matrix.  $\text{Tr}(\mathbf{X})$  and  $|\mathbf{X}|$  denote the trace and determinant of the matrix  $\mathbf{X}$ .  $\mathbf{X} \succ \mathbf{0}$  ( $\mathbf{X} \succeq \mathbf{0}$ ) denotes the positive definiteness (positive semi-definiteness) of the matrix  $\mathbf{X}$ .  $\text{vec}(\mathbf{X})$  is the vectorization operator reshaping a matrix  $\mathbf{X}$  of size  $m \times n$  in a vector of size  $mn \times 1$ .

The sets of real numbers and complex numbers are denoted  $\mathbb{R}$  and  $\mathbb{C}$ .  $\mathbb{R}_+$  is the set of non-negative real numbers.  $\mathbb{S}_+^n$  and  $\mathbb{S}_{++}^n$  represent the sets of positive semi-definite matrices and positive definite matrices of size  $n \times n$ . A subspace is represented by  $\mathcal{X}$ . The indicator function  $I_{\mathbb{X}}(\mathbf{x})$  is zero 0 for all  $\mathbf{x}$  lying in the set  $\mathbb{X}$  and  $\infty$  otherwise.  $\mathcal{N}(\mu, \sigma^2)$  denotes Gaussian distribution with mean  $\mu$  and variance  $\sigma^2$ . The notation  $\mathbf{x} \sim \text{Poisson}(\lambda)$  means that  $\mathbf{x}$  follows a Poisson distribution with mean and variance  $\lambda$ . The natural logarithm is denoted  $\log(\cdot)$  and the logarithm to the base 10 is denoted  $\log_{10}(\cdot)$ .  $f'(x)$  is the derivative of the function  $f(x)$ . The symbols  $\odot$  and  $\oslash$  denote the element-wise multiplication and division of two vectors.  $\otimes$  denotes the Kronecker product of two matrices. Infimal convolution between two functions  $f_1(\mathbf{x})$  and  $f_2(\mathbf{y})$  is defined as:

$$(f_1 \square f_2)(\mathbf{x}) = \min_{\mathbf{y}} f_1(\mathbf{x} - \mathbf{y}) + f_2(\mathbf{y}). \quad (1.9)$$

The proximal operator of a scaled function  $\mathbf{prox}_{\alpha f}(\mathbf{z})$  is defined as follows:

$$\mathbf{prox}_{\alpha f}(\mathbf{z}) = \arg \min_{\mathbf{x}} \left( f(\mathbf{x}) + \frac{1}{2\alpha} \|\mathbf{x} - \mathbf{z}\|_2^2 \right). \quad (1.10)$$

Superscripts  $(\cdot)^\top$ ,  $(\cdot)^H$  and  $(\cdot)^{-1}$  denote the transpose, conjugate transpose and inverse operations respectively.  $(\mathbf{X})^{\frac{1}{2}}$  or  $\sqrt{\mathbf{X}}$  refers to the matrix square root of the positive semi-definite matrix  $\mathbf{X} \in \mathbb{S}_+^n$  such that  $(\mathbf{X})^{\frac{1}{2}}(\mathbf{X})^{\frac{1}{2}} = \mathbf{X}$ . The symbol  $\|\mathbf{x}\|_p$  denotes the  $\ell_p$  norm of vector  $\mathbf{x} \in \mathbb{R}^n$  defined as:

$$\|\mathbf{x}\|_p = (|x_1|^p + |x_2|^p + \cdots + |x_n|^p)^{1/p}, \quad (1.11)$$

which has the following special cases:

$$\|\mathbf{x}\|_1 = |x_1| + \cdots + |x_n|, \quad (1.12)$$

$$\|\mathbf{x}\|_\infty = \max\{|x_1|, \dots, |x_n|\}. \quad (1.13)$$

The dual norm of any norm  $\|\mathbf{x}\|$  is represented by  $\|\mathbf{z}\|_*$  and is defined as:

$$\|\mathbf{z}\|_* = \max\{\mathbf{z}^\top \mathbf{x} \mid \|\mathbf{x}\| \leq 1\}. \quad (1.14)$$

Huber norm  $\|\mathbf{x}\|_H$  of vector  $\mathbf{x}$  is given as follows:

$$\|\mathbf{x}\|_H = \sum_{i=1}^n f_\alpha(x_i); \quad f_\alpha(x_i) = \begin{cases} \frac{|x_i|^2}{2\alpha} & \text{if } |x_i| \leq \alpha \\ |x_i| - \frac{\alpha}{2} & \text{if } |x_i| > \alpha. \end{cases} \quad (1.15)$$

Unless otherwise stated,  $\|\mathbf{x}\|$  will be used to denote the  $\ell_2$  norm of the vector  $\mathbf{x}$ , i.e.,  $\|\mathbf{x}\|_2$ . For matrix  $\mathbf{X} \in \mathbb{R}^{m \times n}$ , the  $\|\mathbf{X}\|_1$  norm is given by:

$$\|\mathbf{X}\|_1 = \max_{j=1, \dots, n} \sum_{i=1}^m |X_{ij}|. \quad (1.16)$$

The nuclear norm, defined as sum of the singular values and denoted  $\|\mathbf{X}\|_{2*}$ , is given by:

$$\|\mathbf{X}\|_{2*} = \sigma_1(\mathbf{X}) + \cdots + \sigma_r(\mathbf{X}) = \text{Tr}(\mathbf{X}^\top \mathbf{X})^{\frac{1}{2}}, \quad (1.17)$$

where  $\{\sigma_i\}$  are the singular values and  $r$  is the rank of the matrix  $\mathbf{X}$ . The spectral norm, defined as the maximum singular value and denoted  $\|\mathbf{X}\|_2$ , is given by:

$$\|\mathbf{X}\|_2 = \sigma_{\max}(\mathbf{X}). \quad (1.18)$$



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