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Abstract

This paper focuses on monotonicity results for dynamic systems that take values in the natural numbers or in more-dimensional lattices. The results are mostly formulated in terms of controlled queueing systems, but there are also applications to maintenance systems, revenue management, and so forth. We concentrate on results that are obtained by inductively proving properties of the dynamic programming value function. We give a framework for using this method that unifies results obtained for different models. We also give a comprehensive overview of the results that can be obtained through it, in which we discuss not only (partial) characterizations of optimal policies but also applications of monotonicity to optimization problems and the comparison of systems.

Contents

1

Introduction

Dynamic programming (DP) is a versatile method that can be used to analyze dynamical systems with a possibly stochastic transition mechanism. Above all, it is a numerical method for so-called Markov decision chains (MDCs), in which there are actions to be taken. In systems without actions DP can also be used to find performance indicators (as an alternative to the method that computes the stationary distribution first). These systems are called Markov reward chains, where the reward refers to the performance measure. However, DP can also be used as a theoretical means to obtain certain properties of Markov decision/reward chains. That is the subject of this paper.

The focus in this paper is on dynamic programming methods to obtain monotonicity results for queueing and other types of systems. The results that we obtain for MDCs can be used to characterize, partially or completely, optimal policies. In the context of Markov reward chains we obtain certain monotonicity results in the states, which can then be used in other optimization problems or to compare systems that, for example, differ in certain parameter values. Relevant results that are not obtained through DP are mentioned (but not proven) to give a complete picture.

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