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Contents

On Robust and Predictive Control Approaches for Linear Parameter Varying Systems: Application to Vehicle Lateral Control

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ABSTRACT

This monograph presents for the first time a unified synthesis on how to design robust and predictive control approaches for (discrete-time) Linear Parameter Varying (LPV) systems. In particular, some recent results concerning LPV state feedback design using the H_∞ framework and Model Predictive Control (MPC) for LPV systems are presented. Then, both approaches are illustrated in two important cases for automotive applications. First, the lateral steering control of autonomous vehicles is considered. Then, an application to Advanced Driver-Assistance Systems is presented, where MPC and LPV approaches are integrated in view of optimal selection of the scheduling parameter.

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1

About LPV Systems and Control

1.1 A Broad Overview

Control of Linear Parameter Varying (LPV) systems has attracted more and more attention in recent decades. Specifically, LPV systems have been shown as a very interesting extension of the robust control theory to a large class of dynamical systems.

Nowadays, the LPV approach is recognised as a well-suited tool to handle system nonlinearities and to adapt control online (during the implementation, in real-time), by means of suitable varying (scheduling) parameters. From this, the synthesis of LPV controllers (also referred to as gain-scheduling) is enabled - for which system stability and performance can be guaranteed for a larger domain of operation.

In particular, the LPV approach has been extensively demonstrated as very efficient for aerospace applications - since the early 1990s. More recently, its potential has been assessed in several other important application cases (in robotics, health, energy, automotive, and so forth). In the recent survey by Hoffmann and Werner (2014), an extensive overview of recent applications is presented. Furthermore, several recent textbooks have also been concerned with the analysis and synthesis problems using LPV tools, such as Toth (2010), Mohammadpour and

Scherer (2012), Sename *et al.* (2013), Briat (2015), and Gáspár *et al.* (2017).

Due to its potential to handle nonlinear systems in a linear-like framework, researchers have been interested in developing robust and optimal control approaches using LPV tools. Accordingly, there have been an important number of works in the last two decades, especially given the fact that embedding real-time control/observation strategies becomes much easier with LPV approaches, and theoretical proofs of stability and performances can be handily generated.

However, we must emphasise that an additional complexity arises in the LPV context due to the varying parameters. Accordingly, specific theoretical tools are required, in particular for stability analysis. Recent studies have been concerned with model identification, stability/stabilisation and control design (predictive and robust approaches), in the context of affine, polynomial or rational LPV systems. One of the main interests of LPV control design is that it allows linear analysis and control synthesis (H_∞, H_2, MPC - i.e. *Model Predictive Control*) through reliable optimisation tools, such as Linear Matrix Inequalities (LMIs) and Quadratic Programming problems (QPs).

Finally, for interested control engineers, we mention some recent interesting toolboxes that enable LPV syntheses and analyses:

- The *LPV Tools* toolbox (Hjartarson *et al.*, 2015), which implements grid-based and LFT methods;
- The *LPVcore* toolbox (den Boef *et al.*, 2021), which is dedicated to modelling, identification, and control of LPV systems.

1.2 Discrete-time LPV Models

The most well-known use of the LPV tool is actually its application to handle nonlinear dynamics. Indeed, there are several different ways to convert a nonlinear system into an LPV one:

- The first one is the *historical* method, referred to as the gain-scheduled control, which aims at doing a Jacobian linearisation of the nonlinear system trajectories around a set of arbitrarily chosen operating conditions. This approach renders a set of LTI models along a grid of parameter values, as addressed by Wu (1995) and Hjartarson *et al.* (2015);

- The second one is to rewrite the nonlinearities as varying parameters. It is worth noting that if the nonlinearities involve state variables, the system is referred to as *quasi-LPV* (and is of course not equivalent to the nonlinear model). Note that this is sometimes called "nonlinearity/qLPV embedding". The clue for qLPV modelling is the application of the *Linear Differential Inclusion* theorem, as stated in Boyd *et al.* (1994b).

This monograph is not concerned with modelling and identification of LPV systems. Concerning these topics, the reader is referred to the book by Toth (2010) or to the PhD thesis by Bruzelius (2004).

Throughout this monograph we will be interested in discrete-time LPV models. In contrast to LTI state space models, LPV systems are characterised by system matrices that include time-varying parameters that evolve over time, and are defined in discrete-time (DT) as follows.

Definition 1.1 (Discrete-time state-space representation of LPV systems)**.** Given a vector of time-varying parameter $\rho \in \mathbb{R}^m$ and matrices $\mathcal{A}(\rho) \in \mathbb{R}^{n_x \times n_x}$, $\mathcal{B}(\rho) \in \mathbb{R}^{n_x \times n_w}$, $\mathcal{C}(\rho) \in \mathbb{R}^{n_z \times n_x}$ and $\mathcal{D}(\rho) \in \mathbb{R}^{n_z \times n_w}$, the DT dynamics of an LPV system $\Xi(\rho)$ are given through the following representation:

$$\Xi(\rho) = \begin{cases} x^+ = \mathcal{A}(\rho)x + \mathcal{B}(\rho)w\,, \\ z = \mathcal{C}(\rho)x + \mathcal{D}(\rho)w\,, \end{cases} \tag{1.1}$$

where $x \in \mathbb{R}^{n_x}$ is the state-vector, $w \in \mathbb{R}^{n_w}$ is the vector of exogenous inputs, and $z \in \mathbb{R}^{n_z}$ is the vector of performance outputs. The time difference elapsed in the transition from state x to its successor (i.e., x^+) is given by a constant sampling time denoted T_s. We also consider an equivalent notation using $k \in (\mathbf{N} \cup \{0\})$ as the discrete-time sample stamp. Thus, $x(k+1)$ represents the successor to $x(k)$.

Remark 1.1. In the LPV setting, the time-varying (scheduling) parameter vector - i.e. $\rho \in \mathbb{R}^{n_\rho}$ in (1.1) - is considered to be known (measured or estimated) and to satisfy several assumptions. The most typical ones (Assumptions 1 and 2) are recapped next.

Assumption 1. Each varying parameter value $\rho_i(k)$ is known and is bounded by extremal values $\underline{\rho}_i$ and $\overline{\rho}_i$ such that $\underline{\rho}_i \leq \rho_i(k) \leq \overline{\rho}_i$, $\forall k$. The joint set of bounds on ρ_i, $i = 1, \ldots, m$, then form the varying parameter admissible space $\Omega \in \mathbb{R}^{n_\rho}$, such that $\rho(k) \in \Omega$, $\forall k$.

Assumption 2. The rate of variation $\delta\rho_i(k) := \rho_i(k) - \rho_i(k-1)$ for each varying parameter ρ_i between two consecutive sampling times k and $k+1$ is bounded by $\underline{\delta\rho}_i$ and $\overline{\delta\rho}_i$ such that $\underline{\delta\rho}_i \leq \delta\rho_i(k) \leq \overline{\delta\rho}_i$, $\forall k$.

In this monograph, we particularly consider the class of LPV systems whose matrices are defined as affine on some basis function with dependency on the varying parameter vector. The definition of an affine LPV representation is recapped below.

Definition 1.2 (Affine LPV description)**.** Consider an LPV system given as in Definition 1.1. We say that it is affine with respect to a basis function $\theta(\rho)$ if the system matrices can be expressed as follows:

$$\begin{array}{ll} \mathcal{A}(\rho) = \mathcal{A}_0 + \sum_{n=1}^{N} \theta_n(\rho)\mathcal{A}_n \ , & \mathcal{B}(\rho) = \mathcal{B}_0 + \sum_{n=1}^{N} \theta_n(\rho)\mathcal{B}_n \\ \mathcal{C}(\rho) = \mathcal{C}_0 + \sum_{n=1}^{N} \theta_n(\rho)\mathcal{C}_n \ , & \mathcal{D}(\rho) = \mathcal{D}_0 + \sum_{n=1}^{N} \theta_n(\rho)\mathcal{D}_n \end{array} \tag{1.2}$$

being $\mathcal{A}_0, \ldots, \mathcal{A}_n$, $\mathcal{B}_0, \ldots, \mathcal{B}_n$, $\mathcal{C}_0, \ldots, \mathcal{C}_n$ and $\mathcal{D}_0, \ldots, \mathcal{D}_n$ are constant matrices. In particular, the vector $\theta(\rho) = (1, \theta_1(\rho), \ldots, \theta_N(\rho))$ forms the parameter-dependent basis function, being$\theta_n(\rho) \in \mathbb{R}$ a scalar function.

1.3 A Brief State-of-the-art

As mentioned previously, LPV systems have attracted more and more attention recently either in robust control approaches or in MPC ones. In addition to the aforementioned books, several survey papers have been concerned with these approaches, for control and/or observation purposes.

1.3.1 Robust Methods

Released in the 1990s, LPV synthesis allows a controller to be sequenced by the parameters of the dynamical system if these parameters can be measured at each instant.[1] Due mainly to their scalability, these controllers can meet an increased level of performance when compared to conventional robust robust controllers (of LTI format). Furthermore, they provide stability and performance in a global perspective, along all possible system trajectories, rather than a local stability only linked to the operating point. These gain-scheduling methods, based on the linearisation at different operating points, have been extensively investigated in the literature, see e.g. Leith and Leithead (2000), Shamma (2012), and Shamma (1988). We emphasise that the robustness property relating to these techniques has been studied in Apkarian and Adams (1998).

The Linear Parameter Varying (LPV) control concept has been successfully developed to achieve stable gain-scheduling schemes (Apkarian and Gahinet, 1995), self-scheduling controllers (Apkarian *et al.*, 1995) and even interpolated controllers, given with respect to (linearised) LTI models obtained at different operating points. However, LPV control methods mainly differ from classical gain-scheduling since, in the general case, the controller depends not only on the varying parameters, but also on the derivative of such parameters (Wu *et al.*, 1996; Apkarian and Adams, 1998). In practice, this issue can render the corresponding implementation quite intricate.

The importance of the LPV approach to control general nonlinear systems comes from the interesting characteristic to rewrite it in the form of a quasi-LPV structure, where the parameters arise as known functions of states, inputs or outputs variables, and not only exogenous inputs. This approach has been followed in a lot of recent studies, such as in the books by Mohammadpour and Scherer (2012) and Sename *et al.* (2013), and in the survey paper by Hoffmann and Werner (2014) - c.f. references cited therein.

Remark 1.2. It is worth noting that, while early synthesis methods have been limited to slow parameter variations (Shamma, 2012), today there are several ways to handle arbitrarily fast as well as rate-limited parameter variations to reduce the design conservatism, c.f. Apkarian *et al.* (1995) and Wu (1995).

[1] We note that the most widely used technique is to consider gain sequencing by interpolating poles and zeros or state matrices.

As known today in the LPV context, the formulation of an LPV control problem - for instance in the H_∞ framework - requires, in general, to solve an infinite number of LMIs, given the scheduling parameter space. Methods have been proposed to reduce the problem to a finite set of LMIs. Accordingly, we mention the three main approaches to handle this issue:

- The use of Linear Fractional Transformations (LFTs) has been largely studied in relation to aerospace applications (Apkarian and Gahinet, 1995; Pfifer, 2013). This technique consists of formulating the LPV system as a Linear Fractional Representation (LFR) containing the nominal LTI system and a model uncertainties block (which, given the LPV setting, encompasses the effects of scheduling parameters). LFT theory makes use of ($\mathcal{D}$, $\mathcal{D}$-$\mathcal{G}$, ...) scalings, application of (full-block) $\mathcal{S}$-procedures and, more recently, the use of Integral Quadratic Constraints (IQCs) for the design of gain-scheduling controllers, c.f. Scherer (2001), Veenman and Scherer (2014), and Morato *et al.* (2023a);

- The polytopic approach, c.f. Apkarian *et al.* (1995), Li *et al.* (2021), and López-Estrada *et al.* (2019), represents, today, the most popular among the LPV control approaches. In particular, it is widely applied due to its simplicity and stability guarantees enabled within the design process. However, the application of this approach is limited to only a few scheduling parameters, due to the exponential growth of the polytope vertices with respect to the number of parameters. Some methods to reduce this over-bounding of parameter regions have been developed, as presented in Li *et al.* (2021) and Casavola *et al.* (2012).

- The *gridding* technique, e.g. Wu (1995), is the historical method. This approach is based on the definition of a mesh over the parameter space. A grid-based LPV system is formulated using a linear or nonlinear interpolation between the corresponding LTI systems at the grid (operating) points. It is known that a higher density of grid points is required for better interpolation performance. An advantage of this technique is that it is applicable for any LPV plants (with general parameter-dependency), requiring neither polytopic nor LFT representations. The implementation of the controller is computationally inexpensive, but may require large amounts of memory in order to store the local controllers.

Remark 1.3. We highlight that, recently, these mentioned techniques have also been extended to the context of interpolation-based LPV controllers, as described in the survey paper by Atoui *et al.* (2022).

1.3.2 Predictive Control Schemes

Dating from the original algorithms proposed by the process industry in the 1980s (e.g. Cutler and Ramaker, 1980; Clarke *et al.*, 1987), MPC has since become a widely used control technique for the regulation of constrained systems (Camacho and Bordons, 2013). Over the last decades, a considerable amount of research has been devoted to the study of MPC algorithms, considering different system models and settings, e.g. Alamir (2012), Allgöwer and Zheng (2012), Limon *et al.* (2018), and Morato *et al.* (2020a). Corresponding theoretical certificates for closed-loop stability (and recursive feasibility of the (recurrent) optimisation problem) have been established since the seminal results provided by Mayne *et al.* (2000) - which recently have been extended to broader settings by means of dissipativity theory in Morato *et al.* (2023a). In fact, predictive control is rather well-established due to a quite simple property: it has the ability to jointly consider performance optimisation and constraint satisfaction under a relatively intuitive synthesis framework.

Before detailing LPV design approaches to MPC, we provide an overview of the predictive control framework: at each (discrete-time) sampling instant, an (optimal) control action is generated through the solution of a constrained optimisation problem, which embeds the performance objectives along a future horizon window, as well as the considered constraints. The general form of this optimisation, at each discrete time sample k, is given, generically, by:

$$
\begin{aligned}
\min_{U_k} \quad & \left(\sum_{j=0}^{N_p-1} \ell(x(k+j|k), u(k+j|k)) \right) + V(x(k+N_p|k)) \,, \\
\text{s.t.: } x(k+j+1|k) \quad &= \quad f\,(x(k+j|k), u(k+j|k)) \,, \forall\, j \in \mathbb{N}_{[0,N_p-1]}, \\
x(k+j|k) \quad &\in \quad \mathcal{X} \,, \forall\, j \in \mathbb{N}_{[1,N_p]}, \\
u(k+j-1|k) \quad &\in \quad \mathcal{U} \,, \forall\, j \in \mathbb{N}_{[1,N_p]}, \\
x(k+N_p|k) \quad &\in \quad \mathbf{X}_f,
\end{aligned}
$$

where x and u denote, respectively, the (predicted) process state and input variables, while

$$
U_k \; := \left[\; u^T(k|k) \quad u^T(k+1|k) \quad \ldots \quad u^T(k+N_p-1|k) \; \right]^T
$$

represents the optimisation[2] decision variable, i.e. the sequence of control actions along the prediction window N_p. From the optimal solution $U_k^\star$, the first entry $u^\star(k|k)$ is applied to the system.

MPC has great theoretical and practical value. Yet, the methodology requires a process *model* at its core - thus "**M**PC". Accordingly, this model[3] is used to map predictions related to the future behaviour of state (or output) variables - which are incorporated within a sampled optimisation problem, considering a rolling prediction horizon window. In broad terms, *nonlinear* MPC (NMPC) schemes are particularly relevant when nonlinear systems are controlled over larger operating conditions - or when the process heavily depends on external parameters. However, the inclusion of nonlinear predictions to the sampled optimisation is not trivial and increases the resulting algorithm's complexity, c.f. Allgöwer and Zheng (2012). In practice, such increased numerical burden becomes an impediment for many real-time applications.

[2]In the (generic) MPC optimisation above, $\mathcal{X}$ and $\mathcal{U}$ are known sets used to represent the process constraints, while $\mathbf{X}_f$ is a *terminal* set used for stability-related features. The optimisation cost $J(x(k), U_k)$ comprises a performance-related stage cost $\ell(\cdot,\cdot)$, summed along the future horizon, and a *terminal* cost $V(\cdot)$, related to the state prediction at the end of the horizon. We note the MPC optimisation procedure is initialised with the current sampled state measurement $x(k)$; accordingly, its solution is the minimiser $U_k^\star$.

[3]If a trustworthy model is not available, the derived control law may simply be unrealistic and thereby the controller may be insufficiently robust to counter-act the uncertainties caused by the prediction mismatches (even stability may be lost, in some dramatic settings).

Accordingly, several recent studies have investigated how LPV models can be used in this regard, as presented in the survey paper by Morato *et al.* (2020a). With regard to the scope of this work - and given that nonlinear mappings can be re-cast as LPV models - recent literature has consistently shown that it is quite natural and direct to develop NMPC algorithms by exploiting LPV realisations. In particular, LPV models are especially interesting (in the context of MPC synthesis) because they retain the linearity property along the inputs-outputs channels, which means that computationally efficient design procedures can be rendered. Conversely, this means that the drawbacks of full-blown NMPC algorithms are avoided (the use of nonlinear programs), without any need to approximate the solution of the optimisation problem - as do the most modern fast NMPC solutions, such as real-time iteration schemes, c.f. Gros *et al.* (2020) and Verschueren *et al.* (2022) and gradient-based methods, c.f. Käpernick and Graichen (2014).

In the context of MPC, a full-horizon prediction model is required.[4] Nevertheless, when an LPV prediction model is used, this problem depends not solely on the future inputs (to be determined by the optimisation), but also on the future scheduling parameters which are, *a priori*, typically unknown.

Therefore, the control community has presented several recent works on the topic of MPC design for LPV systems, handling the scheduling prediction uncertainty issue, as surveyed in Morato *et al.* (2020a). Next, we emphasise the two main classes of LPV MPC algorithms:

- Robust methods, e.g. Jungers *et al.* (2011), Bumroongsri and Kheawhom (2012), Hanema *et al.* (2017), and Abbas *et al.* (2018), which consider the worst-case closed-loop performances implied by the unknown future scheduling parameters. Accordingly, the optimisation is rewritten in order to take into account the bounds of all possible future parameter variations, which can render usually conservative results.

[4] By this, we mean to describe (i.e. predict) the system variables along the future horizon of N_p steps ahead of each discrete-time sample. The notion of a *rolling* prediction horizon is implied: at each sample k, the future $k + N_p$ variables instances are taken into account to generate the sampled predictive control law.

- Gain-scheduling methods, e.g. Brunner *et al.* (2017), Cisneros and Werner (2017), Mate *et al.* (2019), and Alcalá *et al.* (2019). In these works, the LPV model is replaced by an LTV one (or a sequence of LTI realisations) that, at each sampling instant, is evaluated as a local LTI model based on a guess for the scheduling trajectory. In many cases, this guess is simply an assumption that the scheduling parameters will remain constant along the prediction horizon. While these methods operate quite fast (they exhibit reduced numerical burden), sub-optimality may be implied. Nevertheless, when the scheduling trajectory is accurate (as seen for the qLPV case in Cisneros and Werner, 2017; 2019; 2020), an exact nonlinear MPC solution is obtained by the means of quadratic optimisation programs, thus rendering a solution comparable to state-of-the-art solver-based NMPC solutions (such as ACADO; Verschueren *et al.*, 2022 and CasADi; Andersson *et al.*, 2019).

1.4 Monograph Objectives and Organisation

Taking into account the detailed context regarding LPV systems and control, the aim of this monograph is to present some of the advantages of considering LPV approaches. Here, we advocate for the use of LPV design in both robust and predictive control settings. In particular, we focus on the case of the lateral control problem in (automated, assisted and autonomous) vehicles.

For the reason of homogeneity, we will consider here discrete-time LPV systems and present some results on state feedback control approaches in the context of robust LPV control and MPC. With regard to this context, two cases are illustrated:

- First, MPC and LPV state feedback control of the lateral motion of autonomous vehicles is detailed;

- Second, an application to Advanced Driver-Assistance Systems is presented, where MPC and LPV approaches are integrated in order to optimally select scheduling parameters (using MPC) used in an LPV steering control.

It is worth noting that such automotive applications are described in a unified way through the use of the same vehicle across this work, namely, the *Renault Megane* car, detailed in Fergani (2014), for which a realistic validated full car nonlinear model is considered for time-domain simulations.

The remaining content of the monograph is divided into sections on the following topics:

- Section 2: A recap on fundamental results related to robust LPV control synthesis and stability analyses;
- Section 3: A broad overview of LPV predictive control design approaches and problems;
- Section 4: Lateral control: application of LPV approaches to steering control of autonomous vehicles;
- Section 5: LPV control for Advanced Driver-Assistance Systems (ADAS);
- Section 6: Concluding remarks.

1.5 Notations

We denote $\mathbb{N}$ ($\mathbb{N}_0$) as the set of positive (non-negative) integers and abbreviate the set $\{i \in \mathbb{N}_0 \,|\, a \leq i \leq b\}$ by $\mathbb{N}_{[a,b]}$. $\mathbb{S}^n$ stands for the set of symmetric matrices in $\mathbb{R}^{n\times n}$ and ℓ_{2e}^m for the space of sequences with elements in $\mathbb{R}^m$. The $j \times j$ identity matrix is denoted by I_j and $I_{j,\{i\}}$ denotes its i-th column. $\text{col}(v_1, \ldots, v_m) := (v_1^\top, \ldots, v_n^\top)^\top$ denotes the vectorisation operation and $\text{diag}(V_1, \ldots, V_n)$ denotes the block diagonal matrix with $V_1, \ldots, V_n$ on its diagonal. The predicted value of a given variable $v(k)$ at time instant $k+i$, computed based on the information available at instant k, is denoted as $v(k+i|k)$; in particular, $v(k|k) = v(k)$. Furthermore, in matrix inequalities, $(\star)$ denotes the corresponding symmetrical transpose; moreover, $\mathcal{M} > 0$ indicates the positive definiteness of matrix $\mathcal{M}$.

$\mathcal{K} : \mathbb{R}_{\geq 0} \mapsto \mathbb{R}_{\geq 0}$ refers to the class of continuous, positive and strictly increasing scalar functions that pass through the origin. A $\mathcal{C}^1$ function

$f : \mathbb{R}^m \to \mathbb{R}^n$ is such that it is differentiable with continuous derivatives. In this case, $\nabla^T f : \mathbb{R}^m \to \mathbb{R}^{n\times m}$ denotes its Jacobian matrix. Consider sets $\mathcal{A}, \mathcal{B} \subset \mathbb{R}^n$, $\mathcal{C} \subset \mathbb{R}^m$ and a matrix $R \in \mathbb{R}^{n\times m}$. The Minkowski set addition is defined by $A \oplus B := \{a + b \,|\, a \in A\,, b \in B\}$, while the Pontryagin set difference is defined by $A \ominus B := \{a \,|\, a \oplus B \subseteq A\}$. A linear mapping is $R\mathcal{A} = \{y \in \mathbb{R}^n : y = Ra, a \in \mathcal{A}\}$, while the Cartesian product holds as $\mathcal{A} \times \mathcal{C} = \{z \in \mathbb{R}^{n+m} : z = (a^T\ c^T)^T, a \in \mathcal{A}, c \in \mathcal{C}\}$. $\|\cdot\|$ denotes the 2-norm, unless mentioned otherwise.

In terms of (LPV) state-space descriptions, we use x to denote the system states, ρ the scheduling variables, u the control inputs, z the performance outputs, y the measured outputs, and w the system's exogenous inputs (i.e. disturbances). Moreover, for discrete-time realisations, we refer to the successive state either as x^+ or as $x(k+1)$ (analogous).

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