Long Range Dependence

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Long Range Dependence

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Abstract

The notion of long range dependence is discussed from a variety of points of view, and a new approach is suggested. A number of related topics is also discussed, including connections with non-stationary processes, with ergodic theory, self-similar processes and fractionally differenced processes, heavy tails and light tails, limit theorems and large deviations.

Keywords: Long range dependence, rare events, large deviations.

Mathematical Subject Classification:
Primary 60G10, 62M10; Secondary 60F10

Contents

1

Introduction

Long range dependence and *long memory* are synonymous notions, that are arguably very important. This importance can be judged, for example, by the very large number of publications having one of these notions in the title, in areas such as finance [84], econometrics [115], internet modeling [70], hydrology [109], climate studies [142], linguistics [3] or DNA sequencing [71]. These publications address a great variety of issues: detection of long memory in the data, statistical estimation of parameters of long range dependence, limit theorems under long range dependence, simulation of long memory processes, and many others. Surprisingly, very few of these publications address what long range dependence is. When definitions are given, they vary from author to author (the econometric survey [58] mentions 11 different definitions). The notion of long range dependence can also be applied to different aspects of a given stochastic process [63]. More diverse definitions become possible if, instead of looking at the "usual" stationary processes, one studies *stationary point processes*, as in [37], or *random fields*, as in [4].

It is the purpose of this survey to discuss what is meant (often implicitly) by long range dependence, clarify why this notion is important, mention different point of views on the topic and, hopefully, remove some of the mystery that surrounds it.

The notion of long range dependence has, clearly, something to do with memory in a stochastic process. Memory is, by definition, something that lasts. It is the requirement that the memory has to be "long" that is special. Why is it important that in one model the memory is "a bit longer" than in another model? The first serious argument that this can be important is in a series of papers of B. Mandelbrot and his coauthors, e.g. [89] and [93]. It is also due to the influence of these early papers and subsequent publications of Mandelbrot (especially [90]) that long range dependence has also become associated with scaling and fractal behavior. We survey some of the early history in Section 2.

The "specialness" of long memory indicates that most stationary stochastic processes do not have it. This also makes it intuitive that non-stationary processes can provide an alternative explanation to the empirical phenomena that the notion of long range dependence is designed to address. This connection between long memory and lack of stationarity is very important. It is related to such well known phenomena as unit root problem [111] and regime switching [42]. We discuss the connections with non-stationary processes in Section 3.

A very attractive point of view on long range dependence is based on ergodic-theoretical properties of the dynamical system on which a stationary stochastic process is constructed. Many features that are intuitively associated with long memory are automatically found in such an approach. For several reasons this approach has not become widely accepted. We discuss this in Section 4.

Most of the definitions of long range dependence appearing in literature are based on the second-order properties of a stochastic process. Such properties include asymptotic behavior of covariances, spectral density, and variances of partial sums. The reasons for popularity of the second-order properties in this context are both historical and practical: second-order properties are relatively simple conceptually and easy to estimate from the data. This approach to the notion of long memory is discussed in Section 5.

The term "fractional" appears very frequently in the context of long range dependence. This usually refers to a model constructed using a generalized operation of a non-integer order, whereas the "usual" order of the operation has to be integer. The examples include differencing or differentiation "non-integral number of times." Certain features often associated with long memory can sometimes be obtained by doing so. Models obtained in this way are discussed in Section 6.

It is, once again, largely due to the early history that the notion of long range dependence has become closely associated with self-similar processes. Self-similar processes are stochastic models with the property that a scaling in time is equivalent to an appropriate scaling in space. The connection between the two types of scaling is determined by a constant often called *the Hurst exponent*, and it has been argued that the value of this exponent determines whether or not the increments of a self-similar process with stationary increments possess long range dependence. We discuss self-similar processes in Section 7.

The final part of this survey, Section 8, introduces a different approach to understanding long memory, a one that is related to the notion of phase transitions. We argue that this approach makes the notion of long range dependence both intuitive and practical. One should hope for major future research effort in this direction.

References

[1] J. Aaronson, "An introduction to infinite ergodic theory," in volume 50 *of Mathematical Surveys and Monographs*, Providence: American Mathematical Society, 1997.

[2] P. Abry, L. Delbeke, and P. Flandrin, "Wavelet based estimator for the self-similarity parameter of α-stable processes," in *International Conference on Signal Processing, 16th World Computer Congress*, 2000.

[3] E. Alvarez-Lacalle, B. Dorow, J.-P. Eckmann, and E. Moses, "Hierarchical structures induce long-range dynamical correlations in written texts," *PNAS*, vol. 103, pp. 7956–7961, 2006.

[4] V. Anh, J. Angulo, and M. Ruiz-Medina, "Possible long-range dependence in fractional random fields," *Journal of Statistical Planning and Inference*, vol. 80, pp. 95–110, 1999.

[5] S. Asmussen, *Ruin Probabilities*. World Scientific Publishing Company, 2000.

[6] A. Astrauskas, "Limit theorems for quadratic forms of linear processes," *Lithuanian Mathematical Journal*, vol. 23, pp. 355–361, 1983.

[7] A. Astrauskas, "Limit theorems for sums of linearly generated random variables," *Lithuanian Mathematical Journal*, vol. 23, pp. 127–134, 1983.

[8] A. Astrauskas, J. Levy, and M. S. Taqqu, "The asymptotic dependence structure of the linear fractional Lévy motion," *Lietuvos Matematikos Rinkinys (Lithuanian Mathematical Journal)*, vol. 31, pp. 1–28, 1991.

[9] F. Avram and M. S. Taqqu, "Noncentral limit theorems and Appell polynomials," *The Annals of Probability*, vol. 15, pp. 767–775, 1987.

[10] F. Avram and M. S. Taqqu, "Weak convergence of sums of moving averages in the α-stable domain of attraction," *The Annals of Probability*, vol. 20, pp. 483–503, 1992.

[11] F. Avram and M. S. Taqqu, "Robustness of the R/S statistic for fractional stable noises," *Statistical Inference for Stochastic Processes*, vol. 3, pp. 69–83, 2000.

[12] R. Baillie, T. Bollerslev, and H. Mikkelsen, "Fractionally integrated generalized autoregressive conditional heteroscedasticity," *Journal of Econometrics*, vol. 74, pp. 3–30, 1996.

[13] J. Barkoulas, W. Labys, and J. Onochie, "Long memory in future prices," *The Financial Review*, vol. 34, pp. 91–100, 1999.

[14] J. Beran, *Statistics for Long–Memory Processes.* New York: Chapman and Hall, 1994.

[15] K. Berg-Sørensen and H. Flyvberg, "The colour of thermal noise in classical Brownian motion: A feasibility study of direct experimental observation," *New Journal of Physics*, vol. 7, pp. 38–47, 2005.

[16] S. Berman, "Limit theorems for the maximum term in stationary sequencies," *Annals of Mathematical Statistics*, vol. 35, pp. 502–516, 1964.

[17] R. Bhattacharya, V. Gupta, and E. Waymire, "The Hurst effect under trends," *Journal of Applied Probability*, vol. 20, pp. 649–662, 1983.

[18] P. Billingsley, *Convergence of Probability Measures.* New York: Wiley, Second ed., 1999.

[19] N. Bingham, C. Goldie, and J. Teugels, *Regular Variation.* Cambridge University Press, 1987.

[20] T. Bollerslev, "Generalized autoregressive conditional heteroskedasticity," *Journal of Econometrics*, vol. 31, pp. 307–327, 1986.

[21] A. Böttcher and J. Virtanen, "Norms of Toeplitz matrices with Fisher-Hartwig symbols," *SIAM Journal of Matrix Analysis and Applications*, vol. 29, pp. 660–671, 2007.

[22] R. Bradley, "Equivalent mixing conditions for random fields," *Annals of Probability*, vol. 21, pp. 1921–1926, 1993.

[23] R. Bradley, "On the growth of variances in a central limit theorem for strongly mixing sequences," *Bernoulli*, vol. 5, pp. 67–80, 1999.

[24] R. Bradley, "Basic properties of strong mixing conditions. A survey and some open questions," *Probability Surveys*, vol. 2, pp. 107–144, 2005.

[25] P. Breuer and P. Major, "Central limit theorems for non-linear functionals of Gaussian fields," *Journal of Multivariate Analysis*, vol. 13, pp. 425–441, 1983.

[26] P. Brockwell and R. Davis, *Time Series: Theory and Methods.* New York: Springer-Verlag, 1987.

[27] R. Burton and H. Dehling, "Large deviations for some weakly dependent random processes," *Statistics and Probability Letters*, vol. 9, pp. 397–401, 1990.

[28] S. Cambanis, C. Hardin, Jr., and A. Weron, "Ergodic properties of stationary stable processes," *Stochastic Processes and Their Applications*, vol. 24, pp. 1–18, 1987.

[29] S. Cambanis, C. Hardin, Jr., and A. Weron, "Innovations and Wold decompositions of stable sequences," *Probabilistic Theory of Relative Fields*, vol. 79, pp. 1–27, 1988.

[30] S. Cambanis and M. Maejima, "Two classes of self-similar stable processes with stationary increments," *Stochastic Processes and Their Applications*, vol. 32, pp. 305–329, 1989.

[31] S. Cambanis and A. Soltani, "Prediction of stable processes: Spectral and moving average representation," *Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete*, vol. 66, pp. 593–612, 1984.

[32] S. Cohen and G. Samorodnitsky, "Random rewards, fractional Brownian local times and stable self-similar processes," *Annals of Applied Probability*, vol. 16, pp. 1432–1461, 2006.

[33] R. Cont, "Long range dependence in financial markets," in *Fractals in Engineering*, (E. Lutton and J. Vehel, eds.), pp. 159–180, Springer, 2005.

[34] I. Cornfeld, S. Fomin, and Y. G. Sinai, *Ergodic Theory*. Springer-Verlag, 1982.

[35] D. Cox, "Long-range dependence: A review," in *Statistics: An Appraisal*, (H. David and H. David, eds.), pp. 55–74, Iowa State University Press, 1984.

[36] N. Crato and P. Rothman, "Fractional integration analysis of long-run behavior for US macroeconomic time series," *Economics Letters*, vol. 45, pp. 287–291, 1994.

[37] D. Daley and R. Vesilo, "Long range dependence of point processes, with queueing examples," *Stochastic Processes and Their Applications*, vol. 70, pp. 265–282, 1997.

[38] Y. Davydov and V. Egorov, "On convergence of empirical processes," Technical report, IRMA, Lille, vol. 64, no. V, 2005.

[39] L. Decreusefond and A. Üstünel, "Stochastic analysis of the Fractional Brownian motion," *Potential Analysis*, vol. 10, pp. 177–214, 1999.

[40] A. Dembo and O. Zeitouni, *Large Deviations Techniques and Applications*. Boston: Jones and Bartlett Publishers, 1993.

[41] J.-D. Deuschel and D. Stroock, *Large Deviations*. Boston: Academic Press, 1989.

[42] F. Diebold and A. Inoue, "Long memory and regime switching," *Journal of Econometrics*, vol. 105, pp. 131–159, 2001.

[43] H. Djellout and A. Guillin, "Large and moderate deviations for moving average processes," *Annales de la Faculté des Sciences de Toulouse*, vol. X, pp. 23–31, 2001.

[44] R. Dobrushin, "Gaussian and their subordinated self-similar random generalized fields," *Annals of Probability*, vol. 7, pp. 1–28, 1979.

[45] R. Dobrushin and P. Major, "Non-central limit theorems for non-linear functions of Gaussian fields," *Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete*, vol. 50, pp. 27–52, 1979.

[46] R. Douc, F. Roueff, and P. Soullier, "On the existence of some ARCH(∞) processes," *Stochastic Processes and Their Applications*, To appear, 2006.

[47] R. Dudley, "The sizes of compact subsets of Hilbert space and continuity of Gaussian processes," *Journal of Functional Analysis*, vol. 1, pp. 290–330, 1967.

[48] R. Durrett, *Probability: Theory and Examples*. Belmont, California: Duxbury Press, Second ed., 1996.

[49] R. Durrett and S. Resnick, "Functional limit theorems for dependent variables," *Annals of Probability*, vol. 6, pp. 829–846, 1978.

[50] P. Embrechts and M. Maejima, *Selfsimilar Processes*. Princeton and Oxford: Princeton University Press, 2002.

[51] R. Engle, "Autoregressive conditional heteroskedasticity with estimates of the variance of UK inflation," *Econometrica*, vol. 50, pp. 987–1008, 1982.

[52] P. Erdós and A. Rényi, "On a new law of large numbers," *Journal d'Analyse Mathématique*, vol. 22, pp. 103–111, 1970.

[53] W. Feller, *An Introduction to Probability Theory and its Applications*. Vol. 2, New York: Wiley, Second ed., 1971.

[54] S. Ghosh and G. Samorodnitsky, "Large deviations and memory in moving average processes," Technical report, 2007.

[55] L. Gil-Alana, "A fractionally integrated model for the Spanish real GDP," *Economics Bulletin*, vol. 3, pp. 1–6, 2004.

[56] L. Giraitis and D. Surgailis, "On shot noise processes attracted to fractional Lévy motion," in *Stable Processes and Related Topics*, (S. Cambanis, G. Samorodnitsky, and M. Taqqu, eds.), pp. 261–273, Boston: Birkhäuser, 1991. (Volume 25 of *Progress in Probability*).

[57] C. Granger and R. Joyeux, "An introduction to long-memory time series and fractional differencing," *Journal of Time Series Analysis*, vol. 1, pp. 15–30, 1980.

[58] D. Guégan, "How can we define the concept of long memory? An econometric survey," *Econometric reviews*, vol. 24, pp. 113–149, 2005.

[59] P. Hall and C. Heyde, *Martingale Limit Theory and Its Applications*. New York: Academic Press, 1980.

[60] E. J. Hannah, "The central limit theorem for time series regression," *Stochastic Processes and Their Applications*, vol. 9, pp. 281–289, 1979.

[61] M. Hauser and R. Kunst, "Fractionally integrated models with ARCH errors: With an application to the Swiss 1-month Euromarket interest rate," *Review of Quantitative Finance and Accounting*, vol. 10, pp. 95–113, 1998.

[62] H. Helson and D. Sarason, "Past and future," *Mathematica Scandinavica*, vol. 21, pp. 5–16, 1967.

[63] C. Heyde, "On the modes of long-range dependence," *Journal of Applied Probability*, vol. 39, pp. 882–888, 2002.

[64] C. Heyde and Y. Yang, "On defining long range dependence," *Journal of Applied Probability*, vol. 34, pp. 939–944, 1997.

[65] J. Hosking, "Fractional differencing," *Biometrika*, vol. 68, pp. 165–176, 1981.

[66] H. Hult, F. Lindskog, T. Mikosch, and G. Samorodnitsky, "Functional large deviations for multivariate regularly varying random walks," *Annals of Applied Probability*, vol. 15, pp. 2651–2680, 2005.

[67] H. Hurst, "Long-term storage capacity of reservoirs," *Transactions of the American Society of Civil Engineers*, vol. 116, pp. 770–808, 1951.

[68] H. Hurst, "Methods of using long-term storage in reservoirs," *Proceedings of the Institution of Civil Engineers, Part I*, pp. 519–577, 1955.

[69] T. Jiang, M. Rao, and X. Wang, "Large deviations for moving average processes," *Stochastic Processes and Their Applications*, vol. 59, pp. 309–320, 1995.

[70] T. Karagiannis, M. Molle, and M. Faloutsos, "Long-range dependence — Ten years of Internet traffic modeling," *IEEE Internet Computing*, vol. 8, pp. 57–64, 2004.

[71] D. Karmeshu and A. Krishnamachari, "Sequence variability and long-range dependence in DNA: An information theoretic perspective," in *Neural Information Processing*, pp. 1354–1361, Berlin: Springer, 2004. 3316 of Lecture Notes in Computer Science.

[72] Y. Kasahara and M. Maejima, "Weighted sums of i.i.d. random variables attracted to integrals of stable processes," *Probability Theory Related Fields*, vol. 78, pp. 75–96, 1988.

[73] Y. Kasahara, M. Maejima, and W. Vervaat, "Log-fractional stable processes," *Stochastic Processes and their Applications*, vol. 30, pp. 329–339, 1988.

[74] P. Kokoszka and M. Taqqu, "Infinite variance stable ARMA processes," *Journal of Time Series Analysis*, vol. 15, pp. 203–220, 1994.

[75] A. Kolmogorov, "Wienersche Spiralen und einige andere interessante kurven in Hilbertschen raum," *Computes Rendus (Doklady) Academic Sciences USSR* (*N.S.*), vol. 26, pp. 115–118, 1940.

[76] A. Kolmogorov and Y. Rozanov, "On a strong mixing condition for a stationary Gaussian process," *Teoria Veroyatnostei i Primeneniya*, vol. 5, pp. 222–227, 1960.

[77] M. Kotulska, "Natural fluctuations of an elctropore show Fractional Lévy Stable motion," *Biophysical Journal*, vol. 92, pp. 2412–2421, 2007.

[78] U. Krengel, *Ergodic Theorems*. Berlin, New York: De Gruyter, 1985.

[79] H. Künsch, "Discrimination between monotonic trends and long-range dependence," *Journal of Applied Probability*, vol. 23, pp. 1025–1030, 1986.

[80] J. Lamperti, "Semi-stable stochastic processes," *Transaction of the American Mathematical Society*, vol. 104, pp. 62–78, 1962.

[81] M. Leadbetter, G. Lindgren, and H. Rootzén, *Extremes and Related Properties of Random Sequences and Processes*. New York: Springer Verlag, 1983.

[82] W. Leland, M. Taqqu, W. Willinger, and D. Wilson, "On the self-similar nature of Ethernet traffic," *ACM/SIGCOMM Computer Communications Review*, pp. 183–193, 1993.

[83] J. Levy and M. Taqqu, "Renewal reward processes with heavy-tailed inter-renewal inter-renewal times and heavy-tailed rewards," *Bernoulli*, vol. 6, pp. 23–44, 2000.

[84] A. Lo, "Fat tails, long memory, and the stock market since the 1960s," *Economic Notes,* Banca Monte dei Paschi di Siena, 2001.

[85] J. Lopez-Ardao, C. Lopez-Garcia, A. Suarez-Gonzales, M. Fernandes-Vega, and R. Rodriguez-Rubio, "On the use of self-similar processes in network simulation," *ACM Transactions on Modeling and Computer Simulation*, vol. 10, pp. 125–151, 2000.

[86] M. Maejima, "On a class of self-similar processes," *Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete*, vol. 62, pp. 235–245, 1983.

[87] M. Maejima, "A self-similar process with nowhere bounded sample paths," *Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete*, vol. 65, pp. 115–119, 1983.

[88] P. Major, *Multiple Wiener-Itô Integrals*. New York: Springer Verlag, 1981. (Volume 849 of *Springer Lecture Notes in Mathematics*).
[89] B. Mandelbrot, "Une classe de processus stochastiques homothetiques a soi; application a loi climatologique de H. E. Hurst," *Comptes Rendus Academic Sciences Paris*, vol. 240, pp. 3274–3277, 1965.
[90] B. Mandelbrot, *The Fractal Geometry of Nature*. San Francisco: W. H. Freeman and Co., 1983.
[91] B. Mandelbrot and M. Taqqu, "Robust R/S analysis of long-run serial correlation," in *Proceedings of the 42nd Session of the International Statistical Institute*, pp. 69–104, Manila: Bulletin of the I.S.I., 1979.
[92] B. Mandelbrot and J. Van Ness, "Fractional Brownian motions, fractional noises and applications," *SIAM Review*, vol. 10, pp. 422–437, 1968.
[93] B. Mandelbrot and J. Wallis, "Noah, Joseph and operational hydrology," *Water Resources Research*, vol. 4, pp. 909–918, 1968.
[94] P. Mansfield, S. Rachev, and G. Samorodnitsky, "Long strange segments of a stochastic process and long range dependence," *Annals of Applied Probability*, vol. 11, pp. 878–921, 2001.
[95] M. Marcus and G. Pisier, "Characterization of almost surely continuous p-stable random Fourier series and strongly stationary processes," *Acta Mathematica*, vol. 152, pp. 245–301, 1984.
[96] F. Merlevéde, M. Peligrad, and S. Utev, "Recent advances in invariance principles for stationary sequences," *Probability Surveys*, vol. 3, pp. 1–36, 2006.
[97] Z. Michna, "Self-similar processes in collective risk theory," *Journal of Applied Mathematics and Stochastic Analysis*, vol. 11, pp. 429–448, 1998.
[98] T. Mikosch and G. Samorodnitsky, "The supremum of a negative drift random walk with dependent heavy–tailed steps," *Annals of Applied Probability*, vol. 10, pp. 1025–1064, 2000.
[99] T. Mikosch and C. Stărică, "Change of structure in financial time series, long range dependence and the GARCH model," IWI–preprint 99-5-06 of the University of Groningen, 1999.
[100] T. Mikosch and C. Stărică, "Is it long memory or just changing variance we see in financial returns?," *Bulletin of EU & US Inflation and Microeconomic Analysis*, vol. 68, pp. 46–68, 2000.
[101] G. Miller, "Properties of certain symmetric stable distributions," *Journal of Multivariate Analysis*, vol. 8, pp. 346–360, 1978.
[102] A. Mogulski, "Large deviations for trajectories of multidimensional random walks," *Theory of Probability and its Applications*, vol. 21, pp. 300–315, 1976.
[103] P. Moran, "On the range of cumulative sums," *Annals of the Institute of Statistical Mathematics*, vol. 16, pp. 109–112, 1964.
[104] T. Mori and H. Oodaira, "The law of the iterated logarithm for self-similar processes represented by multiple Wiener integrals," *Probability Theory Related Fields*, vol. 71, pp. 367–391, 1986.
[105] S. Nagaev, "Large deviations of sums of independent random variables," *Annals of Probability*, vol. 7, pp. 745–789, 1979.
[106] C. Newman and A. Wright, "An invariance principle for certain dependent sequences," *Annals of Probability*, vol. 9, pp. 671–675, 1981.

[107] I. Norros, E. Valkeila, and J. Virtamo, "An elementary approach to a Girsanov formula and other analytical results on fractional Brownian motion," *Bernoulli*, vol. 5, pp. 571–588, 1999.

[108] D. Nualart, *The Mallavin Calculus and Related Topics*. New York: Springer, 1995.

[109] S. Painter, "Long-range dependence in the subsurface: Empirical evidence and simulation methods," Invited paper at the American Geophysical Union 1998 Fall Meeting, 1998.

[110] M. Peligrad, "Maximum of partial sums and an invariance principle for a class of weak dependent random variables," *Proceedings of American Mathematical Society*, vol. 126, pp. 1181–1189, 1998.

[111] P. Phillips, "Time series regression with a unit root," *Econometrica*, vol. 55, pp. 277–301, 1987.

[112] S. Rachev and G. Samorodnitsky, "Long strange segments in a long range dependent moving average," *Stochastic Processes and Their Applications*, vol. 93, pp. 119–148, 2001.

[113] S. Resnick, "Point processes, regular variation and weak convergence," *Advances in Applied Probability*, vol. 18, pp. 66–138, 1986.

[114] S. Resnick, *Extreme Values, Regular Variation and Point Processes*. New York: Springer-Verlag, 1987.

[115] P. Robinson, ed., *Time Series with Long Memory*. Advanced Texts in Econometrics. Oxford University Press, 2003.

[116] M. Rosenblatt, "A central limit theorem and a strong mixing condition," *Proceedings of the National Academy of Sciences*, vol. 42, pp. 43–47, 1956.

[117] M. Rosenblatt, "Some limit theorems for partial sums of quadratic forms in stationary Gaussian variables," *Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete*, vol. 49, pp. 125–132, 1979.

[118] M. Rosenblatt, "Stochastic processes with short-range and long-range dependence," in *Statistics: An Appraisal*, (H. David and H. David, eds.), pp. 509–520, Iowa State University Press, 1984.

[119] M. Rosenblatt, *Stationary Sequences and Random Fields*. Boston: Birkhauser, 1985.

[120] J. Rosiński, "On the structure of stationary stable processes," *The Annals of Probability*, vol. 23, pp. 1163–1187, 1995.

[121] J. Rosiński, G. Samorodnitsky, and M. Taqqu, "Sample path properties of stochastic processes represented as multiple stable integrals," *Journal of Multivariate Analysis*, vol. 37, pp. 115–134, 1991.

[122] J. Rosiński and T. Żak, "Simple conditions for mixing of infinitely divisible processes," *Stochastic Processes and Their Applications*, vol. 61, pp. 277–288, 1996.

[123] J. Rosiński and T. Żak, "The equivalence of ergodicity and weak mixing for infinitely divisible processes," *Journal of Theoretical Probability*, vol. 10, pp. 73–86, 1997.

[124] G. Samorodnitsky, "Extreme value theory, ergodic theory, and the boundary between short memory and long memory for stationary stable processes," *Annals of Probability*, vol. 32, pp. 1438–1468, 2004.

[125] G. Samorodnitsky, "Long memory and self-similar processes," *Annales de la Faculté des Sciences de Toulouse*, vol. 15, pp. 107–123, 2006.
[126] G. Samorodnitsky and M. Taqqu, "The various linear fractional Lévy motions," in *Probability, Statistics and Mathematics: Papers in Honor of Samuel Karlin*, (K. A. T. W. Anderson and D. Iglehart, eds.), pp. 261–270, Boston: Academic Press, 1989.
[127] G. Samorodnitsky and M. Taqqu, "$(1/\alpha)$-self-similar processes with stationary increments," *Journal of Multivariate Analysis*, vol. 35, pp. 308–313, 1990.
[128] G. Samorodnitsky and M. Taqqu, *Stable Non-Gaussian Random Processes*. New York: Chapman and Hall, 1994.
[129] K. Sato, *Lévy Processes and Infinitely Divisible Distributions*. Cambridge University Press, 1999.
[130] J. Smit, "Solution to Problem 130," *Statistica Neerlandica*, vol. 37, p. 87, 1983.
[131] A. Stegeman, "Non-stationarity versus long-range dependence in computer network traffic measurements," Technical report, University of Groningen, 2001.
[132] S. Stoev and M. Taqqu, "Simulation methods for linear fractional stable motions and FARIMA using the Fast Fourier Transform," *Fractals*, vol. 1, pp. 95–121, 2004.
[133] D. Surgailis, "Convergence of sums of nonlinear functions of moving averages to selfsimilar processes," *Soviet Mathematique Doklady*, vol. 23, pp. 247–250, 1981.
[134] D. Surgailis, "On L^2 and non-L^2 multiple stochastic integration," in *Stochastic Differential Systems*, (Lecture Notes in Control and Information Sciences), pp. 212–226, New York: Springer-Verlag, 1981.
[135] D. Surgailis, J. Rosiński, V. Mandrekar, and S. Cambanis, "Stable mixed moving averages," *Probability Theory Related Fields*, vol. 97, pp. 543–558, 1993.
[136] M. Taqqu, "Weak convergence to fractional Brownian motion and to the Rosenblatt process," *Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete*, vol. 31, pp. 287–302, 1975.
[137] M. Taqqu, "A representation for self-similar processes," *Stochastic Processes and their Applications*, vol. 7, pp. 55–64, 1978.
[138] M. Taqqu, "Convergence of integrated processes of arbitrary Hermite rank," *Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete*, vol. 50, pp. 53–83, 1979.
[139] M. Taqqu, "A bibliographical guide to self-similar processes and long-range dependence," in *Dependence in Probability and Statistics*, (E. Eberlein and M. Taqqu, eds.), pp. 137–162, Boston: Birkhäuser, 1986.
[140] M. Taqqu, "Random processes with long-range dependence and high variability," *Journal of Geophysical Research*, vol. 92, pp. 9683–9686, 1987.
[141] M. Taqqu and R. Wolpert, "Infinite variance self-similar processes subordinate to a Poisson measure," *Zeitschrift für Wahrscheinlichkeitstheorie und verwandte Gebiete*, vol. 62, pp. 53–72, 1983.

[142] C. Varotsos and D. Kirk-Davidoff, "Long-memory processes in global ozone and temperature variations at the region 60° S–60° N," *Atmospheric Chemistry and Physics*, vol. 6, pp. 4093–4100, 2006.

[143] W. Vervaat, "Sample paths of self-similar processes with stationary increments," *The Annals of Probability*, vol. 13, pp. 1–27, 1985.

[144] N. Wiener, "The homogeneous chaos," *American Journal of Mathematics*, vol. 60, pp. 897–936, 1938.

[145] W. Willinger, V. Paxson, and M. Taqqu, "Self-similarity and heavy tails: Structural modeling of network traffic," in *A Practical Guide to Heavy Tails*, (R. Adler, R. Feldman, and M. Taqqu, eds.), pp. 27–54, Boston: Birkhäuser, 1998.

[146] W. Willinger, M. Taqqu, R. Sherman, and D. Wilson, "Self–similarity through high variability: Statistical analysis of ethernet LAN traffic at the source level (extended version)," *IEEE/ACM Transactions on Networking*, vol. 5, pp. 71–96, 1997.

[147] L. Wu, "On large deviations for moving average processes," in *Probability, Finance and Insurance*, pp. 15–49, River Edge, NJ: World Scientific Publishers, 2004.

[148] A. Yaglom, "Correlation theory of processes with stationary random increments of order n," *Matematica Sbornik*, vol. 37, pp. 141–196, English translation in *American Mathematical Society of Translations Series 2*, vol. 8, pp. 87-141, 1958, 1955.

[149] P. Zaffaroni, "Stationarity and memory of ARCH(∞) models," *Econometric Theory*, vol. 20, pp. 147–160, 2004.

[150] A. Zygmund, *Trigonometric Series.* Vol. I,II, Cambridge: Cambridge University Press, 1968.