

Economic Nonlinear Model Predictive Control

Stability, Optimality and Performance

Other titles in Foundations and Trends® in Systems and Control

Nonlinear Model Reduction by Moment Matching

Giordano Scarciootti and Alessandro Astolfi

ISBN: 978-1-68083-330-0

Logical Control of Complex Resource Allocation Systems

Spyros Reveliotis

ISBN: 978-1-68083-250-1

Observability of Hybrid Dynamical Systems

Elena De Santis and Maria Domenica Di Benedetto

ISBN: 978-1-68083-220-4

Operator Splitting Methods in Control

Giorgos Stathopoulos, Harsh Shukla, Alexander Szucs, Ye Pu and
Colin N. Jones

ISBN: 978-1-68083-174-0

Sensor Fault Diagnosis

Vasso Reppa, Marios M. Polycarpou and Christos G. Panayiotou

ISBN: 978-1-68083-128-3

Economic Nonlinear Model Predictive Control

Stability, Optimality and Performance

Timm Faulwasser

Karlsruhe Institute of Technology (KIT)
timm.faulwasser@kit.edu

Lars Grüne

University of Bayreuth
lars.gruene@uni-bayreuth.de

Matthias A. Müller

University of Stuttgart
matthias.mueller@ist.uni-stuttgart.de

now

the essence of knowledge

Boston — Delft

Foundations and Trends[®] in Systems and Control

Published, sold and distributed by:

now Publishers Inc.
PO Box 1024
Hanover, MA 02339
United States
Tel. +1-781-985-4510
www.nowpublishers.com
sales@nowpublishers.com

Outside North America:

now Publishers Inc.
PO Box 179
2600 AD Delft
The Netherlands
Tel. +31-6-51115274

The preferred citation for this publication is

T. Faulwasser, L. Grüne and M. A. Müller. *Economic Nonlinear Model Predictive Control*. Foundations and Trends[®] in Systems and Control, vol. 5, no. 1, pp. 1–98, 2018.

ISBN: 978-1-68083-393-5

© 2018 T. Faulwasser, L. Grüne and M. A. Müller

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, mechanical, photocopying, recording or otherwise, without prior written permission of the publishers.

Photocopying. In the USA: This journal is registered at the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923. Authorization to photocopy items for internal or personal use, or the internal or personal use of specific clients, is granted by now Publishers Inc for users registered with the Copyright Clearance Center (CCC). The 'services' for users can be found on the internet at: www.copyright.com

For those organizations that have been granted a photocopy license, a separate system of payment has been arranged. Authorization does not extend to other kinds of copying, such as that for general distribution, for advertising or promotional purposes, for creating new collective works, or for resale. In the rest of the world: Permission to photocopy must be obtained from the copyright owner. Please apply to now Publishers Inc., PO Box 1024, Hanover, MA 02339, USA; Tel. +1 781 871 0245; www.nowpublishers.com; sales@nowpublishers.com

now Publishers Inc. has an exclusive license to publish this material worldwide. Permission to use this content must be obtained from the copyright license holder. Please apply to now Publishers, PO Box 179, 2600 AD Delft, The Netherlands, www.nowpublishers.com; e-mail: sales@nowpublishers.com

Foundations and Trends[®] in Systems and Control

Volume 5, Issue 1, 2018

Editorial Board

Editor-in-Chief

Panos J. Antsaklis
University of Notre Dame
United States

Alessandro Astolfi
Imperial College London, United Kingdom
University of Rome "Tor Vergata", Italy

Editors

John Baillieuli
Boston University

Dragan Nesic
The University of Melbourne

Peter Caines
McGill University

Marios Polycarpou
University of Cyprus

Christos Cassandras
Boston University

Jörg Raisch
Technical University Berlin

Denis Dochain
UC Louvain

Arjan van der Schaft
University of Groningen

Magnus Egerstedt
Georgia Institute of Technology

M. Elena Valcher
University of Padova

Karl Henrik Johansson
KTH Stockholm

Richard Vinter
Imperial College London

Miroslav Krstic
University of California, San Diego

George Weiss
Tel Aviv University

Jan Maciejowski
University of Cambridge

Editorial Scope

Topics

Foundations and Trends[®] in Systems and Control publishes survey and tutorial articles in the following topics:

- Control of:
 - Hybrid and Discrete Event Systems
 - Nonlinear Systems
 - Network Systems
 - Stochastic Systems
 - Multi-agent Systems
 - Distributed Parameter Systems
- Delay Systems
- Filtering, Estimation, Identification
- Optimal Control
- Systems Theory
- Control Applications

Information for Librarians

Foundations and Trends[®] in Systems and Control, 2018, Volume 5, 4 issues. ISSN paper version 2325-6818. ISSN online version 2325-6826. Also available as a combined paper and online subscription.

Contents

1	Introduction	2
1.1	Outline	4
1.2	Notation	4
2	Revisiting Stabilizing NMPC	5
2.1	Main Idea of NMPC	5
2.2	Stabilizing NMPC with Terminal Constraints	8
2.3	Stabilizing NMPC without Terminal Constraints	10
3	Economic MPC with Terminal Constraints	14
3.1	Dissipativity and Optimal Operation at Steady State	15
3.2	Closed-loop Stability	19
3.3	Example – Chemical Reactor with Dissipativity	21
3.4	Example – Chemical Reactor without Dissipativity	24
4	EMPC without Terminal Constraints and Penalties	29
4.1	The Turnpike Property	30
4.2	Recursive Feasibility	33
4.3	Practical Stability	35
4.4	Example – Chemical Reactor with Dissipativity	41

5	Performance Bounds	44
5.1	Averaged Performance	46
5.2	Non-averaged Performance	46
5.3	Transient Performance	47
5.4	Example – Chemical Reactor with Dissipativity	49
6	EMPC with Averaged Constraints	51
6.1	Asymptotic Average Constraints	52
6.2	Simple Example	58
6.3	Transient Average Constraints	59
6.4	Extensions	60
7	EMPC with Generalized Terminal Constraints	62
7.1	Problem Formulation and Performance Analysis	62
7.2	Self-tuning Terminal Weight	66
7.3	Discussion and Extensions	68
8	Lyapunov-based Approach	71
8.1	Basics of the Scheme	71
8.2	Closed-loop Properties	73
9	Multi-objective Approach	76
9.1	Derivation of the Scheme	76
9.2	Closed-loop Properties	79
9.3	Example – Chemical Reactor without Dissipativity	80
10	Conclusions and Outlook	83
10.1	Discussion	83
10.2	Further Results and Open Problems	86
	Acknowledgements	89
	References	90

Economic Nonlinear Model Predictive Control

Timm Faulwasser¹, Lars Grüne² and Matthias A. Müller³

¹*Karlsruhe Institute of Technology, Germany; timm.faulwasser@kit.edu*

²*University of Bayreuth, Germany; lars.gruene@uni-bayreuth.de*

³*University of Stuttgart, Germany;
matthias.mueller@ist.uni-stuttgart.de*

ABSTRACT

In recent years, Economic Model Predictive Control (EMPC) has received considerable attention of many research groups. The present tutorial survey summarizes state-of-the-art approaches in EMPC. In this context EMPC is to be understood as receding-horizon optimal control with a stage cost that does not simply penalize the distance to a desired equilibrium but encodes more sophisticated economic objectives. This survey provides a comprehensive overview of EMPC stability results: with and without terminal constraints, with and without dissipativity assumptions, with averaged constraints, formulations with multiple objectives and generalized terminal constraints as well as Lyapunov-based approaches. Moreover, we compare different performance criteria for some of the considered approaches and comment on the connections to recent research on dissipativity of optimal control problems. We consider a discrete-time setting and point towards continuous-time variants. We illustrate the different EMPC schemes with several examples.

1

Introduction

The principle idea of Model Predictive Control (MPC) can be dated back to the 1960s, when [76] as well as [56] suggested receding-horizon solutions of Optimal Control Problems (OCP). While MPC saw its first applications in petro-chemical industries in the 1970s, by now a mature body of knowledge encompasses stability and robustness of linear and nonlinear MPC,¹ strategies and tools for efficient numerical implementation ranging from sub-microseconds for small scale linear-quadratic MPC to handling of strong nonlinearities, differential-algebraic dynamics and partial-differential equations in real-time feasible implementations. Several monographs provide overviews on the field of MPC, see [77, 39, 20]. In other words, nowadays MPC can be regarded as mature control method, which has *had significant impact on industrial process control*, cf. [61, p. *xi*].

Standard control tasks frequently solved with NMPC include *setpoint regulation* and *trajectory tracking*, whereby the former refers to the stabilization of known setpoints defined in the state-space or some

¹In the literature, MPC often refers to the a setting with linear systems, convex quadratic objective and linear constraints while NMPC, which stands for Nonlinear Model Predictive Control, highlights the presence of nonlinear dynamics and non-convex constraints.

output space and the latter refers to the task of tracking time-dependent reference trajectories. However, even before first stability results on NMPC with state and input constraints were available, it has been observed by [66] that

[in] attempting to synthesize a feedback optimizing control structure, our main objective is to translate the economic objective into process control objectives.

The classical way to tackle this problem of designing economically beneficial control schemes is by means of the so-called control pyramid, wherein real-time optimization is used to compute economically desirable targets, which are then passed to the advanced process control, i.e. the MPC layer, [21]. In other words, classically economic targets are translated into setpoints and reference trajectories, which are in turn stabilized by control techniques such as MPC. If indeed MPC is used to track these targets, then it is natural that the MPC objective penalizes mainly the deviation from the desired setpoint. It is this setting of setpoint regulation and tracking in which the vast majority of results on MPC stability and robustness of are formulated, cf. [64, 39, 77], and which is used frequently in industrial practice. At the same time, in process systems engineering and other fields of application, one aims at economic process operation. Hence, in the view of the quote from [66] given above, the question of closed-loop properties of receding-horizon optimal control with generic or economic objectives is very natural. In the process control community this issue has been addressed using the label *Dynamic Real Time Optimization* [50], while in [3, 4] the term *Economic Model Predictive Control* (EMPC) has been coined.

The present survey provides a concise overview of different approaches on the question of stability and optimality in different formulations of EMPC. In contrast to previous overviews on the same topic [19], we cover approaches both with and without terminal constraints and end penalties, and turnpike/dissipativity-based settings as well as Lyapunov-based approaches.

1.1 Outline

In Section 2 we recall important stability results for stabilizing NMPC. Section 3 analyzes the stability of EMPC based on dissipativity properties and terminal constraints. Section 4 investigates the counterpart without terminal constraints and penalties. In Section 5 we provide an overview of performance bounds for the EMPC schemes from Section 3 and Section 4.

EMPC with averaged constraints is discussed in Section 6, while Section 7 revisits generalized terminal constraints. Lyapunov-based approaches and multi-objective approaches are presented in Section 8 and Section 9, respectively. This survey ends with conclusions and an outlook on open issues in Section 10.

1.2 Notation

Throughout this review, we use the following notation: Real vectors are denoted by Latin letters, i.e. $x \in \mathbb{R}^{n_x}, u \in \mathbb{R}^{n_u}$. The two-norm of any vector $x \in \mathbb{R}^{n_x}$ is $\|x\|$.

Consider a discrete-time system $x(t+1) = f(x(t), u(t))$ with $f : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \rightarrow \mathbb{R}^{n_x}$. The trajectory originating from x_0 driven by the input $u(\cdot)$ is written as $x(\cdot; x_0, u(\cdot))$. Whenever the control sequence is clear from context, we write simply $x(\cdot; x_0)$.

We will use the following standard classes of comparison functions:

- $\mathcal{L} := \left\{ \gamma : \mathbb{R}_0^+ \rightarrow \mathbb{R}_0^+ \mid \gamma \text{ continuous and decreasing with } \lim_{k \rightarrow \infty} \gamma(k) = 0 \right\}$
- $\mathcal{K} := \{ \alpha : \mathbb{R}_0^+ \rightarrow \mathbb{R}_0^+ \mid \alpha \text{ continuous and strictly increasing with } \alpha(0) = 0 \}$
- $\mathcal{K}_\infty := \{ \alpha \in \mathcal{K} \mid \alpha \text{ unbounded} \}$
- $\mathcal{KL} := \{ \beta : \mathbb{R}_0^+ \times \mathbb{R}_0^+ \rightarrow \mathbb{R}_0^+ \mid \beta(\cdot, k) \in \mathcal{K}, \beta(r, \cdot) \in \mathcal{L} \}$.

We refer to [52] for a detailed overview of properties of these comparison functions.

References

- [1] Alessandretti, A., A. Aguiar, and C. Jones. 2014. “An Economic Model Predictive Control scheme with terminal penalty for continuous-time systems”. In: *Proc. of the 53rd IEEE Conference on Decision and Control (CDC)*. 2728–2733.
- [2] Alessandretti, A., A. Aguiar, and C. Jones. 2016. “On convergence and performance certification of a continuous-time economic model predictive control scheme with time-varying performance index”. *Automatica*. 68: 305–313.
- [3] Amrit, R., J. Rawlings, and D. Angeli. 2011. “Economic optimization using model predictive control with a terminal cost”. *Annual Reviews in Control*. 35(2): 178–186. ISSN: 1367-5788.
- [4] Angeli, D., R. Amrit, and J. Rawlings. 2012. “On Average Performance and Stability of Economic Model Predictive Control”. *IEEE Transactions on Automatic Control*. 57(7): 1615–1626.
- [5] Bailey, J., F. Horn, and R. Lin. 1971. “Cyclic operation of reaction systems: Effects of heat and mass transfer resistance”. *AIChE Journal*. 17(4): 818–825.
- [6] Bayer, F. A., M. Lorenzen, M. A. Müller, and F. Allgöwer. 2016. “Robust economic Model Predictive Control using stochastic information”. *Automatica*. 74: 151–161.

- [7] Bayer, F. A., M. A. Müller, and F. Allgöwer. 2014. “Tube-based robust economic model predictive control”. *Journal of Process Control*. 24(8): 1237–1246.
- [8] Bertsekas, D. 1999. *Nonlinear Programming*. 2nd. Athena Scientific, Belmont, Massachusetts.
- [9] Böhm, C., R. Findeisen, and F. Allgöwer. 2008. “Avoidance of Poorly Observable Trajectories: A predictive control perspective”. In: *Proc. 17th IFAC World Congress*. Vol. 41. No. 2. Elsevier. 1952–1957.
- [10] Braun, P., L. Grüne, C. M. Kellett, S. R. Weller, and K. Worthmann. 2016. “A distributed optimization algorithm for the predictive control of smart grids”. *IEEE Transactions on Automatic Control*. 61(12): 3898–3911.
- [11] Broomhead, T. J., C. Manzie, R. C. Shekhar, and P. Hield. 2015. “Robust periodic economic MPC for linear systems”. *Automatica*. 60: 30–37.
- [12] Carlson, D., A. Haurie, and A. Leizarowitz. 1991. *Infinite Horizon Optimal Control: Deterministic and Stochastic Systems*. Springer Verlag.
- [13] Chen, H. and F. Allgöwer. 1998. “A quasi-infinite horizon nonlinear model predictive control scheme with guaranteed stability”. *Automatica*. 34(10): 1205–1217.
- [14] Damm, T., L. Grüne, M. Stieler, and K. Worthmann. 2014. “An exponential turnpike theorem for dissipative optimal control problems”. *SIAM Journal on Control and Optimization*. 52(3): 1935–1957.
- [15] Diehl, M., R. Amrit, and J. Rawlings. 2011. “A Lyapunov function for economic optimizing model predictive control”. *IEEE Transactions on Automatic Control*. 56(3): 703–707.
- [16] Dorfman, R., P. Samuelson, and R. Solow. 1958. *Linear Programming and Economic Analysis*. McGraw-Hill, New York.
- [17] Driessen, P. A. A., R. M. Hermans, and P. P. J. van den Bosch. 2012. “Distributed economic model predictive control of networks in competitive environments”. In: *Proceedings of the 51st IEEE Conference on Decision and Control*. 266–271.

- [18] Ebenbauer, C., T. Raff, and F. Allgöwer. 2009. “Dissipation inequalities in systems theory: An introduction and recent results”. In: *R. Jeltsch and G. Wanner (ed.), Invited Lectures of the International Congress on Industrial and Applied Mathematics 2007*. 23–42.
- [19] Ellis, M., H. Durand, and P. Christofides. 2014. “A tutorial review of economic model predictive control methods”. *Journal of Process Control*. 24(8): 1156–1178.
- [20] Ellis, M., M. Liu, and P. Christofides. 2017. *Economic Model Predictive Control: Theory, Formulations and Chemical Process Applications*. Springer, Berlin.
- [21] Engell, S. 2007. “Feedback control for optimal process operation”. *Journal of Process Control*. 17: 203–219.
- [22] Fagiano, L. and A. R. Teel. 2013. “Generalized terminal state constraint for model predictive control”. *Automatica*. 49(9): 2622–2631.
- [23] Faulwasser, T. and D. Bonvin. 2015a. “On the Design of Economic NMPC Based on an Exact Turnpike Property”. In: *Proceedings of the 9th IFAC Symposium on Advanced Control of Chemical Processes — ADCHEM 2015*. Vol. 48. No. 8. IFAC-PapersOnLine. 525–530.
- [24] Faulwasser, T. and D. Bonvin. 2015b. “On the Design of Economic NMPC based on Approximate Turnpike Properties”. In: *54th IEEE Conference on Decision and Control*. Osaka, Japan. 4964–4970.
- [25] Faulwasser, T. and D. Bonvin. 2017. “Exact Turnpike Properties and Economic NMPC”. *European Journal of Control*. 35(Feb.): 34–41.
- [26] Faulwasser, T., V. Hagenmeyer, and R. Findeisen. 2014a. “Constrained Reachability and Trajectory Generation for Flat Systems”. *Automatica*. 50(4): 1151–1159.

- [27] Faulwasser, T., M. Korda, C. Jones, and D. Bonvin. 2014b. “Turnpike and Dissipativity Properties in Dynamic Real-Time Optimization and Economic MPC”. In: *Proc. of the 53rd IEEE Conference on Decision and Control*. Los Angeles, California, USA. 2734–2739.
- [28] Faulwasser, T., M. Korda, C. Jones, and D. Bonvin. 2017. “On Turnpike and Dissipativity Properties of Continuous-Time Optimal Control Problems”. *Automatica*. 81: 297–304.
- [29] Ferramosca, A., D. Limon, I. Alvarado, T. Alamo, and E. F. Camacho. 2009. “MPC for tracking with optimal closed-loop performance”. *Automatica*. 45(8): 1975–1978.
- [30] Ferramosca, A., D. Limon, and E. F. Camacho. 2014. “Economic MPC for a Changing Economic Criterion for Linear Systems”. *IEEE Transactions on Automatic Control*. 59(10): 2657–2667.
- [31] Gaitsgory, V., A. Parkinson, and I. Shvartsman. 2017. “Linear programming formulations of deterministic infinite horizon optimal control problems in discrete time”. arXiv Preprint, arxiv:1702.00857.
- [32] Gaitsgory, V., L. Grüne, and N. Thatcher. 2015. “Stabilization with discounted optimal control”. *Systems & Control Letters*. 82: 91–98.
- [33] Grimm, G., M. J. Messina, S. E. Tuna, and A. R. Teel. 2005. “Model predictive control: for want of a local control Lyapunov function, all is not lost”. *IEEE Transactions on Automatic Control*. 50(5): 546–558.
- [34] Grüne, L. 2009. “Analysis and design of unconstrained nonlinear MPC schemes for finite and infinite dimensional systems”. *SIAM Journal on Control and Optimization*. 48(2): 1206–1228.
- [35] Grüne, L. 2013. “Economic receding horizon control without terminal constraints”. *Automatica*. 49(3): 725–734.
- [36] Grüne, L. 2016. “Approximation properties of receding horizon optimal control”. *Jahresbericht DMV*. 118(1): 3–37.
- [37] Grüne, L. and M. Müller. 2016. “On the relation between strict dissipativity and turnpike properties”. *Systems & Control Letters*. 90: 45–53.

- [38] Grüne, L. and A. Panin. 2015. “On non-averaged performance of economic MPC with terminal conditions”. In: *Proc. of the 54th IEEE Conference on Decision and Control*. 2740–2745.
- [39] Grüne, L. and J. Pannek. 2017. *Nonlinear Model Predictive Control. Theory and Algorithms*. 2nd ed. London: Springer-Verlag.
- [40] Grüne, L., W. Semmler, and M. Stieler. 2015. “Using nonlinear model predictive control for dynamic decision problems in economics”. *Journal of Economic Dynamics and Control*. 60: 112–133.
- [41] Grüne, L. and M. Stieler. 2014a. “A Lyapunov function for economic MPC without terminal conditions”. In: *Proc. of the 53rd IEEE Conference on Decision and Control*. 2740–2745.
- [42] Grüne, L. and M. Stieler. 2014b. “Asymptotic stability and transient optimality of economic MPC without terminal conditions”. *Journal of Process Control*. 24(8): 1187–1196.
- [43] Heidarinejad, M., J. Liu, and P. D. Christofides. 2012. “Economic model predictive control of nonlinear process systems using Lyapunov techniques”. *AIChE Journal*. 58(3): 855–870.
- [44] Hill, D. J. and P. J. Moylan. 1980. “Dissipative Dynamical Systems: Basic Input-Output and State Properties”. *Journal of the Franklin Institute*. 309(5): 327–357.
- [45] Houska, B., H. Ferreau, and M. Diehl. 2011. “ACADO toolkit – An open-source framework for automatic control and dynamic optimization”. *Optimal Control Applications and Methods*. 32(3): 298–312.
- [46] Houska, B. and M. A. Müller. 2017. “Cost-to-travel functions: a new perspective on optimal and model predictive control”. *System & Control Letters*. submitted.
- [47] Huang, R., L. T. Biegler, and E. Harinath. 2012. “Robust stability of economically oriented infinite horizon NMPC that include cyclic processes”. *Journal of Process Control*. 22(1): 51–59.
- [48] Jadbabaie, A. and J. Hauser. 2005. “On the stability of receding horizon control with a general terminal cost”. *IEEE Transactions on Automatic Control*. 50(5): 674–678.

- [49] Jadbabaie, A., J. Yu, and J. Hauser. 2001. "Unconstrained receding-horizon control of nonlinear systems". *IEEE Transactions on Automatic Control*. 46(5): 776–783.
- [50] Kadam, J. and W. Marquardt. 2007. "Integration of Economical Optimization and Control for Intentionally Transient Process Operation". English. In: *Assessment and Future Directions of Nonlinear Model Predictive Control*. Ed. by R. Findeisen, F. Allgöwer, and L. Biegler. Vol. 358. *Lecture Notes in Control and Information Sciences*. Springer Berlin Heidelberg. 419–434.
- [51] Keerthi, S. S. and E. G. Gilbert. 1988. "Optimal infinite horizon feedback laws for a general class of constrained discrete-time systems: stability and moving horizon approximations". *Journal of Optimization Theory and Applications*. 57: 265–293.
- [52] Kellett, C. 2014. "A compendium of comparison function results". *Mathematics of Control, Signals, and Systems*. 26(3): 339–374.
- [53] Klatt, K., S. Engell, A. Kremling, and F. Allgöwer. 1995. "Testbeispiel: Rührkesselreaktor mit Parallel- und Folgereaktion". In: *Entwurf nichtlinearer Regelungen*. Ed. by S. Engell. Oldenbourg-Verlag. 425–432.
- [54] Köhler, P. N., M. A. Müller, and F. Allgöwer. 2016. "A distributed economic MPC scheme for coordination of self-interested systems". In: *Proceedings of the American Control Conference*. 889–894.
- [55] Lee, C. K. and J. E. Bailey. 1980. "Modification of Consecutive-Competitive Reaction Selectivity by Periodic Opera". *Ind. Eng. Chem. Proc. Des. Dev.* 19: 160–166.
- [56] Lee, E. and L. Markus. 1967. *Foundations of Optimal Control Theory. The SIAM series in applied mathematics*. John Wiley & Sons New York, London, Sydney.
- [57] Lee, J. and D. Angeli. 2011. "Cooperative distributed model predictive control for linear plants subject to convex economic objectives". In: *Proceedings of the 50th IEEE Conference on Decision and Control and European Control Conference*. 3434–3439.
- [58] Limon, D., I. Alvarado, T. Alamo, and E. F. Camacho. 2008. "MPC for tracking piecewise constant references for constrained linear systems". *Automatica*. 44(9): 2382–2387.

- [59] Limon, D., M. Pereira, D. Muñoz de la Peña, T. Alamo, and J. M. Grosso. 2014. “Single-layer economic model predictive control for periodic operation”. *Journal of Process Control*. 24(8): 1207–1224.
- [60] Lucia, S., J. A. E. Andersson, H. Brandt, M. Diehl, and S. Engell. 2014. “Handling uncertainty in economic nonlinear model predictive control: A comparative case study”. *Journal of Process Control*. 24(8): 1247–1259. ISSN: 0959-1524.
- [61] Maciejowski, J. 2002. *Predictive control: with constraints*. Pearson Education Limited.
- [62] Maree, J. and L. Imsland. 2016. “Combined economic and regulatory predictive control”. *Automatica*. 69: 342–347.
- [63] Marquez, A., J. Patiño, and J. Espinosa. 2014. “Min-max Economic Model Predictive Control”. In: *Proceedings of the 53rd IEEE Conference on Decision and Control*. 4410–4415.
- [64] Mayne, D., J. Rawlings, C. Rao, and P. Scokaert. 2000. “Constrained model predictive control: Stability and optimality”. *Automatica*. 36(6): 789–814.
- [65] Michalska, H. and R. Vinter. 1994. “Nonlinear stabilization using discontinuous moving-horizon control”. *IMA Journal of Mathematical Control and Information*. 11(4): 321–340.
- [66] Morari, M., Y. Arkun, and G. Stephanopoulos. 1980. “Studies in the synthesis of control structures for chemical processes: Part I: Formulation of the problem. Process decomposition and the classification of the control tasks. Analysis of the optimizing control structures”. *AIChE Journal*. 26(2): 220–232.
- [67] Moylan, P. 2014. *Dissipative Systems and Stability*. Available from <http://www.pmoylan.org>.
- [68] Müller, M. A., D. Angeli, and F. Allgöwer. 2014a. “On the performance of economic model predictive control with self-tuning terminal cost”. *Journal of Process Control*. 24(8): 1179–1186.
- [69] Müller, M. A., D. Angeli, and F. Allgöwer. 2014b. “Transient average constraints in economic model predictive control”. *Automatica*. 50(11): 2943–2950.

- [70] Müller, M. A., D. Angeli, F. Allgöwer, R. Amrit, and J. B. Rawlings. 2014c. “Convergence in economic model predictive control with average constraints”. *Automatica*. 50(12): 3100–3111.
- [71] Müller, M. A. and L. Grüne. 2016. “Economic model predictive control without terminal constraints for optimal periodic behavior”. *Automatica*. 70: 128–139.
- [72] Müller, M. A., L. Grüne, and F. Allgöwer. 2015a. “On the role of dissipativity in economic model predictive control”. In: *Proceedings of the 5th IFAC Conference on Nonlinear Model Predictive Control*. 110–116.
- [73] Müller, M., D. Angeli, and F. Allgöwer. 2013. “Economic model predictive control with self-tuning terminal cost”. *European Journal of Control*.
- [74] Müller, M., D. Angeli, and F. Allgöwer. 2015b. “On Necessity and Robustness of Dissipativity in Economic Model Predictive Control”. *IEEE Transactions on Automatic Control*. 60(6): 1671–1676. ISSN: 0018-9286.
- [75] Polushin, I. G. and H. J. Marquez. 2005. “Conditions for the existence of continuous storage functions for nonlinear dissipative systems”. *Systems Control Lett.* 54(1): 73–81. ISSN: 0167-6911.
- [76] Propoi, A. 1963. “Application of linear programming methods for the synthesis of automatic sampled-data systems”. *Avtomat. i Telemekh.* 24(7): 912–920.
- [77] Rawlings, J. and D. Mayne. 2009. *Model Predictive Control: Theory & Design*. Nob Hill Publishing, Madison, WI.
- [78] Renken, A. 1972. “The use of periodic operation to improve the performance of continuous stirred tank reactors”. *Chemical Engineering Science*. 27: 1925–1932.
- [79] Rothfuß, R., J. Rudolph, and M. Zeitz. 1996. “Flatness based control of a nonlinear chemical reactor model”. *Automatica*. 32: 1433–1439.
- [80] Sokoler, L. E., B. Dammann, H. Madsen, and J. B. Jørgensen. 2014. “A mean-variance criterion for economic model predictive control of stochastic linear systems”. In: *Proceedings of the 53rd Conference on Decision and Control*. 5907–5914.

- [81] von Neumann, J. 1945. “A model of general economic equilibrium”. *The Review of Economic Studies*. 13(1): 1–9.
- [82] Weiss, L. 1972. “Controllability, realization and stability of discrete-time systems”. *SIAM Journal on Control*. 10(2): 230–251.
- [83] Willems, J. C. 2007. “Dissipative dynamical systems”. *European Journal of Control*. 13(2-3): 134–151.
- [84] Willems, J. 1972. “Dissipative dynamical systems part I: General theory”. *Archive for rational mechanics and analysis*. 45(5): 321–351.
- [85] Zanon, M., S. Gros, and M. Diehl. 2014. “Indefinite linear MPC and approximated economic MPC for nonlinear systems”. *Journal of Process Control*. 24(8): 1273–1281.
- [86] Zanon, M., L. Grüne, and M. Diehl. 2017. “Periodic optimal control, dissipativity and MPC”. *IEEE Transactions on Automatic Control*. to appear.
- [87] Zavala, V. M. 2015. “A multiobjective optimization perspective on the stability of economic MPC”. In: *Proceedings of the 9th IFAC Symposium on Advanced Control of Chemical Processes — ADCHEM 2015*. Vol. 48. IFAC Papers OnLine. 974–980.