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Stability, Optimality and Performance

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Contents

Economic Nonlinear Model Predictive Control

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ABSTRACT

In recent years, Economic Model Predictive Control (EMPC) has received considerable attention of many research groups. The present tutorial survey summarizes state-of-the-art approaches in EMPC. In this context EMPC is to be understood as receding-horizon optimal control with a stage cost that does not simply penalize the distance to a desired equilibrium but encodes more sophisticated economic objectives. This survey provides a comprehensive overview of EMPC stability results: with and without terminal constraints, with and without dissipativtiy assumptions, with averaged constraints, formulations with multiple objectives and generalized terminal constraints as well as Lyapunov-based approaches. Moreover, we compare different performance criteria for some of the considered approaches and comment on the connections to recent research on dissipativity of optimal control problems. We consider a discrete-time setting and point towards continuous-time variants. We illustrate the different EMPC schemes with several examples.

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1

Introduction

The principle idea of Model Predictive Control (MPC) can be dated back to the 1960s, when [76] as well as [56] suggested receding-horizon solutions of Optimal Control Problems (OCP). While MPC saw its first applications in petro-chemical industries in the 1970s, by now a mature body of knowledge encompasses stability and robustness of linear and nonlinear MPC,[1] strategies and tools for efficient numerical implementation ranging from sub-microseconds for small scale linear-quadratic MPC to handling of strong nonlinearities, differential-algebraic dynamics and partial-differential equations in real-time feasible implementations. Several monographs provide overviews on the field of MPC, see [77, 39, 20]. In other words, nowadays MPC can be regarded as mature control method, which has *had significant impact on industrial process control*, cf. [61, p. *xi*].

Standard control tasks frequently solved with NMPC include *setpoint regulation* and *trajectory tracking*, whereby the former refers to the stabilization of known setpoints defined in the state-space or some

[1] In the literature, MPC often refers to the a setting with linear systems, convex quadratic objective and linear constraints while NMPC, which stands for Nonlinear Model Predictive Control, highlights the presence of nonlinear dynamics and non-convex constraints.

output space and the latter refers to the task of tracking time-dependent reference trajectories. However, even before first stability results on NMPC with state and input constraints were available, it has been observed by [66] that

> *[in] attempting to synthesize a feedback optimizing control structure, our main objective is to translate the economic objective into process control objectives.*

The classical way to tackle this problem of designing economically beneficial control schemes is by means of the so-called control pyramid, wherein real-time optimization is used to compute economically desirable targets, which are then passed to the advanced process control, i.e. the MPC layer, [21]. In other words, classically economic targets are translated into setpoints and reference trajectories, which are in turn stabilized by control techniques such as MPC. If indeed MPC is used to track these targets, then it is natural that the MPC objective penalizes mainly the deviation from the desired setpoint. It is this setting of setpoint regulation and tracking in which the vast majority of results on MPC stability and robustness of are formulated, cf. [64, 39, 77], and which is used frequently in industrial practice. At the same time, in process systems engineering and other fields of application, one aims at economic process operation. Hence, in the view of the quote from [66] given above, the question of closed-loop properties of receding-horizon optimal control with generic or economic objectives is very natural. In the process control community this issue has been addressed using the label *Dynamic Real Time Optimization* [50], while in [3, 4] the term *Economic Model Predictive Control* (EMPC) has been coined.

The present survey provides a concise overview of different approaches on the question of stability and optimality in different formulations of EMPC. In contrast to previous overviews on the same topic [19], we cover approaches both with and without terminal constraints and end penalties, and turnpike/dissipativity-based settings as well as Lyapunov-based approaches.

1.1 Outline

In Section 2 we recall important stability results for stabilizing NMPC. Section 3 analyzes the stability of EMPC based on dissipativity properties and terminal constraints. Section 4 investigates the counterpart without terminal constraints and penalties. In Section 5 we provide an overview of performance bounds for the EMPC schemes from Section 3 and Section 4.

EMPC with averaged constraints is discussed in Section 6, while Section 7 revisits generalized terminal constraints. Lyapunov-based approaches and multi-objective approaches are presented in Section 8 and Section 9, respectively. This survey ends with conclusions and an outlook on open issues in Section 10.

1.2 Notation

Throughout this review, we use the following notation: Real vectors are denoted by Latin letters, i.e. $x \in \mathbb{R}^{n_x}, u \in \mathbb{R}^{n_u}$. The two-norm of any vector $x \in \mathbb{R}^{n_x}$ is $\|x\|$.

Consider a discrete-time system $x(t+1) = f(x(t), u(t))$ with $f : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \to \mathbb{R}^{n_x}$. The trajectory originating from x_0 driven by the input $u(\cdot)$ is written as $x(\cdot; x_0, u(\cdot))$. Whenever the control sequence is clear from context, we write simply $x(\cdot; x_0)$.

We will use the following standard classes of comparison functions:

- $\mathcal{L} := \Big\{\gamma : \mathbb{R}_0^+ \to \mathbb{R}_0^+ \,|\, \gamma \text{ continuous and decreasing with } \lim_{k\to\infty} \gamma(k) = 0\Big\}$
- $\mathcal{K} := \{\alpha : \mathbb{R}_0^+ \to \mathbb{R}_0^+ \,|\, \alpha \text{ continuous and strictly increasing with } \alpha(0) = 0\}$
- $\mathcal{K}_\infty := \{\alpha \in \mathcal{K} \,|\, \alpha \text{ unbounded}\}$
- $\mathcal{KL} := \{\beta : \mathbb{R}_0^+ \times \mathbb{R}_0^+ \to \mathbb{R}_0^+ \,|\, \beta(\cdot, k) \in \mathcal{K}, \beta(r, \cdot) \in \mathcal{L}\}$.

We refer to [52] for a detailed overview of properties of these comparison functions.

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