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Achieving Ecological Resilience Through Regime Shift Management

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Contents

Achieving Ecological Resilience Through Regime Shift Management

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ABSTRACT

Resilience has come to mean several things in the system sciences. The term is used to describe systems whose function is robust with respect to external disturbances. It is used to describe systems that tolerate faults in some internal component. It is also used to describe systems that have the capacity to recover after a complete collapse of system function. This monograph defines resilience in the last sense. This notion of resilience is often called *ecological resilience*. For a system to be ecologically resilient, one first accepts system collapse as being inevitable and then requires that the resources for subsequent system recovery be buried in the wreckage of that collapse.

The ecological concept of a *regime shift* plays an important role in describing abrupt shifts in ecosystem behavior. In particular, the collapse of system function may be seen as an instance of a regime shift. Restoring a collapsed ecosystem often involves walking the system through a sequence of alternative states before full recovery can be achieved. The transitions between these alternative states are also regime shifts. But in this case, these regime shifts are purposefully

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triggered to speed up the recovery process. Managing regime shifts that trigger system collapse provides a way to *conserve* existing system function. Managing regime shifts that systematically rebuild lost system function provides a way to *restore* lost system function. Regime shifts, therefore, play an important role in enhancing the ecological resilience of a dynamical system.

This monograph formalizes the ecologist's regime shift concept by identifying regimes with components (also known as basic sets) of a Morse decomposition of the system's chain recurrent set. This formalization allows the identification of two distinct regime shift mechanisms; one triggered by external shocks (shock-induced regime shift) and the other triggered by bifurcations of the system flow (bifurcation-induced regime shift). This allows one to define ecological resilience in terms of a discrete abstraction (called the *regime transition system*) that characterizes the sequences of shock-induced regime shifts that can be triggered in the system. This monograph shows how that transition system can be constructed from the system's differential equation model and demonstrates how it can be used to enhance the resilience of ecosystem foodwebs through the careful management of the system's regime shifts.

1

Introduction – What is a Resilient System?

Different scientific communities take different viewpoints of what it means for a dynamical system to be *resilient*. Engineers often use the term to describe systems that are *fault-tolerant* or *robust* to disturbances. Ecologists, on the other hand, use the term *ecological resilience* (Holling, 1996) to describe ecosystems that have the capacity to spring back after a catastrophic collapse. This monograph focuses on ecological resilience; where system collapse is taken as being inevitable, though the seeds for eventual system restoration are contained within the wreckage of that collapse. The dictionary states that resilience is the capacity to quickly recover from difficulties. This definition focuses on the ability to restore lost system function and seems to be in greater harmony with how ecologists use the term. This monograph, therefore, adopts the ecologist's view of system resilience.

Ecological resilience may be graphically illustrated by the bouncing ball in Figure 1.1. Hitting the floor is the collapse experienced by the falling ball. The rubber ball is resilient because it flies back into the air after hitting the floor. On the other hand, if the ball had been a wad of wet dough, it would have hit the floor and stuck there, thereby illustrating that a wet ball of dough is not resilient. The bouncing rubber

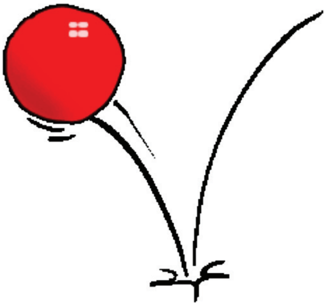

Figure 1.1: Bouncing ball.

ball can also be used to illustrate another aspect of resilient systems; namely that their behaviors are *recurrent*. After the rubber ball hits the ground, it bounces back into the air and then falls back to earth where it hits the ground again. This generates a sequence of bounces that continue for several cycles. In other words, resilient systems traverse a cycle of modes in which the system function (i.e., the ball's height above ground) declines until, upon hitting the floor, it redirects its kinetic energy and bounces back up. For the ball this decline is inevitable. What makes the ball resilient is that it can redirect or reorganize itself in a manner that allows recovery of some of its original height. This means that resilience refers not only to a system's ability to *withstand* collapse, it also refers to the ability of that system to *restore* lost system function.

Ecologically resilient systems therefore cycle back and forth between a *nominal* regime and an *alternative* regime. The transition between regimes is called a *regime shift*. Figure 1.2 illustrates these shifts graphically by plotting an energy (Lyapunov) function, $V(x)$, for a multi-stable system whose two stable equilibria are local minima of V. The region of attraction for one of the equilibria is the nominal regime and the region of attraction for the alternative equilibrium is the alternative regime. A regime shift occurs if an external disturbance causes the system state to jump from the nominal regime into the alternative regime. Once the system state enters the alternative regime, it remains in that regime until a future disturbance forces it back across the energy barrier separating the two regimes. We refer to this first shift as a *collapse*

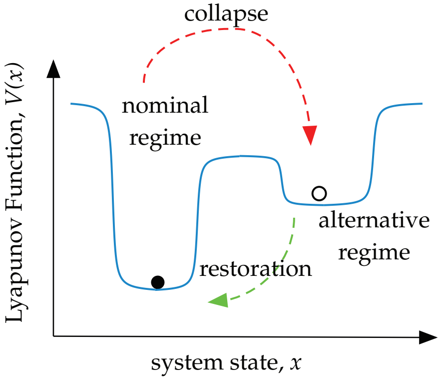

Figure 1.2: Regime shifts in a multi-stable system.

since the alternative regime is usually treated as being undesirable. System restoration occurs when the system state shifts back to the nominal regime. This restoration of system function will always occur in an ecologically resilient system, though it may take quite some time before the restoration is complete. For this reason, one may *intervene* by intentionally triggering regime shifts that force the system to jump back more quickly. Achieving ecological resilience for the system in Figure 1.2 therefore requires the *active management of regime shifts*. With regard to system collapse, regime shift management seeks to reduce the likelihood of an impending collapse. This management policy may also be called a *conservation* policy. For a collapsed system, regime shift management seeks to hasten the restoration of lost system function. This is done through a sequence of intentionally triggered regime shifts that form a restoration plan for the system.

This monograph introduces a mathematical framework for regime shift management that can be used to enhance a system's ecological resilience. There are at least three challenges to be overcome in the development of a framework for ecological resilience. The first challenge stems from the equilibrium-based view of regime shifts that was popularized in Scheffer *et al.* (2001). Real life ecosystems are non-equilibrium processes for which an equilibrium based notion of a regime is too restrictive. So our first problem involves finding a precise formalization of the regime shift concept that extends the equilibrium-based notion in

Scheffer *et al.* (2001) to non-equilibrium processes. In the second place we need to identify specific mechanisms driving a system state to jump into the alternative regime. A good understanding of the mechanisms triggering a regime shift will allow one to develop measures characterizing how close a system is to collapse. These measures play an important role in developing conservation policies seeking to forestall an impending shift. Finally, while system collapse is often sudden and catastrophic, restoration is often more complicated. It is usually impossible to jump back to the nominal regime with a single regime shift. In many cases, the system's resources need to be released and re-organized before a shift back to the nominal regime can be realized. This means that complex systems may need to follow a sequence of intermediate regimes (a.k.a. order of succession) before restoring full system function. So the third problem involves identifying a sequence of intermediate regimes leading back to full restoration of system function.

The following sections address these three challenges with regard to compartmental systems used in modeling ecosystem food webs. These system models are sometimes referred to as *consumer-resource systems* because their dynamics are governed by the flow of resources to consumers. In particular, we take the equilibrium-based notion of a regime and generalize it to define regimes in terms of the basic sets of a Morse decomposition for the system's chain recurrent set. A regime shift then occurs when the system state jumps between basic sets. This formalization allows us to identify two distinct regime shift mechanisms. There is a *bifurcation-induced* regime shift that is generated by local bifurcations of the system and there is a *shock-induced* regime shift that is triggered by impulsive shock like disturbances. For each of these regime shift mechanisms we show how semidefinite programming tools can be used to measure how close a system is to an impending regime shift. The monograph then turns to the problem of system restoration. In particular it uses a discrete abstraction of the system to characterize sequences of shock-induced regime shifts that the system can generate. This abstraction is then used to develop plans that realize the full restoration of a collapsed system through active management of its regime shifts.

The monograph draws on mathematical methods in systems/control theory as well as mathematical ecology. While its methods are demonstrated on models for ecosystem foodwebs, the approach can be used to describe the resilience of engineering infrastructure found in traffic networks, power distribution systems, and wastewater management. The results in this monograph, therefore, should be of interest to scientists and engineers interested in the mathematical foundations of resilient dynamical systems.

The remainder of the monograph is organized as follows. Section 2 defines regime shifts in terms of the system's basic sets. Section 3 introduces a measure of how close a system is to triggering a bifurcation-induced regime shifts, also called the D2B or distance-to-bifurcation. Section 4 uses a stochastic reachability problem to estimate the likelihood of a shock-induced regime shift. This likelihood provides a probabilistic measure of a system's susceptibility to shock-induced regime shifts. Section 5 uses algorithmic analysis of dynamical systems to construct a discrete abstraction of a system's regime shift sequences and then shows how this abstraction could be used to formulate restoration plans for a collapsed system. Section 6 closes with some comments on future directions for this work.

Further Readings: Control engineering approaches to resilience are frequently confined to fault detection and accommodation (Polycarpou and Helmicki, 1995) and fault tolerant control (Blanke *et al.*, 2006). This view of resilience has been important in reconfigurable flight control systems (Bodson and Groszkiewicz, 1997; Egbert and Halley, 2001). The ecological view of resilience, on the other hand, emphasizes renewal over an engineering desire for predictability. This perspective was raised in C.S. Holling's review essay (Holling, 1996) and was based on insights drawn from his earlier work concerning the resilience of terrestrial (forest) systems (Holling, 1973). This ecological viewpoint stresses the recurrent nature of resilient systems and later work sought to apply this notion to complex socio-ecological systems Holling (2001). A popularized extension of this recurrence idea was referred to as *panarchy* in Folke (2006) and Gunderson (2001) and has motivated the founding of international research organizations (*Resilience Alliance* and *Stockholm Resilience Centre*) that study the ecological resilience of real-life socio-ecological systems.

Note on Mathematical Notation: The set of integers, real numbers, and complex numbers are denoted as $\mathbb{Z}$, $\mathbb{R}$, and $\mathbb{C}$, respectively. The vector space of n-dimensional real valued vectors (a.k.a. Euclidean n-space) is denoted as $\mathbb{R}^n$. The set of n by m real valued matrices is $\mathbb{R}^{n\times m}$. The set of non-negative n-dimensional real vectors is denoted as $\mathbb{R}^n_{\geq 0}$. A function $f\colon X \to Y$ is a rule that associates each element of the set X to at most one element of the set Y. The value that f takes at $x \in X$ is denoted as $f(x)$. A function with n continuous derivatives is said to be C^n.

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