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Control and State Estimation for max-plus Linear Systems

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Contents

Control and State Estimation for max-plus Linear Systems

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ABSTRACT

Max-plus linear systems theory was inspired by and originated from classical linear systems theory more than three decades ago, with the purpose of dealing with nonlinear synchronization and delay phenomena in timed discrete event systems in a linear manner. Timed discrete event systems are driven by discrete events, are equipped with a notion of time, and their temporal evolution is entirely characterized by the occurrence of events over time. If their behavior is completely governed by synchronization and delay phenomena, timed discrete event systems can be modeled as max-plus linear systems. On appropriate levels of abstraction, such systems adequately describe many problems in diverse areas such as manufacturing, communication, or transportation networks. The aim of this paper is to provide a thorough survey of current research work in max-plus linear systems. It summarizes the main mathematical concepts required for a theory of max-plus linear systems, including idempotent semirings, residuation theory, fixed point equations in the max-plus algebra, formal power series, and timed-event

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graphs. The paper reviews some recent major achievements in control and state estimation of max-plus linear systems. These include max-plus observer design, max-plus model matching by output or state feedback and observer-based control synthesis. Control is required to be optimal with respect to the so-called just-in-time criterion, which is a common standard in industrial engineering. It implies that the time for all input events is delayed as much as possible while guaranteeing that all output events occur, at the latest, at pre-specified reference times.

1

Introduction

Discrete event systems (DESs) are typically understood as event-driven systems whose state evolutions are completely characterized by the occurrence of discrete events over time. They often provide an adequate level of abstraction when modeling manufacturing systems (e.g., Cohen *et al.*, 1983; Cohen *et al.*, 1985), computer networks (e.g., Cruz, 1991; Boudec and Thiran, 2002) or transportation systems (e.g., Braker, 1993; Farhi *et al.*, 2005; Heidergott *et al.*, 2006; Lotito *et al.*, 2001; Olsder *et al.*, 1998). The diversity of phenomena observed in this class of systems led to the emergence of different modeling frameworks such as finite automata (e.g., Hopcroft *et al.*, 2006), Markov chains (e.g., Norris, 1997), and Petri nets (e.g., Reisig, 1985; Murata, 1989). In the context of control of DESs, Cassandras and Lafortune, 2006 and Seatzu *et al.*, 2012 provide extensive surveys on different modeling paradigms. Timed Event Graphs (TEGs) are a subclass of timed Petri nets where the occurrence of events only depends on delay and synchronization phenomena. The latter, when described in standard algebra, are highly nonlinear. Motivated by this, a special algebra, called max-plus algebra, has been suggested, in which these phenomena are linear. For more than three decades, researchers (e.g., Baccelli *et al.*, 1992; Cohen *et al.*, 1998) have been working to establish a linear systems and control theory in this algebra. Probably the first work on manufacturing systems described in this algebraic framework is due to R.A. Cuninghame-Green (Cuninghame-Green, 1962). In 1981 (see Cohen *et al.*, 1999 for a historical review), the Max-Plus working group of the INRIA started to develop a control theory for dynamical systems that are linear in the max-plus algebra. The underlying idea behind these developments is that, by changing the algebra, the behavior of certain discrete event systems can be described by linear equations. This, in turn, can then be exploited for analysis and control synthesis purposes. Hence, metaphorically speaking, by changing one's glasses, it is possible to reexamine a nonlinear world in a linear way. However, there is a price to be paid. Classical control theory is built on powerful mathematical concepts such as linear algebra and vector spaces. In contrast, the max-plus algebra is a weaker structure, namely an idempotent semiring, or dioid. This implies that addition in this algebra (which corresponds to the standard maximum operation) is not invertible. Despite this detriment, it has been possible to develop a rather elegant control theory for dynamical systems that are linear in the max-plus algebra, and several control strategies have been proposed for this class of systems. Examples are optimal open loop control (Cohen *et al.*, 1999; Lhommeau *et al.*, 2005; Menguy *et al.*, 2000) and optimal state and output feedback control in order to solve the model matching problem (Cottenceau *et al.*, 2001b; Lhommeau *et al.*, 2003a; Maia *et al.*, 2003; Maia *et al.*, 2005; Maia *et al.*, 2011), as well as control strategies forcing the state to stay in a specified set (Amari *et al.*, 2012; Katz, 2007; Maia *et al.*, 2005; Necoara *et al.*, 2009).

This paper provides an overview of the max-plus linear systems theory elaborated in the past three decades, especially with respect to the just-in-time criterion, a common standard in industrial engineering. Optimality, in this criterion, means that all input events are delayed as much as possible while ensuring that the output events occur at or before pre-specified reference times.

The paper is organized as follows: in Section 2, a motivational example is introduced. It represents a simple manufacturing system, and it will be used throughout this paper to illustrate the main concepts developed in subsequent sections. Section 3 briefly summarizes timed event graphs, the class of discrete event systems that is investigated in this paper. In this class, the occurrence of discrete events is only governed by delay and synchronization phenomena. Using the example introduced in Section 2, it is shown how to derive equations that describe the temporal evolution of timed event graphs.

In the following sections, the main mathematical foundations for developing a systems and control theory for max-plus linear systems are summarized. Section 4 provides the necessary algebraic background.

Section 5 investigates maps between idempotent semirings and their properties. Section 6 presents useful mathematical results dealing with fixed point equations in the max-plus algebra. Section 7 reviews residuation theory, which plays an essential role in the process of establishing a max-plus linear systems and control theory. Section 8 presents idempotent semirings of formal power series in the event domain. They prove particularly useful for deriving compact models for TEGs. This is discussed in some detail in Section 9.

The main part of this paper reviews some recent major achievements in control and estimation of max-plus linear systems. Section 10 is dedicated to the state estimation problem in max-plus linear systems, and an observer design inspired by Luenberger's approach (Luenberger, 1971) is presented. Section 11 discusses how to synthesize open-loop and closed-loop (both output and state feedback) control by solving an optimization problem with constraints. Optimality is in the sense of the well-known just-in-time criterion while the constraints reflect requirements imposed by a model matching, or model reference, problem (Hardouin *et al.*, 2011; Maia *et al.*, 2003; Maia *et al.*, 2005). Section 12 introduces an observer-based controller for the case when the state of the plant is not completely measurable or when it is too expensive to measure all the states. The resulting observer-based controller is compared with the output feedback and state-feedback controllers described in Section 11. It turns out that the proposed observer-based controller in general indeed provides better performance than an output feedback controller. Section 13 discusses how various control problems can be posed as specific model matching problems by setting up appropriate reference models. Finally, Section 14 illustrates the main results of this paper for the running manufacturing system example.

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