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Learning-Based Control: A Tutorial and Some Recent Results

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ABSTRACT

This monograph presents a new framework for learning-based control synthesis of continuous-time dynamical systems with unknown dynamics. The new design paradigm proposed here is fundamentally different from traditional control theory. In the classical paradigm, controllers are often designed for a given class of dynamical control systems; it is a model-based design. Under the learning-based control framework, controllers are learned online from real-time input–output data collected along the trajectories of the control system in question. An entanglement of techniques from reinforcement learning and model-based control theory is advocated to find a sequence of suboptimal controllers that converge to the optimal solution as learning steps increase. On the one hand, this learning-based design approach attempts to overcome the well-known “curse of dimensionality” and the “curse of modeling” associated with Bellman’s Dynamic Programming. On the other hand, rigorous stability and robustness analysis can be derived for

the closed-loop system with real-time learning-based controllers. The effectiveness of the proposed learning-based control framework is demonstrated via its applications to theoretical optimal control problems tied to various important classes of continuous-time dynamical systems and practical problems arising from biological motor control, connected and autonomous vehicles.

1

Introduction

The idea of learning-based control can be traced back at least to the Ph.D. dissertation (Minsky, 1954), where Minsky for the first time introduced the concept of reinforcement learning (RL) motivated by the problem of gaining further insight into the learning, memorizing, and thinking processes in human brain. Borrowing the words from Sutton *et al.* (1992), RL is direct adaptive optimal control. The field of RL is vibrant and is far from being saturated as clearly shown in numerous review articles and books (Bertsekas, 2011, 2013; Schmidhuber, 2015; Silver, 2015; Sutton and Barto, 2018; Szepesvári, 2010). Sixty years later after Minsky's original work, Google DeepMind developed perhaps one of the most advanced artificial intelligence (AI) system based on RL, and defeated the human world champion in the game of Go (Silver *et al.*, 2016, 2017). Indeed, besides Google DeepMind's AI system, RL has demonstrated its advantage in multiple industry applications (Barto *et al.*, 2017; Lorica, 2017). The recent success of RL and related methods can be attributed to several key factors. First, RL is driven by reward signals obtained through the interaction with the environment. Different from other machine learning (ML) techniques, this learning architecture is especially useful when the learning objective is to find the optimal

behavior or policy over a time interval. Second, RL is closely related to the human learning behavior. It has been identified in a number of papers that the learning behavior in the frontal cortex and the basal ganglia is driven by the neuron spikes in dopamine neurons. These spikes encode the temporal difference error signal (Dayan and Balleine, 2002; Doya, 2002; Glimcher, 2011; Lo and Wang, 2006; Wang *et al.*, 2018; Wise, 2004), which is a key element in the RL theory (Sutton and Barto, 2018, Chapter 6). Hence, it is not surprising that we can achieve human-level intelligence through RL. Third, RL has a solid mathematical foundation. The main theoretical result behind RL is the dynamic programming (DP) theory (Bellman, 1957), which is a powerful tool for solving sequential decision making problems. The mathematical guarantee from DP theory gives the advantage of RL over other heuristic AI methods. Finally, RL can be incorporated with other ML and optimization methods to build a sophisticated learning system. For example, the learning performance of RL methods can be significantly improved by incorporating the recently developed deep neural network technique (Mnih *et al.*, 2015, 2016; Schmidhuber, 2015; Silver *et al.*, 2016, 2017). Because of these important features, RL and its extensions have become one of the most active research topics in AI and ML communities. Nonetheless, conventional RL theory exhibits some shortcomings. A common feature of most RL algorithms is that they are only applicable for discrete environments described by Markov decision processes (MDP) or discrete-time systems. To overcome this limitation, several researchers Baird, III (1993, 1994), Munos (2000), Doya (2000), Doya *et al.* (2002), van Hasselt and Wiering (2007), Theodorou *et al.* (2010), and van Hasselt (2012), have made significant efforts in adapting RL into the continuous environment, by discretizing and interpolating the time-state-action spaces. Alternatively, Bradtke and Duff (1994), Sutton *et al.* (1999), and Das *et al.* (1999) investigated RL for the semi-Markov process, a continuous-time dynamical system equipped with discrete state space. It should be mentioned that these methods may suffer from high computational burden when performing the discretization and approximation for continuous-time dynamical systems evolving in continuous state and action spaces. More recently, RL-based

methods, mostly known under the name of adaptive dynamic programming (ADP), have been developed for continuous learning environments (Russell and Norvig, 2010, Chapter 2) described by ordinary differential equations (ODEs) or stochastic differential equations (SDEs). Another limitation of traditional RL methods is that the stability and robustness of the controlled process is usually not considered. In fact, a common assumption in the convergence analysis of various RL methods is that the underlying MDP always has a steady state distribution (Bhatnagar *et al.*, 2009; Nedić and Bertsekas, 2003; Sutton *et al.*, 2000; Tsitsiklis, 1994; Tsitsiklis and Van Roy, 1997). However, few results have been proposed to guarantee this assumption, especially when there exist policies under which the MDP does not have steady state distribution. In contrast with these limitations, experimental results have demonstrated that biological systems exhibit the ability of learning complicated motor movements in an unstable environment composed with high-dimensional continuous state space (Adams, 1971; Shadmehr and Mussa-Ivaldi, 2012; Wolpert *et al.*, 2011). Traditional RL theory is insufficient in explaining this type of learning process.

The purpose of this tutorial is to present a learning-based approach to control dynamical systems from real-time data and to review some major developments in this relatively young field. Due to space limitation, we will focus on continuous-time dynamical systems described by ODEs and SDEs. With input–output data at hand, we can certainly opt for the indirect route as in model-based control theory, that is, first build a mathematical model and then design controllers for the practical system in question. This indirect method has proven successful for a variety of problems arising in the contexts of engineering and sciences. However, it is widely known that building precise mathematical models that can describe the motion of dynamical systems is time-consuming and costly. For certain classes of optimal control problems, especially when the dynamical systems under consideration are strongly nonlinear, it is very hard, if not impossible, to solve the Bellman equation. This observation has led Bellman (1957) to state: “Turning to the succor of modern computing machines, let us renounce all analytic tools.” In this monograph, we aim to develop a framework for learning-based control theory that shows how to learn directly suboptimal controllers

from input–output data. Ultimately, these suboptimal controllers are expected to converge to the (unknown) optimal solution to the Bellman equation. Besides the benefit of direct vs indirect control methods, the learning-based control theory overcomes the curse of modeling tied to the traditional DP. There are three main challenges on the development of learning-based control. First, there is a need to generalize existing recursive methods, known under the names of policy iteration (PI) and value iteration (VI), from model-based to data-driven contexts when the system dynamics are completely unknown. Previous RL-based learning algorithms are not directly extendable to the setting of continuous-time dynamical systems, let alone convergence and sensitivity analyses. Second, as a fundamental difference between learning-based control and RL, stability and robustness are important issues that must be addressed for the safety-critical engineering systems such as self-driving cars. Therefore, there is a need to develop new tools and methods, beyond the present literature of RL, that can provide theoretic guarantees on the stability and robustness of the controller learned from real-time data collected online along the trajectories of the control system under consideration. Third, data efficiency of RL algorithms need be addressed for safety-critical engineering systems. In this monograph, we will address the first two issues and only discuss the third issue from the perspective of numerical and experimental studies by means of some case studies. The learning-based control theory as reviewed in this monograph is closely tied to the literature of safe RL and ADP, and is a new direction in control theory that is still in its infancy and especially so for continuous-time dynamical systems described by differential equations. For prior work of others on ADP-based optimal control, the reader may consult (Jiang and Jiang, 2017; Lewis and Vrabie, 2009; Lewis *et al.*, 2012b; Liu *et al.*, 2017; Luo *et al.*, 2014; Song *et al.*, 2015; Vrabie *et al.*, 2013; Wang *et al.*, 2009; Werbos, 1968) and many references therein. For recent developments in learning-based control for other types of systems and problems, see Antsaklis *et al.* (1991), Antsaklis and Rahnama (2018), Rahnama and Antsaklis (2019), Werbos (2013, 2014, 2018), Kiumarsi *et al.* (2017), He and Zhong (2018), Recht (2019), Bertsekas (2019), Kamalapurkar *et al.* (2018), Chen *et al.* (2019), Pang *et al.* (2020), and references therein.

The rest of the monograph is organized as follows. Section 2 describes the learning-based optimal control of continuous-time linear and nonlinear systems described by (ordinary or stochastic) differential equations. Section 3 is concerned with the learning-based optimal control of a class of large-scale dynamical systems. Section 4 deals with the learning-based adaptive optimal tracking with disturbance rejection, the so-called adaptive optimal output regulation problem, for classes of linear and nonlinear control systems. Applications of the presented learning-based control theory to autonomous vehicles and human motor control are given in Section 5. Finally, some concluding remarks and discussions on future work are provided in Section 6.

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